

Evaluation of Plotting Position Formulae for Pearson Type 3 Distribution in Three Hydrological Stations on the Niger River

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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ABSTRACT

In this study, eight unbiased plotting position formulae recommended for Pearson Type 3 distribution were evaluated by comparing the simulated series of each formula with the annual maximum series (AMS) of River Niger at Baro, Koroussa and Shintaku hydrological stations, each having data length of 51 years, 53 years and 58 years respectively.

The parameters of Pearson Type 3 distribution were computed by the method of moments with corrections for skewness. While the fitting of Pearson Type 3 distribution proceeds with the development of flood – return period (Q-T) relationship, followed by application of the derived Q- T relation to compute simulated discharges for comparison with AMS of the study stations.

The plotting position formulae were evaluated on the basis of optimum values of the statistically goodness-of-fit of probability plot correlation coefficient (PPCC), relative root mean square error (RRMSE), percent bias (PBIAS), mean absolute error (MAE) and Nash-sutcliffe efficiency (NSE), across the stations.

The plotting position formulae were ranked on scale of 1 to 8. Thus a plotting formula that best simulates the empirical observations using the goodness-of-measures was scored “1” and so on. The individual scores per plotting position were summed across the gof tests to obtain the total score.

The study show that Chegodayev is the best plotting position formula for Baro, Weibull is the best

plotting position Formula for Kourassou and Shintaku hydrological stations. The overall performances of the eight plotting position formulae across the hydrological stations show that weibull distribution is the overall best having scored 27, seconded by Chegodayev with 30 and thirdly, Beard with 38.

The Pearson Type 3 distribution had been found one of the best probability distribution model of flood flow in Nigeria and this study was conducted to gain in-depth knowledge of the distribution. Finally, this study recommends extension of the studies to Log-Pearson Type 3 distribution.

Keywords: Plotting positions; exceedance probability; performance measures; quantile relations; and pearson type 3 distribution.

1. INTRODUCTION

The design of hydraulic structures, river basin development projects, and planning and the prevention of flood damage, all require accurate estimation of flood quantiles. The estimation must be as accurate as possible to avoid severe implications on project economy, damages and loss of human lives.

A crucial step in frequency analysis is establishment of quantile relation between the magnitude of extreme event and their exceedance (or non-exceedance) probability. Subsequently apply the derived quantile as a standard engineering tool for extrapolation of flood events beyond the length of data. In practice, Q_T relation is estimated by fitting a statistical distribution to a sample of flood series, after satisfying the assumption of independence, homogeneity, stationarity and absence of outliers. The task of fitting a probability distribution model may be simple, but the difficulty lies in selecting the appropriate model to be used for making design or management decision. In countries that have no guidelines and manuals for hydrologic frequency analysis. Finding appropriate model is a subject of rigorous research using several parameter estimation methods, namely; graphical, method of moments, maximum likelihood estimate, probability weighted moments and L-moments, etc.,

For example, China uses Pearson Type 3 distribution, the United States of America uses log Pearson Type 3, while the United Kingdom uses General pareto distribution (GPA) and most countries in Europe prefer Generalised Extreme Value distribution [1]. The cumulative distribution function (CDF) may be expressed in terms of the plotting position formulae, from which the exceedance probability and the return period are computed.

Plotting position formulae are useful graphical tool for evaluating the adequacy of a particular probability distribution fitting a data sample, Linearising probability relationship for interpolation, extrapolation and comparison purposes among others. In spite of significant progress made in development of plotting position formulae, the selection of an appropriate distribution for any given flood record from among the candidate distributions is still a subject of continuing research investigation [2]. Review of pertinent literature on plotting positions may be found in [3] to [13]. From the reviewed works, there are numerous claims, counter claims and recommendations among researchers. For example [7] studies the various plotting position methods on the criteria of unbiasedness and minimum variance, and concluded that Weibull's formula is biased and plots the largest values of a sample at too small return periods. Also [12] studies estimation of plotting position for flood frequency analysis and proposed that plotting positions are examined in more details and advocated collaboration amongst researchers. Furthermore, [11] has shown that an underestimation of the Probability of extreme events has resulted from the use of many other plotting positions, and that this has had an adverse impact on building codes and other means for optimum design against extreme – weather events. This study evaluates seven unbiased plotting positions formulae derived for Pearson Type 3 distribution. This study will enhance the use of Pearson Type 3 distribution for hydrological frequency analysis in Nigeria.

2. STUDY AREA

The data of three hydrological stations on the Niger river basin at Baro, Kouroussa and Shintaku were employed in this study. The stations and their geographic attributes and descriptive statistics are shown in Table 1. The Niger River basin covers a total area of

Table 1. Geographic characteristics and descriptive statistic

Parameter	Hydrological Stations		
	Baro	Kouroussa	Shintaku
Latitude	08° 35'	08° 51'	07° 10'
Longitude	08° 23'	10° 47'	06° 45'
Minimum Flow	423 m3/sec	275 m3/sec	7730.59 m3/sec
Maximum Flow	1150 m3/sec	1185 m3/sec	16480 m3/sec
Mean Flow	716.34 m3/sec	713.98 m3/sec	13, 320.8 m3/sec
Standard Deviation	188.71 m3/sec	223.643 m3/sec	2015.43 m3/sec
Coefficient of Variation	0.264	0.313	0.151
Skewness	0.122	0.242	-0.671
Data length	1948 – 2000 (53 years)	1950 – 2000 (51 years)	1957 – 2014 (58 years)

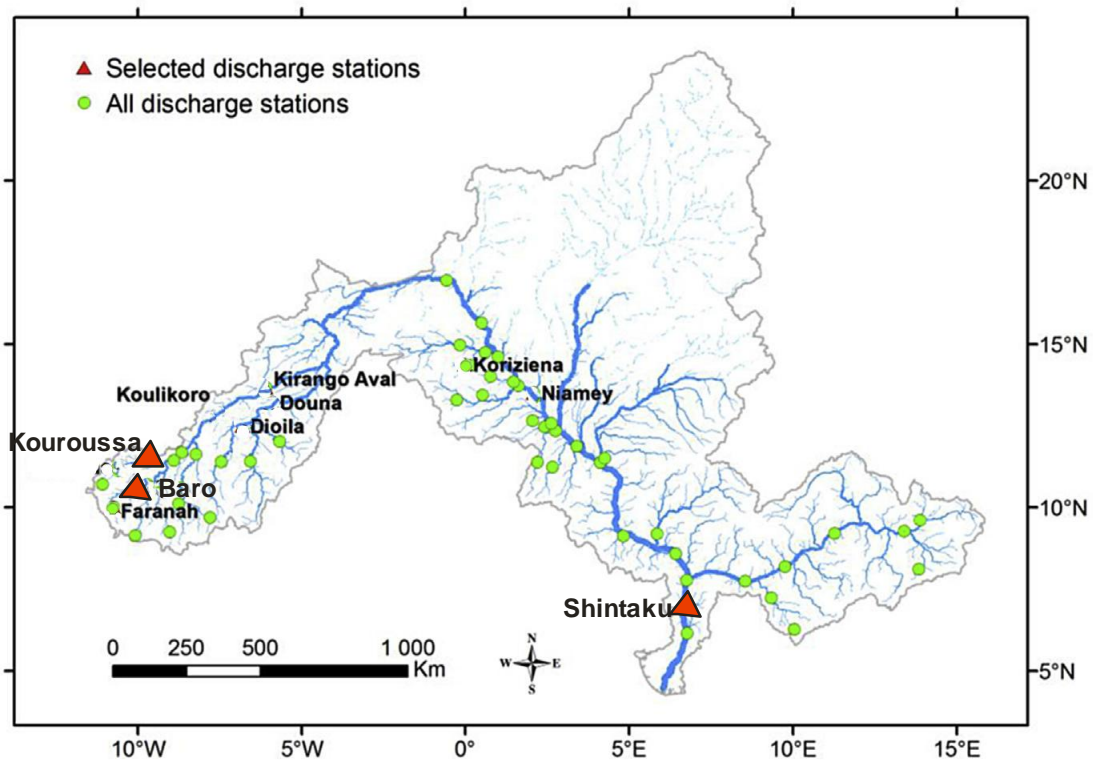


Fig. 1. Map of Niger River Basin showing hydrologic stations

approximately 2,156,000km², only about 1,270,000km² actively contribute to runoff and river discharge. The whole basin is spread over the territory of ten countries. Table 1 shows the selected hydrological stations along the Niger River. The coefficient of variation shows that the flow is moderately variable. Secondly, the annual maximum discharges are generally skewed and for this reason, normal distribution will not be a suitable probability distribution model. Fig. 1 shows the map of the Niger River Basin showing the hydrological stations.

3. METHODOLOGY

3.1 Parameter Estimation Technique

3.1.1 Method of Moments (MOM)

The MOM, the moments of a distribution function in terms of its parameters are set equal to the moments of the observed sample. For details, see [14] and [15]. According to the central moment of distribution are given by:

$$\mu_r = E(X - \mu)^r = \int (x - \mu)^r f_x(x) dx \quad (1)$$

$$\text{Variances: } \sigma^2 = \mu_2 \quad (2)$$

$$\text{Skewness: } \sqrt{\beta_1} = \frac{\mu_3}{\mu_2^{3/2}} \quad (3)$$

$$\text{Kurtosis: } \beta = \frac{\mu_4}{\mu_2^2} \quad (4)$$

The sample moments are given by:

$$\bar{x} = n^{-1} \sum x_1 \quad (5)$$

$$m_r = n^{-1} \sum (x_1 - \bar{x})^r \quad (6)$$

The sample mean \bar{x} is a natural estimator for μ . The higher sample moments m_r , are reasonable estimators of the μ_r , but they are not unbiased. Unbiased estimators are often used. In particular σ^2 , μ_3 and the fourth cumulant $k_4 = \mu_4$ are unabashedly estimated by:

$$S^2 = (n - 1)^{-1} \sum (x_1 - \bar{x})^2 \quad (7)$$

$$m_3^* = \frac{n^2}{(n-1)(n-2)} m_3 \quad (8)$$

$$k_4^* = \frac{n^2}{(n-2)(n-3)} \left\{ \frac{n+1}{n-1} m_4 - 3m_2^2 \right\} \quad (9)$$

The sample standard deviation $s = \sqrt{S^2}$ is an estimator of σ but is not unbiased. The sample estimators or cv (Coefficient of Variance), skewness and kurtosis are respectively:

$$cv = S / \bar{X} \quad (10)$$

$$g = m_3^* / s^3 \quad (11)$$

$$k = k_4^* / s^4 + 3 \quad (12)$$

The population parameters of Person type 3 distribution were estimated from the sample statistics; mean, variance and coefficient of skewness. Accordingly, using the method of moments, the sample characteristics are equated to the population parameters. The parameters of the Pearson type 3 distribution are computed using Equations 13 to 15.

$$\bar{Q} = \alpha \beta + \zeta \quad (13)$$

$$\sigma_Q^2 = \alpha^2 \beta \quad (14)$$

$$\text{and skewness (Cs)} = \frac{2}{\sqrt{\beta}} \quad (15)$$

Where α , β and ζ are scale, shape and location parameters respectively. [16] showed that the method of moments with correction of skewness generally give better results than the method of maximum likelihood estimation (MLE). The proposed correction of skewness is given in Equation 16.

$$\gamma = \bar{C}_s \left[\left(1 + \frac{6.51}{N} + \frac{20.20}{N^2} \right) + \left(\frac{1.48}{N} + \frac{6.77}{N^2} \right) c_s^2 \right] \quad (16)$$

Equation 16 was shown to give better estimation of the skewness of the population under the following conditions (i) $0.25 \leq Cs \leq 5.0$ and (ii) $20 \leq N \leq 90$. The maximum relative error due to Equation 16 is 1.5%, see [16] and [17].

3.1.2 Pearson Type 3 distribution

The Pearson type 3 distribution belongs to the generalized gamma distribution and one of the most popular distributions for the hydrologic frequency analysis. It is a three - parameters distribution which have been estimated by various authors using method of moments, maximum likelihood estimate, Probability - Weighted moments and principle of maximum entropy [18] and [19].

If a random variable Q has a Pearson type 3 distribution then its probability density function (pdf) is given by:

$$f_Q(Q) = \frac{1}{\alpha \Gamma(\beta)} \int_{\zeta}^Q \left(\frac{Q - \zeta}{\alpha} \right)^{\beta-1} \exp \frac{Q - \zeta}{\alpha} \quad (17)$$

The variable Q is defined in the range $\zeta < Q < \infty$. In general, the scale parameter “ α ” can take negative or positive values. If $\alpha < 0$, the PR3 variate is upper bounded, thus unsuitable for frequency analysis of floods. The cdf of PR3 can be expressed as ;

$$FQ(Q) = \frac{1}{\alpha\Gamma(\beta)} \int_{\zeta}^Q \left(\frac{Q-\zeta}{\alpha}\right)^{\beta-1} \exp\left(\frac{Q-\zeta}{\alpha}\right) (18)$$

The quantile expression for the PR3 distribution is given by;

$$Q_T = \alpha\beta + \gamma + K_T * \sqrt{\alpha^2\beta} \quad (19)$$

Where K_T is the frequency factor corresponding to the return periods and can be evaluated by using the Wilson-Hilferty transformation for $1.0 \leq Cs \leq 2.0$.

3.2 Plotting Position Formulae

The plotting position formulae used in the study may be represented by

$$P = (Q \leq q_i) = \frac{i-b}{N+1-2b} \quad (20)$$

Equation 20 expresses the probability of non-exceedence or the cumulative probability of non-exceedence. The values of “Q” varies from 0 to 0.5. For example, when $b = 0.44$. Equation is called Gringorten Plotting position. Different position formulae are obtained by changing the values of “b” and so on, Table 2 shows the selected plotting position formulae.

3.3 Performance Measure

3.3.1 Probability plot correlation coefficient test

The probability plot correlation coefficient (PPCC) test is one of the goodness – of – fit tests for determining a suitable probability distribution for a sample. The PPCC test determines whether an assumed probability distributions are acceptable for the sample data using the correlation coefficients between the sample data and theoretical quantiles of assumed probability distribution. [20] and [21] reported that PPCC test provides a conceptual simple attractive, and powerful alternative to other possible hypothesis tests. The PPCC test statistics is formulated on the basis of the linear correlation coefficient r between the data ranked in ascending order, denoted by Q_i and the

Table 2. Plotting position formulas

S/N	Class	Plotting Position Formula (PPF)	Recommended Probability Distribution
1.	Cunnane (1978)	$P_i = \frac{i - 0.4}{N + 0.2}$	GEV, log – Gumbel, PR3; LP3
2.	Chegodayev (1965)	$\frac{i - 0.3}{N + 0.4}$	GEV, log – Gumbel, PR3; LP3
3.	Hazen (1914)	$\frac{i - 0.5}{n}$	
4.	Weibull (1939)	$\frac{i}{N + 1}$	All Distributions
5.	Nguyen et al., (1989)	$P_i = \frac{i - 0.42}{N + 0.3Cs + 0.05}$	Pearson Type 3 (PR3) -3 ≤ Cs ≤ 3 and 5 ≤ N ≤ 100
6.	Beard (1945)	$\frac{i - 0.3175}{N + 0.365}$	All Distributions
7.	Hosking (1990)	$P_i = \frac{i - 0.35}{N}$	Some 3-parameter Distribution

theoretical quantiles W_i , which is calculated as $W_m = \Phi^{-1}(P_i)$ of the normal distribution. The inverse function $\Phi^{-1}(P_i)$ is calculated using the MS Excel built-in function NORM-INV ($P_i, \bar{Q}; \sigma$), which returns corresponding W_i values with \bar{Q} ; and σ the mean and standard deviation of the observed data series. The PPCC test statistics is calculated as:

$$r = \frac{\sum_{i=1}^n (Q_i - \bar{Q})(W_i - \bar{W})}{\sqrt{\sum_{m=1}^N (Q_i - \bar{Q})^2 * (W_i - \bar{W})^2}} \quad (21)$$

Where $\bar{Q} = \frac{1}{N} \sum_{i=1}^N Q$ and $\bar{W} = \sum_{i=1}^N W_i$. In the alternative the value of r in Equation 21, may be computed through MS Excel built in function CORREL(.).

3.2.2 Statistical performance measures

The statistical performance measure of root mean square error (PMSE), maximum absolute error (MAE), percent bias (PBIAS) and Nash-Sutcliffe efficiency (NSE) were applied in evaluating the best plotting position for Pearson Type 3 distribution. RMSE, MAE, PBIAS and NSE are among the performance criteria commonly used for frequency analysis. The above statistical criteria are expressed mathematically in Equation 22 – 25. NSE index determines the relative magnitude of the residual variance (“noise”) compare to the measured data variance (“data”). The RMSE and MAE are measurement in the same units as the model output response of interest and are representative of the size of a typical error. The PBIAS measures the average tendency of the simulated data to be larger or smaller than observed counter parts. It also measures over and under estimation of bias and expresses it as a percentage according to [22] and [26].

The 5% critical value of PPCC Statistic of GEV distribution can be approximated according to [23]:

$$\ln\left(\frac{1}{1-r_{crit,\alpha}}\right) = (a+b\gamma) * N^{c+d\gamma+e\gamma^2} \text{ for } 0.5 \leq \gamma \leq 5.0 \quad (22)$$

For $\alpha = 0.05$; $a = 1.73$, $b = -0.0827$; $c = 0.207$; $d = -0.020$ and $e = 0.00223$.

The critical region for H_0 , at the significance level of α , begin at $r_{crit, \alpha}$ below which, if $r < r_{crit,\alpha}$ the hypothesized pearson type 3 distribution is rejected.

The relative root-mean square error (RRMSE) computes the relative error between the observed and simulated values. The RRMSE statistics calculates each error in proportion to the size of the overall fit. Other statistical error indices used in this study are percent bias (PBIAS), maximum absolute error (MAE), and Nash–Sutcliffe efficiency (NSE). Details about the selected may be found in [23]. The MS Excel software implemented the calculations of Equations 23 – 26.

$$RRMSE = \sqrt{\frac{1}{N} \sum_{i=1}^n \left(\frac{Q_i - P_i}{Q_i}\right)^2} \quad (23)$$

where Q_i is ordered set of observation value and P_i is predicted values for given values of Q_i .

$$PBIAS = \left[\frac{\sum_{i=1}^N (Q_i - P_i)}{\sum_{i=1}^n Q} \right] * 100 \quad (24)$$

$$NSE = 1 - \frac{\sum_{i=1}^N (Q_i - P_i)^2}{\sum_{i=1}^N (Q_i - \bar{Q}_i)^2} \quad \cdot \quad -\infty \leq NSE \leq 1.0 \quad (25)$$

$$MAE = \frac{1}{N} \sum_{i=1}^N |Q_i - P_i| \quad \cdot \quad -\infty \leq MAE \leq 1.0 \quad (26)$$

4. RESULTS AND DISCUSSION

Eight unbiased plotting position formulae recommended for Pearson type 3 distributions were evaluated to determine the best formula using data of three hydrological stations on the Niger river. The solutions to all calculations and graphical plots were performed using MS Excel software. The MS Excel has built – in functions which implemented the calculations. For example, the function EXP((GAMMALN(w))) returns the natural logarithm of the gamma functions used, and so on. First using the sample data, with the method of moments corrected for skewness, as proposed by [16], quantile equation were derived as shown in Table 3. Each plotting position formula was inputted into

each $Q_T - T$ model to compute the predicted discharges. The predicted discharges of each plotting position formula were compared with the observed annual maximum series. The strength and weakness of each plotting position formula, were evaluated according to the statistical performance criteria of PPCC, PBIAS, NSE, RMSE and MAE stated in Equations 23 – 26. Ranking scores are assigned to each plotting position formulae according to the optimal value of the test statistic. For example, a plotting position formula with MAE, RMSE, and PBIAS values, of zero and NSE value of 1.0 is scored 1. In case of a tie, equal scores are given to the contending plotting position formulae. The plotting position formulae were ranked on a scale of 1 to 8, with a score of “1” being the best and 8, the least. Finally, for each plotting position formula, the total score is obtained by summing the individual scores across the goodness-of-fit tests. The ranking of the eight plotting position formulae are shown in Tables 4 – 6, while Figs. 2 – 4 show the performance for Shintaku, Kouroussa and Baro stations respectively.

Fig. 5 show the overall ranking across the three hydrological stations. Figs. 6 – 9 show the scattered plots of simulated and observed discharges against the year of occurrences for selected plotting position formulae.

According to ranking scheme adopted, the plotting position with the least total score is adjudged the best for the station. Consequently, for Baro, the best formula is Chegodayev with a

score of 9, seconded closely by Beard formula with a score of 10. For Kouroussa; Weibull is the best with a score of 10, seconded closely by Chegodayev with a score of 11. Finally, at Shintaku, Weibull is the best with a score of 4, seconded by Chegodayev with a score of 10.

The total scores for each station were summed across the three study stations and the resulting plots are presented in Figs. 2 – 4. Figure 4 shows the overall ranking of the plotting position with Weibull, was adjudged the best, seconded by Chegodayev and thirdly by Beard. The impetus for this research is that Pearson Type 3 distribution was found one of the best probability distribution model for Nigeria and the study was undertaken to find the best probability plotting position, that is most appropriate for the Pearson Type 3 distribution. The method of moments (MoM) with correction for skewness according to [16] was use for parameter estimation. They claimed that MoM with correction skewness gave better result than Maximum likelihood estimate (MLE). The findings of the study is in agreement with [2] who studied position formulae for Surma basin in Bangladash and found Weibull the best. Also [5] compared eight unbaised plotting formulae for Pearson Type 3 distribution using annual maximum series of Malaysia Peninsular and found Weibull performed better than other formulae. Furthermore, [24] evaluated seven plotting position formulae for Pearson Type 3 distribution and also found Weibull plotting position formula.

Table 3. Quantile Relations for the Study Stations

Station	$Q_T - T$ Model	β	α	γ
Baro	$Q_T = 716.34 + 188.71K_T$	268.56	11.52	-2376.23
Kouroussa	$Q_T = 713.98 + 223.64K_T$	68.65	26.99	-1139.05
Shintaku	$Q_T = 13320.80 + 2015.43K_T$	8.90	675.72	7309.44

Table 4. Ranking of plotting position formulae for Baro

PPF	Computed Performance Measures					PPCC Statistics		
	MAE	NSE	PBIAS	RMSE	TOTAL	PPCC	PPC _{crit}	Decision
Cunnane	4	3	2	4	13	0.982	0.979	Accept(A)
Weibull	6	1	5	1	13	0.985	0.979	A
Chegodayev	1	2	4	2	9	0.983	0.979	A
Beard	2	2	3	3	10	0.983	0.979	A
Hazen	8	6	1	7	22	0.983	0.979	A
Nguyen	5	4	7	6	22	0.982	0.979	A
In-na	7	4	6	5	22	0.984	0.979	A
Hosking	3	5	8	8	24	0.981	0.979	A
*Range:	0.0 to ∞	$-\infty$ to 1.0	$-\infty$ to ∞	0.0 to ∞				
*Optimal Value:	0.0	1.0	0.0	0.0				

*Moriassi et al. (2015), PPF: Plotting Position formulae

Table 5. Ranking of plotting position formulae for Kouroussa

PPF	Computed Performance Measures					PPCC Statistics		
	MAE	NSE	PBIAS	RMSE	Total	PPCC	PPC _{crit}	Decision
Cunnane	4	3	3	4	14	0.992	0.979	A
Weibull	1	1	7	1	10	0.994	0.979	A
Chegodayev	2	2	5	2	11	0.993	0.979	A
Beard	3	4	4	3	14	0.993	0.979	A
Hazen	8	7	2	7	24	0.992	0.979	A
Nguyen	6	6	6	5	23	0.981	0.979	A
In-na	7	5	1	6	19	0.992	0.979	A
Hosking	5	5	8	6	24	0.992	0.979	A
Range:	0.0 to ∞	-∞ to 1.0	-∞ to ∞	0.0 to ∞				
Optimal Value:	0.0	1.0	0.0	0.0				

**Moriasi et al. (2015). PPF: Plotting Position formulae*

Table 6. Ranking of Plotting Position Formulae for Shintaku

PPF	Computed Performance Measures					PPCC Statistics		
	MAE	NSE	PBIAS	RMSE	TOTAL	PPCC	PPC _{crit}	Decision
Cunnane	2	5	7	5	19	0.979	0.981	Reject (R)
Weibull	1	1	1	1	4	0.987	0.981	A
Chegodayev	4	2	2	2	10	0.984	0.981	A
Beard	5	3	3	3	14	0.984	0.981	A
Hazen	3	6	6	6	21	0.981	0.981	A
Nguyen	6	8	4	8	26	0.983	0.981	A
In-na	7	4	8	4	23	0.979	0.981	R
Hosking	8	7	5	7	27	0.981	0.981	A
*Range:	0.0 to ∞	-∞ to 1.0	-∞ to ∞	0.0 to ∞				
*Optimal Value:	0.0	1.0	0.0	0.0				

**Moriasi et al. (2015). PPF: Plotting Position formulae*

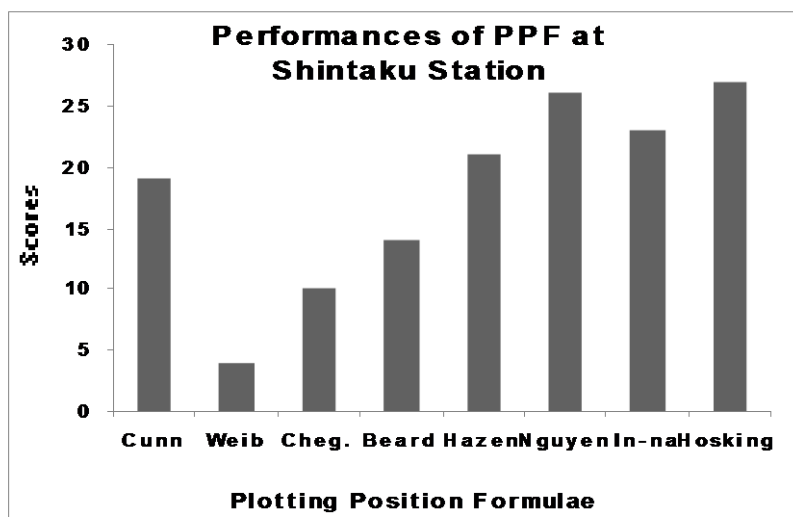


Fig. 2. Plot of Total Scores vs. PPF Flow at Shintaku Station

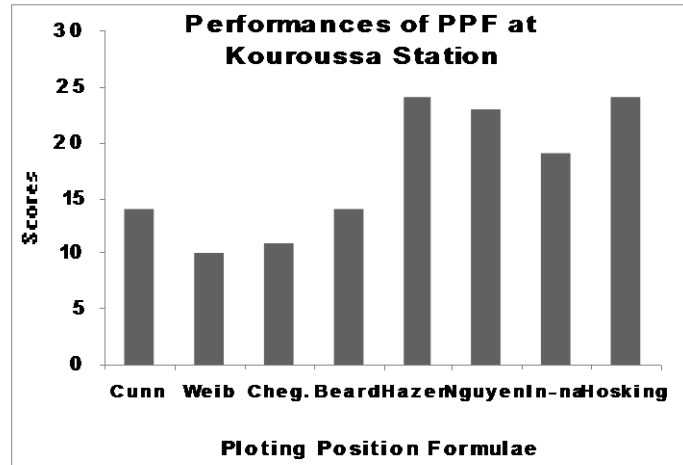


Fig. 3. Plot of Total Scores vs. PPF at Kouroussa Station

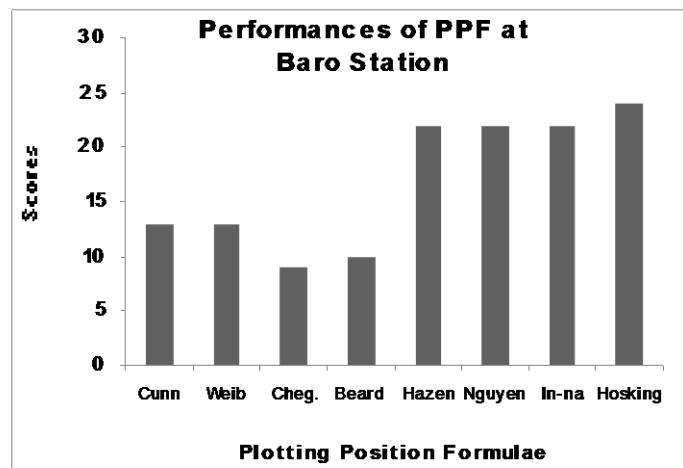


Fig. 4. Plot of Total Scores vs. PPF at Baro Station

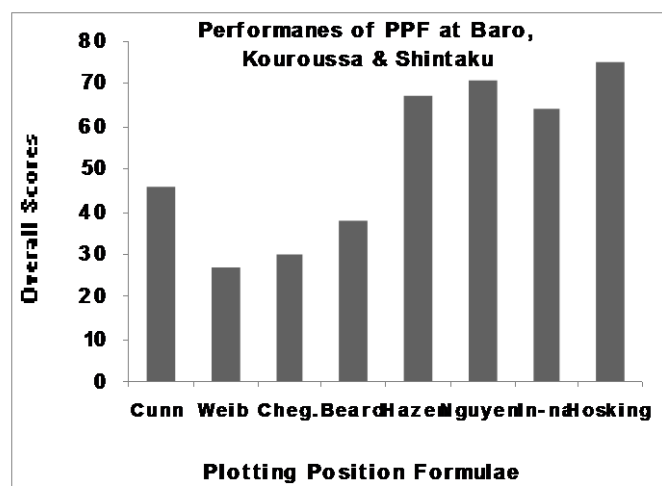


Fig. 5. Plot of Overall Scores vs. PPF, All Stations

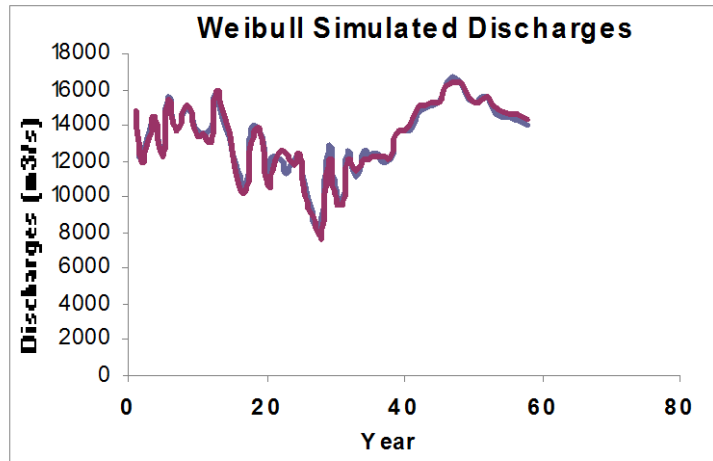


Fig. 6. Observed and Simulated Discharges at Shintaku Station

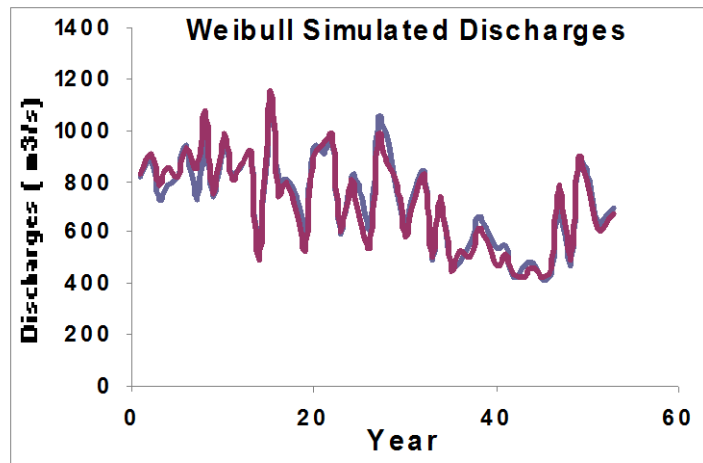


Fig. 7. Observed and Simulated Discharges at Baro Station

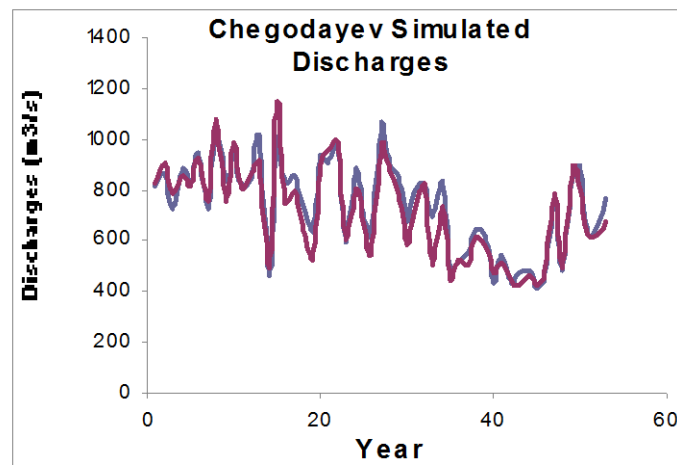


Fig. 8. Observed and Simulated Discharges atc Baro Station

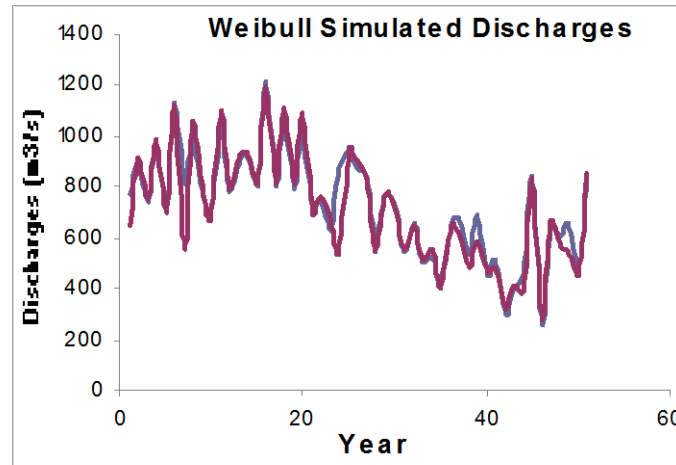


Fig. 9. Observed and Simulated Discharges at Kouroussa Station
 Note: in Figures 6 – 9, red lines indicate AMS, blue lines indicate simulated series

5. RECOMMENDATION FOR FURTHER RESEARCH

Further studies are required to evaluate the recommended plotting position formulae to determine the best plotting positions formula for Pearson Type III distributions. The Pearson type 3 distributions has been found satisfactorily to model the distribution of annual floods in Nigeria [25]. Furthermore [26] recommended that the study on unbiased plotting position formulae be extended to Pearson type (3) distribution.

6. CONCLUSION

This paper evaluated plotting positions formulae recommended for Pearson Type 3 Distribution at three hydrological stations on the Niger River in Nigeria. The following conclusions can be derived from present research work on hydrological station basis. First at Shintaku station; Weibull plotting position formula ranked best with a score of “4”, seconded by Chegodayev with a distant score of “10”. At Kouroussa, the best plotting position formula is Weibull with a score of “10”, seconded closely by Chegodayev with a score of “11”. Finally, at Baro, Chegodayev is the best plotting position for formula, with a score of “10”, seconded closely by Beard plotting position formula with a score of “10”. The overall ranking is shown in Figure 4; with Weibull; the best with overall score of “27” seconded by Chegodayev with score of “30” thirdly, Beard plotting position formula with overall score of 38.

This study adopted a simulation approach to compute the predicted series from the eight

unbiased plotting position formulae for Pearson type 3 distribution which were compared using the statistical goodness-of-fit criteria of PPCC, PBIAS, NSE, RMSE and MAE.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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