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Alternate Approach (Penalty Approach) to Assignment Problem Solving and Comparison to Existing Approaches

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

Assignment problem is an important area in Operation Research and is also discussed in real physical world. In this paper an attempt has been made to solve the assignment problem using a new Method called the Penalty method. We discuss a numerical example by using the new Method and compare it with standard existing method which is the Hungarian method. We compare the optimal solution of the new Method and the Hungarian method. The new method is a simple procedure, easy to apply for solving assignment problem.

Keywords: Assignment problem; Hungarian method; alternative method; operation research.

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1 Introduction

The assignment problem is one of the most important areas in the area of allocation of resource to activity on one-to-one basis. It is one of the most studied areas in Combinatorial Optimization problem or Operation research in Mathematics [1-3]. It is a special case of transportation problems. The assignment problem finds numerous applications in various diverse business situations, such as assigning machines to factory orders, assigning sales people to sales territories, assigning contract to contract bidders, assigning teachers to classes and assigning accountants to accounts of the clients. In this paper we developed another method for solving assignment problem [4-8]. Finally, we compare the Optimal Solution of the new method and the Hungarian method.

2 Mathematical Formulation of Assignment Problem

Like any transportation problem, the assignment problem has a tableau which is matrix associated with it. Consider the problem of assigning resources to activity on one-to-one basis in such a way that cost or time is minimized. The cost matrix (C_{ij}) is given below [9-13]:

Table 1. Representation of the assignment problem

		ACTIVITY					
		A_1	A_2	…		A_n	
	R_1	$C_{1,1}$	$C_{1,2}$	…	$C_{1,n-1}$	$C_{1,n}$	1
	R_2	$C_{2,1}$	$C_{2,2}$	…	$C_{2,n-1}$	$C_{2,n}$	1
		⋮	⋮	…	⋮	⋮	
Resource		$C_{n-1,1}$	$C_{2,n-1}$	…	$C_{n-1,n-1}$	$C_{n-1,n-1}$	1
	R_n	$C_{n,1}$	$C_{2,n}$	…	$C_{n,n-1}$	$C_{n,n}$	1
Required		1	1	…		1	

The cost matrix (C_{ij}) is the same as that of a transportation problem except that the availability at each of the resource and the requirement at each of the destinations is unity. Let X_{ij} denote the assignment of the i^{th} resource to the j^{th} activity such that:

$$X_{ij} = \begin{cases} 1: if \ \ resource i \ is \ assigned \ to \ activity \ \ j \\ 0, \qquad\qquad\qquad\qquad otherwise \end{cases}$$

Then the mathematical formulation of the assignment problem is

$$\text{Minimize } Z = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} X_{ij}$$

subject to the constraints

$$\sum_{i=1}^{n} X_{ij} = 1$$

$$\sum_{j=1}^{n} X_{ij} = 1$$

X_{ij} = 0 or 1

For all I =1,2…n and j = 1,2,…n.

I. Alternative Method for Solving Assignment Problem

In this section we introduce a new method for solving assignment problem. The algorithm for the alternative method is as follows:

STEP 1: Compute the row and column penalties by subtracting the least cost from the next least cost. Where we have the same least cost on a row or column, assign 0 to indicate no penalty.
STEP 2: Identify the highest penalty from the row or column penalties and assign the least cost. Cross out all entries in the row and column penalties of the assigned cost.
STEP 3: When there is a tie in the penalties, assign the penalty with the least cost in the cost matrix.
STEP 4: When there is a tie in the assigned cost, search through the row or column of each of the tied entry and assign the cost with the highest penalty.
STEP 5: Stopping Criteria: Stop when all rows and columns of the cost matrix have been crossed out.

II. Numerical Examples using the Algorithms of the Alternative Method

1. Solve the following assignment problem using the Alternative Method.

Consider the problem of assigning six jobs to six persons. The assignment table is given below

	jobs					
persons	1	2	3	4	5	6
A	73	21	10	9	86	95
B	73	55	72	91	14	82
C	80	18	39	11	73	27
D	44	99	90	36	100	14
E	71	33	73	42	37	86
F	34	11	39	35	52	9

Determine the optimum assignment schedule and minimum assignment cost.

STEP 1: Computing row and column Penalties by subtracting the least from the next least on a row and column.

	JOBS						
persons	1	2	3	4	5	6	
A	~~73~~	~~21~~	~~10~~	~~9~~	~~86~~	~~95~~	1
B	73	55	72	91	14	82	41
C	80	18	39	11	73	27	7
D	44	99	90	36	100	14	22
E	71	33	73	42	37	86	4
F	34	11	39	35	52	9	2
	10	7	29	2	23	5	

The highest Penalty is 41, we assign 14 which is the least on it row.

STEP 2: Compute the row and column penalties on the reduced matrix in Step 1.

	Jobs							
persons	1	2	3	4	5	6		
A	73	21	10	9	86	95	1	1
B	73	55	72	91	14	82	41	-
C	80	18	39	11	73	27	7	7
D	44	99	90	36	100	14	22	22
E	71	33	73	42	37	86	4	9
F	34	11	39	35	52	9	2	2
	10	7	29	2	23	5		
	10	7	29	2	-	5		

The highest penalty is 29. The lowest on it column is 10, so we assign 10 and cross out all elements under it row and column.

STEP 3: Compute the row and column penalties of the reduced matrix in Step 2.

	Jobs								
persons	1	2	3	4	5	6			
A	73	21	10	9	86	95	1	1	-
B	73	55	72	91	14	82	41	-	-
C	80	18	39	11	73	27	7	7	7
D	44	99	90	36	100	14	22	22	22
E	71	33	73	42	37	86	4	9	9
F	34	11	39	35	52	9	2	2	2
	10	7	29	2	23	5			
	10	7	29	2	-	5			
	10	7	-	24	-	5			

The highest penalty is 24, we assign 11, which is the lowest in its column and cross out all rows and columns under it.

STEP 4: compute the row and column penalties on the reduced matrix in step 3.

	Jobs									
persons	1	2	3	4	5	6				
A	73	21	10	9	86	95	1	1	-	-
B	73	55	72	91	14	82	41	-	-	-
C	80	18	39	11	73	27	7	7	7	-
D	44	99	90	36	100	14	22	22	22	22
E	71	33	73	42	37	86	4	9	9	38
F	34	11	39	35	52	9	2	2	2	2
	10	7	29	2	23	5				
	10	7	29	2	-	5				
	10	7	-	24	-	5				
	10	22	-	-	-	5				

The highest penalty is 38, we assign 33 which is the smallest on it row and cross out all rows and columns under it.

STEP 5: We are left with two-by-two matrix: $\begin{bmatrix} 44 & 14 \\ 34 & 9 \end{bmatrix}$
44 + 9 > 34 + 14, so we assign 34 and 14 and cross out all rows and columns under them.

	Jobs									
persons	1	2	3	4	5	6				
A	73	21	10	9	86	95	1	1	-	-
B	73	55	72	91	14	82	41	-	-	-
C	80	18	39	11	73	27	7	7	7	-
D	44	99	90	36	100	14	22	22	22	22
E	71	33	73	42	37	86	4	9	9	38
F	34	11	39	35	52	9	2	2	2	2
	10	7	29	2	23	5				
	10	7	29	2	-	5				
	10	7	-	24	-	5				
	10	22	-	-	-	5				

STEP 6: Stop since all rows and columns have been crossed out.
optimal assignment = A3 + B5 + C4 + D6 + E2 + F1
= 10 + 14 + 11 + 14 + 33 + 34
= 106

2. Use the alternative method to solve the following assignment problem shown in table. The matrix entries represent the time it takes for each job to be processed by each machine in hours.

J/M	I	II	III	IV	V	VI	VII
1	**18**	**24**	**12**	**6**	**30**	**7**	**13**
2	**14**	**7**	**14**	**22**	**20**	**15**	**8**
3	**10**	**27**	**8**	**28**	**24**	**7**	**25**
4	**8**	**23**	**21**	**16**	**9**	**19**	**17**
5	**4**	**2**	**24**	**18**	**4**	**14**	**9**
6	**10**	**2**	**23**	**9**	**9**	**30**	**12**
7	**3**	**7**	**3**	**26**	**9**	**17**	**11**

STEP 1: Subtract the least entry from the next least entry on a row and column. The highest penalty is 7. The smallest entry on it row is 2. So, we assign 2 and cross all rows and columns under it.

								RP
	18	**24**	**12**	**6**	**30**	**7**	**13**	**1**
	14	**7**	**14**	**22**	**20**	**15**	**8**	**1**
	10	**22**	**8**	**28**	**24**	**7**	**25**	**1**
	8	**23**	**21**	**16**	**9**	**19**	**17**	**1**
	4	**2**	**24**	**18**	**4**	**14**	**9**	**2**
	10	**2**	**23**	**9**	**9**	**30**	**12**	**7**
	3	**7**	**3**	**26**	**9**	**17**	**11**	**0**
CP	**1**	**0**	**5**	**3**	**5**	**0**	**1**	

STEP 2: Subtract the least entry from the next least on the row and column of the reduced matrix in step 1.

	18	24	12	6	30	7	13	1	1
	14	7	14	22	20	15	8	1	6
	10	22	8	28	24	7	25	1	1
	8	23	21	16	9	19	17	1	1
	4	2	24	18	4	14	9	2	0
	10	2	23	9	9	30	12	7	-
	3	7	3	26	9	17	11	0	0
CP	1	0	5	3	5	0	1		
	1	-	5	10	5	0	1		

The highest penalty is 10. The least on it column is 6, so we assign 6 and cross out rows and columns under it.

STEP 3: Subtract the least entry from the next least entry on a row and column of the reduced matrix in step 2.

	18	24	12	6	30	7	13	1	1	-
	14	7	14	22	20	15	8	1	6	6
	10	22	8	28	24	7	25	1	1	1
	8	23	21	16	9	19	17	1	1	1
	4	2	24	18	4	14	9	2	0	0
	10	2	23	9	9	30	12	7	-	-
	3	7	3	26	9	17	11	0	0	0
CP	1	0	5	3	5	0	1			
	1	-	5	10	5	0	1			
	1	-	5	-	5	7	1			

The highest penalty is 7 and the least on its column is 7, so we assign 7 and cross out all rows and columns under it.

STEP 4: Subtract the least entry from the next least entry on a row and column of the reduced matrix in step 3.

									RP		
	18	24	12	6	30	7	13	1	1	-	-
	14	7	14	22	20	15	8	1	6	6	6
	10	22	8	28	24	7	25	1	1	1	-
	8	23	21	16	9	19	17	1	1	1	1
	4	2	24	18	4	14	9	2	0	0	0
	10	2	23	9	9	30	12	7	-	-	-
	3	7	3	26	9	17	11	0	0	0	0
CP	1	0	5	3	5	0	1				
	1	-	5	10	5	0	1				
	1	-	5	-	5	7	1				
	1	-	11	-	5	-	1				

The highest penalty is 11, we assign 3 which is lowest on its column and cross out all rows and columns under it.

STEP 5: Compute row and column penalties of the reduced matrix in step 4.

								RP			
18	24	12	6	30	7	13	1	1	-	-	-
14	7	14	22	20	15	8	1	6	6	6	6
10	22	8	28	24	7	25	1	1	1	-	-
8	23	21	16	9	19	17	1	1	1	1	1
4	2	24	18	4	14	9	2	0	0	0	0
10	2	23	9	9	30	12	7	-	-	-	-
3	2	3	26	9	17	11	0	0	0	0	-
CP											
1	0	5	3	5	0	1					
1	-	5	10	5	0	1					
1	-	5	-	5	7	1					
1	-	11	-	5	-	1					
4	-	-	-	5	-	1					

The highest penalty is 6, we assign 8 which is the lowest on it row and cross rows and columns under it.

STEP 6: We are left with the 2x2 matrix in the cost matrix: $\begin{bmatrix} \mathbf{8} & \mathbf{9} \\ \mathbf{4} & \mathbf{4} \end{bmatrix}$
4 + 8 < 4 + 9

So, we assign 4 and 8 and cross out all rows and columns under 4 and 8.

	I	II	III	IV	V	VI	VII	RP				
	18	24	12	6	30	7	13	1	1	-	-	-
	14	7	14	22	20	15	8	1	6	6	6	6
	10	22	8	28	24	7	25	1	1	1	-	-
	8	23	21	16	9	19	17	1	1	1	1	1
	4	2	24	18	4	14	9	2	0	0	0	0
	10	2	23	9	9	30	12	7	-	-	-	-
	3	2	3	26	9	17	11	0	0	0	0	-
CP	1	0	5	3	5	0	1					
	1	-	5	10	5	0	1					
	1	-	5	-	5	7	1					
	1	-	11	-	5	-	1					
	4	-	-	-	5	-	1					

Optimal
Processing Time = 6 + 8 + 7 + 8 + 4 + 2 + 3
= 38 hours

3. Use the alternative method to solve the following assignment problem shown in table.

43	**56**	**91**	**89**	**33**
57	**66**	**53**	**60**	**34**
64	**20**	**84**	**68**	**91**
5	**79**	**71**	**81**	**58**
60	**11**	**19**	**48**	**28**

STEP 1: Compute row and column penalties by subtracting the least and the next least cost on each column and row.

43	56	91	89	33	10
57	66	53	60	34	23
64	20	84	68	91	44
5	~~79~~	~~71~~	~~81~~	~~58~~	53
60	11	19	48	28	8
38	9	44	12	5	

The highest penalty is 53. We assign 5 on it row since it is the lowest and cross out all rows and columns under it.

STEP 2: Compute the least and the next least on each row and column of the reduced matrix in step 1.

43	56	91	89	33	10	23
57	66	53	60	34	23	26
64	20	~~84~~	~~68~~	~~91~~	44	48
5	~~79~~	~~71~~	~~81~~	~~58~~	53	-
60	11	19	48	28	8	9
38	9	44	12	5		
-	9	44	12	5		

The highest penalty is 48. We assign 20 which is the lowest on it rows and cross out all rows and columns under it.

STEP 3: Compute the least entry from the next least entry of the reduced matrix in step 2.

43	56	~~91~~	~~89~~	33	10	23	56
57	66	53	60	34	23	26	19
64	20	~~84~~	~~68~~	~~91~~	44	48	-
5	~~79~~	~~71~~	~~81~~	~~58~~	53	-	-
60	11	19	48	28	8	9	9
38	9	44	12	5			
-	9	44	12	5			
-	-	34	12	5			

The highest penalty is 56. We assign 33 which is the lowest on it row and cross out rows and columns under it.

STEP 4: We are now left with the two-by-two matrix: $\begin{bmatrix} 53 & 60 \\ 19 & 48 \end{bmatrix}$

19 + 60 < 53 + 48, therefore we assign 60 and 19, and cross out all rows and columns under them.

43	56	91	89	33	10	23	56
57	66	53	60	34	23	26	19
64	20	84	68	91	44	48	-
5	79	71	81	58	53	-	-
60	11	19	48	28	8	9	9
38	9	44	12	5			
-	9	44	12	5			
-	-	34	12	5			

STEP 5: Stop since all rows and columns have been crossed out.

Optimal Assignment = 33+60+20+5+19
=137

3 Conclusion

In this paper, we proposed a new method called the Alternative method for solving Assignment problem. We explained the algorithm for the alternative method and showed it computational efficiency by using numerical examples. The Optimal Solution is the same as that of the Optimal Solution of the Hungarian method. We have therefore introduced a different algorithm for solving the Assignment problem.

Competing Interests

Authors have declared that no competing interests exist.

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