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Analysis of Drought Severity and Duration Using Copulas in Anuradhapura, Sri Lanka

E. M. R. S. B. Ekanayake[1*] and Kanthi Perera[1]

[1]*Department of Engineering Mathematics, Faculty of Engineering, University of Peradeniya, Sri Lanka.*

Authors' contributions

This work was carried out in collaboration between both authors. Author EMRSBE designed the study, performed the statistical analysis, results interpretation, managed the literature searches and wrote the first draft of the manuscript. Author KP also involved in design the study, managed the analyses of the study and manuscript preparation. Both authors read and approved the final manuscript.

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ABSTRACT

Anuradhapura district is one of the largest agricultural crop production areas in Sri Lanka. But it is often affected by droughts and droughts caused severe damage for agricultural industry. Thus it is very important to identify the drought characteristics (drought duration and drought severity) and their joint probability distribution to minimize the adverse effects of droughts. Drought characteristics were defined using 3-month standard precipitation index (SPI). It is calculated using monthly rainfall data from 1951 to 2007 in Anuradhapura. Occurrences of 46 drought events were indentified using the calculated SPI.

Since dependency nature of the drought variables, copula based joint distribution was used to calculate the joint distribution. The joint distribution could be obtained by combining the marginal distributions using copula. Five copulas were examined and compared to find the best fitted copula to represent the joint distribution. The best marginal distributions were identified as the gamma distributions for drought durations and drought severity using AIC, BIC and Kolmogorov-Smirnov test. Frank copula was

**Corresponding author: Email: rukmanek@gmail.com;*

identified as the best copula based on AIC, BIC and Cramer-Von Mises statistics. The joint distribution was derived combining the gamma distributions using the Frank copula. The univariate and joint returns periods of droughts were calculated.
A drought event occurred in 1974 was identified as a major drought event. Drought duration and severity in 1974 were 9 months and 10.95 respectively. Using the indentified univariate and multivariate return periods, drought risks could be minimized by pre-planning and making decision against adverse effects.

Keywords: SPI; copula; joint distribution; return periods.

1. INTRODUCTION

Drought caused severe economic losses in Sri Lanka as an island located close to the equator is prone to warm weather conditions. This can be seen by the annual occurrences of droughts in the country. In addition, a noticeable severe drought also occurred in Sri Lanka this year (2014), over one million people have been affected by the drought in Polonnaruwa, Anuradhapura, Moneragala, Ampara, Hambantota, Puttalam, Trincomalee, Mannar, Vavuniya, Mullaitivu and Kilinochchi districts and it is also immense damage done on agricultural industry. Thus, droughts are of great importance in the planning and management of water resources as out of the total population in Sri Lanka, 32% engage in agricultural activities. Therefore, it is important to identify the drought characteristics to minimize the drought risk.

Drought is a complex phenomenon and its impacts vary from region to region. Drought can therefore be difficult for people to understand. In the most general sense, drought originates from a deficiency of precipitation over an extended period of time [1]. Therefore many indices have been developed (Table 1) to identify for modeling the droughts, such as Palmer Drought Severity Index [2] and the standardized precipitation index (SPI) [3].

Table 1. Drought indices

Drought Index	Description
PDSI (Palmer Drought Severity Index)	Calculation is based on precipitation and temperature data and available Water content (AWC) of the soil [2]
PHDI (Palmer Hydrological Drought Index)	Gauges groundwater and reservoirs, for even longer timescales than PDSI itself.
PCMI (Palmer Crop Moisture Index)	For short-term drought on order of weeks. It can rapidly vacillate and is poor tool for monitoring long-term drought
SPI (Standard Precipitation Index)	Calculation is based on precipitation data

The SPI is the most widely used index, because it is simple, spatially invariant and only needs monthly precipitation data to calculate it, which is used to describe droughts in different parts of the world. Therefore drought properties are investigated by using SPI in numerous studies. Since drought is complex phenomena, one variable cannot provide a comprehensive evaluation of droughts [4]. Therefore drought duration and drought severity derived from SPI to describe the droughts.

But joint multivariate models of droughts are difficult to establish because different distribution functions are often used to various attributes of droughts [5]. Further, Copula based models represent the best joint distributions for droughts than traditional by bivariate models [6]. To overcome such difficulties copula based joint multivariate distributions can be used. The advantage of the copula method is that no assumption is needed for the variables to be independent or normal or having the same type of the marginal distributions [7].

Multivariate distributions using copulas have been used for drought, rainfall and flood analysis in recent past. They have been used for rainfall frequency analysis [7,8,9,10,11,12], flood frequency analysis [13,14,15,16,17], drought frequency analysis [18,19,20,21] and both rainfall and flood frequency analysis [22,23,24,25]. Further in 2006 and 2007 Shiau modeled copula based joint distribution for drought frequency using drought duration and severity [4,18]. In 2009 Shiau compared the two regions in Iran to find out the driest region using copula approach [5]. L. Chen et al. have used two-variable, three-variable and four variable copulas based joint distribution to identify the return periods in droughts [26]. P. Ganguli et al. applied BB1, Gumbel-Hourgaard and Student's t Copula to build the joint distribution for drought severity and duration and compared with the traditional bivariate distribution of severity and duration [6].

The objective of this paper is to find out meteorological drought events in Anuradhapura, which is one of the largest agricultural districts in Sri Lanka. Thereafter, to identify the best joint probability distribution function of drought duration and drought severity based on Gaussian copula and four Archimedean copulas. The best joint distribution will be used to identify the return periods of drought events to minimize the drought risks.

2. METHODOLOGY

2.1 Drought Modeling Using SPI

The SPI is based on the cumulative probability of a given rainfall event occurring at a particular station. The historic rainfall data of the station is fitted to a gamma distribution, as the gamma distribution has been found to fit the precipitation distribution quite well. Parameters of the distribution are estimated by maximum likelihood method [27]. Then cumulative probabilities of the gamma distribution were calculated and then those probabilities approximated to a standard normal distribution. We referred those standard normal values as SPI. Usually SPI values were calculated at different time scales such as 1, 3, 6, 9 and 12 months [6].

$$g(x;\alpha,\beta) = \frac{\beta^{\alpha} x^{\alpha-1} e^{-x\beta}}{\Gamma(\alpha)} \; for\; x \geq 0 \; and \; \alpha,\beta > 0 \tag{1}$$

Generally 12-month SPI is used for analyze medium term droughts and 24-month SPI is used for long term drought analysis. Further 6-month time scale droughts may be useful for seasonal drought identification [28]. In this study 3-month scale SPI is used to identify the short term seasonal drought events.

<u>2.1.1 Drought definition using SPI</u>

There are several definitions for drought event based on SPI series. One definition is consecutive number of time intervals where SPI less than 0 [18]. This does not reflect the actual drought scenario, since SPI between -1 to 1 represents normal condition according to the Table 2. Thus, drought event defined as when SPI value reaches -1.0 and ends when SPI becomes positive again [29]. Since Sri Lanka is in South Asian region this definition could be appropriate than other definitions.

The drought characteristics are defined by drought duration and drought severity. Drought duration denoted by D, is the continuous SPI periods in particular drought event, while drought severity denoted by S is the sum of cumulative values of SPI within drought event (Eq. 2).

$$S = -\sum_{i=1}^{D} SPI_i \tag{2}$$

Table 2. Description for SPI values

SPI	Description
≤ -2.00	Extremely dry
-1.99 to -1.50	Very dry
-0.99 to 0.99	Normal
0.99 to 1.50	Moderately wet
1.49 to 2.00	Very wet
>2.00	Extremely wet

Inter-arrival time (I_d) of drought is also defined in the analysis as the time gap between beginning of two consecutive drought events [6].

2.2 Modeling the Joint Probability Distribution

The copulas provide the opportunity to study and measure relationships between random variables. The Copula method firstly found by Sklar [30], to describe the function that joins univariate distribution functions to multivariate distribution. Let a joint distribution function $H(x,y)$ with the marginal distributions $F_X(x)$ and $F_Y(y)$ which are cumulative distribution functions of X and Y. Then there exists a function such that $H(x,y) = C(F_X(x), F_Y(y))$; where C is the copula. To construct the copula, Let $U = F_X(x)$ and $V = F_Y(y)$. If X and Y are the marginal distributions then C can be defined by $C\,(u,v) = F(\,F_X^{-1}(u), F_Y^{-1}(v)\,)$. This gives

$$C\big(F_X(x), F_Y(y)\big) = F\left(F_X^{-1}\big(F_X(x)\big), F_Y^{-1}\big(F_Y(y)\big)\right) = H(x,y) \tag{3}$$

For modeling the bivariate joint probability distribution, first we identified the marginal distributions for drought duration and drought severity. Exponential, Normal, Log-normal, Weibull, Gamma and Logistic distributions were fitted to identify the best marginal distribution for both drought duration and drought severity. The Akaike's information criteria (AIC) and Bayesian information criteria (BIC) were used to identify the best fitted marginal distribution. Then Kolomogorov-smirnov (KS) test was used for further confirmation.

Tail dependency of drought characteristics is checked using ranked scatter plot of drought duration, drought severity and then selected five copulas namely Gumbel-Hourgaard, Joe, Clayton, Frank and Gaussian to identify the best copula. (See Appendix) Where, there is no tail dependency in theoretical Gaussian and Frank Copulas while there is upper/lower tail dependency in Joe, Clayton, Gumbel-Hourgaard Copulas. Pearson correlation coefficient and Kendall's tau (τ) used to identify the relationship between the drought characteristics.

Copula parameter is estimated by the Kendall's τ, where Kendall's τ is the most commonly used non-parametric dependence measure to estimate copula parameter. Then AIC and BIC are used to compute the goodness-of-fit measures between fitted copula and empirical joint distribution. For further confirmation Cramer-von Mises S_n statistic based on Genest-Remillard [31] goodness of fit test was applied. Estimation method for S_n was found by inverting the tau parameter. There after we evaluate the best joint distribution function for the drought duration and drought severity, using the best copula and best marginal distributions. Then the univariate and joint return periods are calculated accordingly.

2.3 Return Periods for Droughts

Return periods are important because it can provide useful information about proper use of water under drought conditions. Generally return period is defined as the average elapsed time between occurrences of critical events in hydrology and water resources engineering. Here, we calculated univariate and multivariate return periods for particular drought events.

2.3.1 The univariate return period

The univariate return period of drought duration (T_D) and drought severity (T_S) can be expressed as follows.

$$T_D = \frac{E(I_d)}{1 - F_D(d)} \tag{4}$$

$$T_S = \frac{E(I_d)}{1 - F_S(s)} \tag{5}$$

Where $F_D(d)$ and $F_S(s)$ are cumulative marginal distribution functions of drought duration and drought severity respectively. $E(I_d)$ is the expected time of inter-arrival time of drought events.

2.3.2 The joint return period

Since drought events are characterized by drought duration and drought severity, univariate return periods may give under or over estimated results. Therefore it is important to calculate joint return periods. The joint return periods are very important in hydrological designs [32]. Bivariate return periods T_{DS} and $T_{DS}^{'}$ were computed using copula-based approach [18], which are given by,

$$T_{DS} = \frac{E(I_d)}{P(D \geq d \; or \; S \geq s)} = \frac{E(I_d)}{1 - F_{DS}(d,s)} = \frac{E(I_d)}{1 - C\big(F_D(d), F_S(s)\big)} \tag{6}$$

$$T_{DS}^{'} = \frac{E(I_d)}{P(D \geq d \; and \; S \geq s)} = \frac{E(I_d)}{1 - F_D(d) - F_S(s) + F_{DS}(d,s)}$$
$$= \frac{E(I_d)}{1 - F_D(d) - F_S(s) + C\big(F_D(d), F_S(s)\big)} \tag{7}$$

3. RESULTS AND DISCUSSION

Monthly rainfall data collected from meteorology department, Sri Lanka from year 1951 to 2007 in Anuradhapura used for the analysis. Forty six drought events were identified using 3-month SPI. The 3-month SPI series obtained from the gamma distribution for monthly precipitation data is shown in the Fig. 1. According to the Fig. 1 the most intensive droughts occurred in 1974 and 1976 as SPI values are below -3.0. Further 18 extremely dry situations were identified during the entire period.

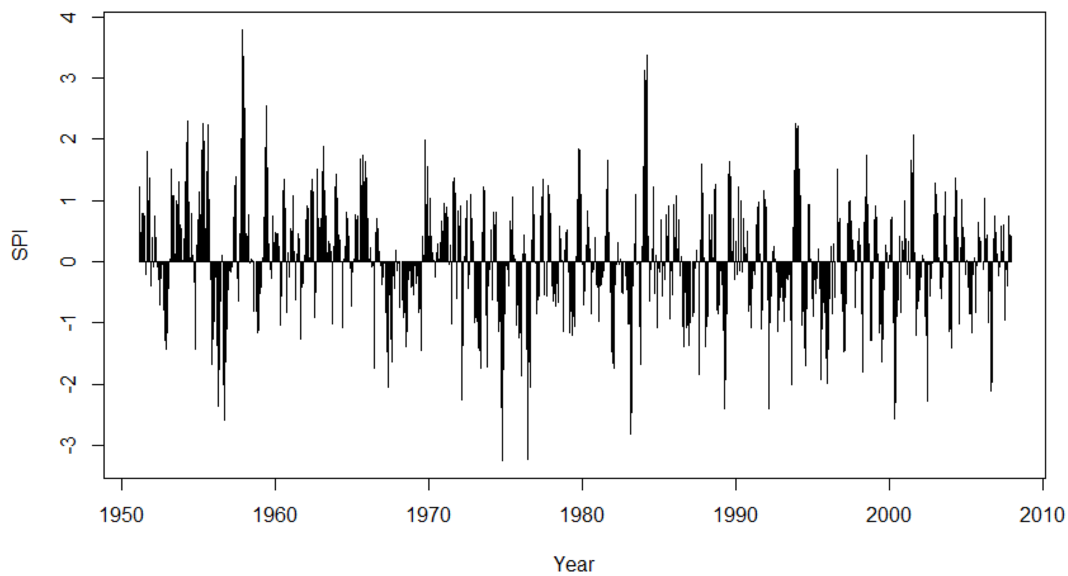

Fig. 1. The 3-month SPI series using precipitation data (1951-2007) in Anuradhapura

According to Figs. 2 and 3, drought events occurred twice in 5 years during the time period. (Spikes and dots in Figs. 2 and 3 shows Occurrences of droughts) They were occurred in the years 1973, 1983, 1991, 1992 and 1998. Anuradhapura region has experienced the longest drought durations in 1968, 1986 and 1992 while the most severe droughts in 1955, 1974, 1986, 1992 and 1995. Altogether, mainly 4 droughts are identified in year 1955-1956, 1974, 1986 and 1992.

Table 3 indicates the descriptive statistics of annual precipitation, drought duration and drought severity. The annual minimum precipitation is 742.1 mm and maximum precipitation is 2419.2 mm while the annual mean precipitation is 1298.4 mm in the study period.

Fig. 4 shows the scatter plot for the described drought characteristics. Calculated Kendall's tau and Pearson's coefficient correlations for drought duration and drought severity are 0.733 and 0.854 respectively with their corresponding p-values less than 0.0001 (i.e, P<0.0001). Results confirmed that variables showed positive association and a highly correlated relationship.

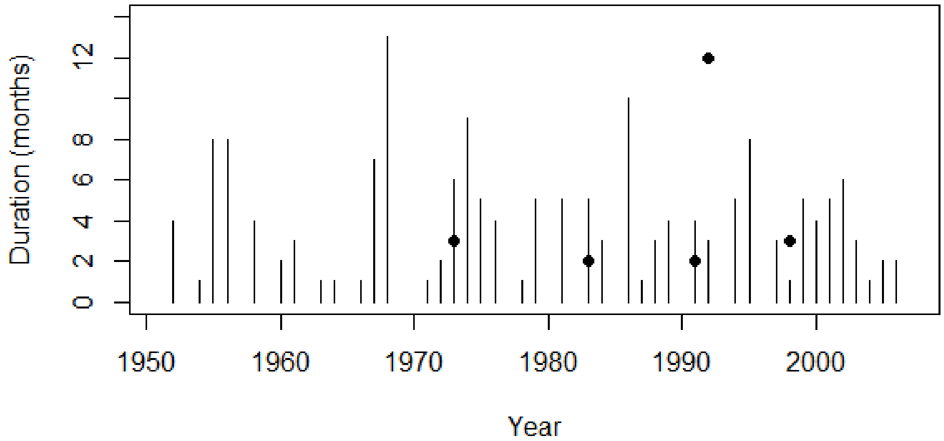

Fig. 2. Drought duration and its properties in Anuradhapura. Spikes and dots (if there is more than one drought per year) show occurrence of droughts

Table 3. Descriptive statistics of rainfall and drought characteristics for Anuradhapura

Climate variables	Statistics	Value
Precipitation	Mean annual precipitation (mm)	1298.4
	Standard deviation (mm)	298.4
	Coefficient of variation (%)	22.98
	Minimum (mm)	742.1
	Maximum (mm)	2419.2
Drought	Number of droughts	46
	Mean inter-arrival time(months)	10.13
Drought duration	Mean	4.15
	Standard deviation	2.94
	Minimum	1
	Maximum	13
Drought severity	Mean	4.44
	Standard deviation	2.93
	Minimum	1.01
	Maximum	10.95

The most informative tool for the tail dependency is ranked scatter plots for drought characteristics to identify the suitable copula [33]. Fig. 5 shows the ranked scatter for drought severity and drought duration. According to the Fig. 5 we cannot figure out any upper tail or lower tail dependency. Therefore, Gaussian and Frank are to be the best suitable copulas for the joint distribution.

3.1 Estimation of Marginal Distributions

Table 4 shows AIC, BIC and KS test values for selected distributions of drought duration and drought severity. Gamma distribution and lognormal distributions are found to be the best distribution for drought duration by AIC and BIC values (Table 4). In both distributions AIC and BIC values are as the same. Then by looking at KS values we selected the gamma distribution for the drought duration. For the drought severity we selected the best distribution as gamma distribution from AIC and BIC values.

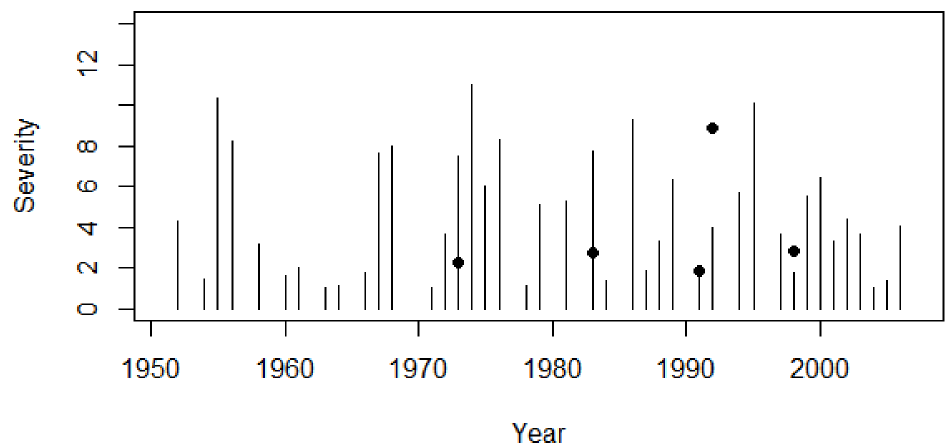

Fig. 3. Drought severity and its properties in Anuradhapura. Spikes and dots (if there is more than one drought per year) show occurrence of droughts

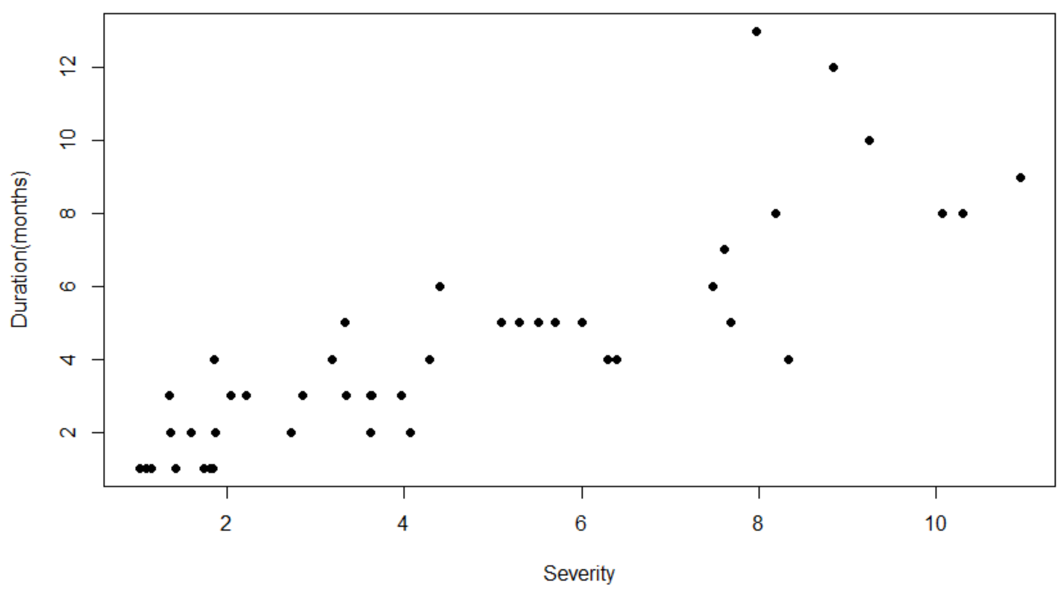

Fig. 4. Scatter plot of drought severity and drought duration

Fitted gamma distribution function with empirical distribution and CDF of for drought characteristics of histogram are shown in Figs. 6(a and b). These graphs shows good association between theoritical gamma distribution and empirical distributions. Shape parameter (α) and scale parameter (β) for drought duration and drought severity 2.128, 0.513 and 2.212, 0.498 respectively. These parameters were estimated by using maximum likelihood method.

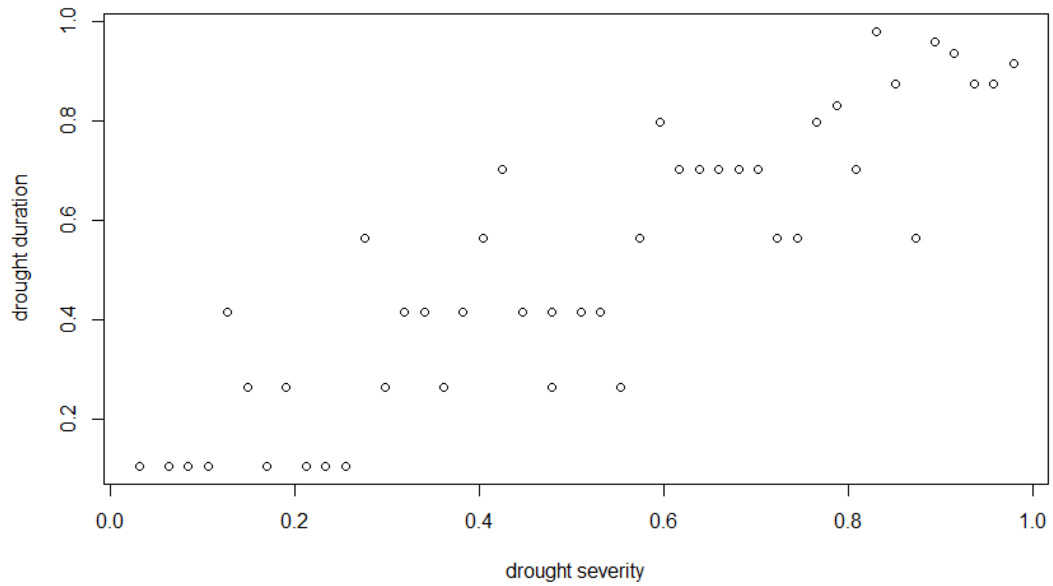

Fig. 5. Scatter plot of ranked observations of drought severity and drought duration

Table 4. Performance of different probability distributions in modeling marginal drought variables

Drought variables	Distribution	KS statistic	AIC	BIC
Duration	exponential	0.214	224.97	226.80
	normal	0.168	232.86	236.52
	lognormal	0.132	214.61	218.26
	logistics	0.145	230.59	234.25
	gamma	0.119	214.61	218.27
	weibull	0.108	216.01	219.67
Severity	exponential	0.203	231.22	233.04
	normal	0.132	232.47	236.13
	lognormal	0.108	220.39	224.05
	logistics	0.136	235.32	238.98
	gamma	0.117	219.76	223.41
	weibull	0.115	220.21	223.86

3.2 Estimation of Joint Distribution

Table 5 shows the estimated parameters, AIC and BIC values for the selected copulas. According to the Table 5, Frank and Gaussian copula are the best among other copulas' as they have low AIC and BIC values and no upper or lower tail dependency (Fig. 5). Calculated Cramer-von Mises S_n statistics for Frank copula and Gaussian copula are 0.0796 and 0.0816. Frank copula is selected as the S_n value is lesser in Frank copula than that of Gaussian copula. Here, S_n values are obtained by parametric bootstrapping and inverting the Kendall's tau.

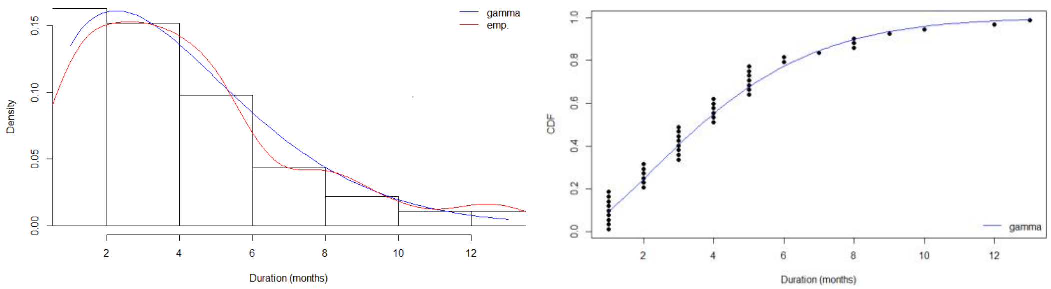

Fig. 6 (a). Theoretical and empirical distributions of drought duration

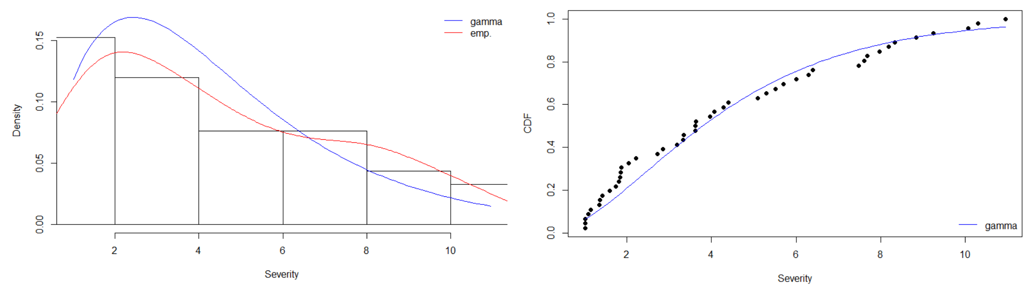

Fig. 6 (b). Theoretical and empirical distributions of drought severity

Table 5. Estimated parameters, AIC and BIC measures of the fitted copula models

Copula	Parameter	AIC values	BIC values
Gaussian	0.913	-63.537	-61.709
Clayton	5.490	-32.578	-30.749
Gumbel-Hourgaard	3.745	-52.547	-50.719
Frank	13.099	-63.626	-61.797
Joe	6.277	-22.602	-20.774

The selected gamma marginal distributions and Frank copula were used to construct the copula based joint distribution of drought duration and drought severity. Fig. 7 displays the surface plots of joint probability density function (PDF) (Fig. 7a), joint cumulative distribution function (CDF) (Fig. 7b) and contour plots of joint CDF (Fig. 7c) for Frank copula at different probability levels. The peak of the density plot at middle of the square indicates presence of strong positive dependence between drought variables.

3.3 Return Periods

Univariate return levels of drought duration and severity for return periods of 2, 5, 10, 20 and 50 years calculated according to the Eq. 4, Eq. 5 separately and are shown in the Table 6. According to data in Table 6 for calculated joint return periods for given drought duration and severity (according to Eq. 6 and Eq. 7) are shown in the Table 6.

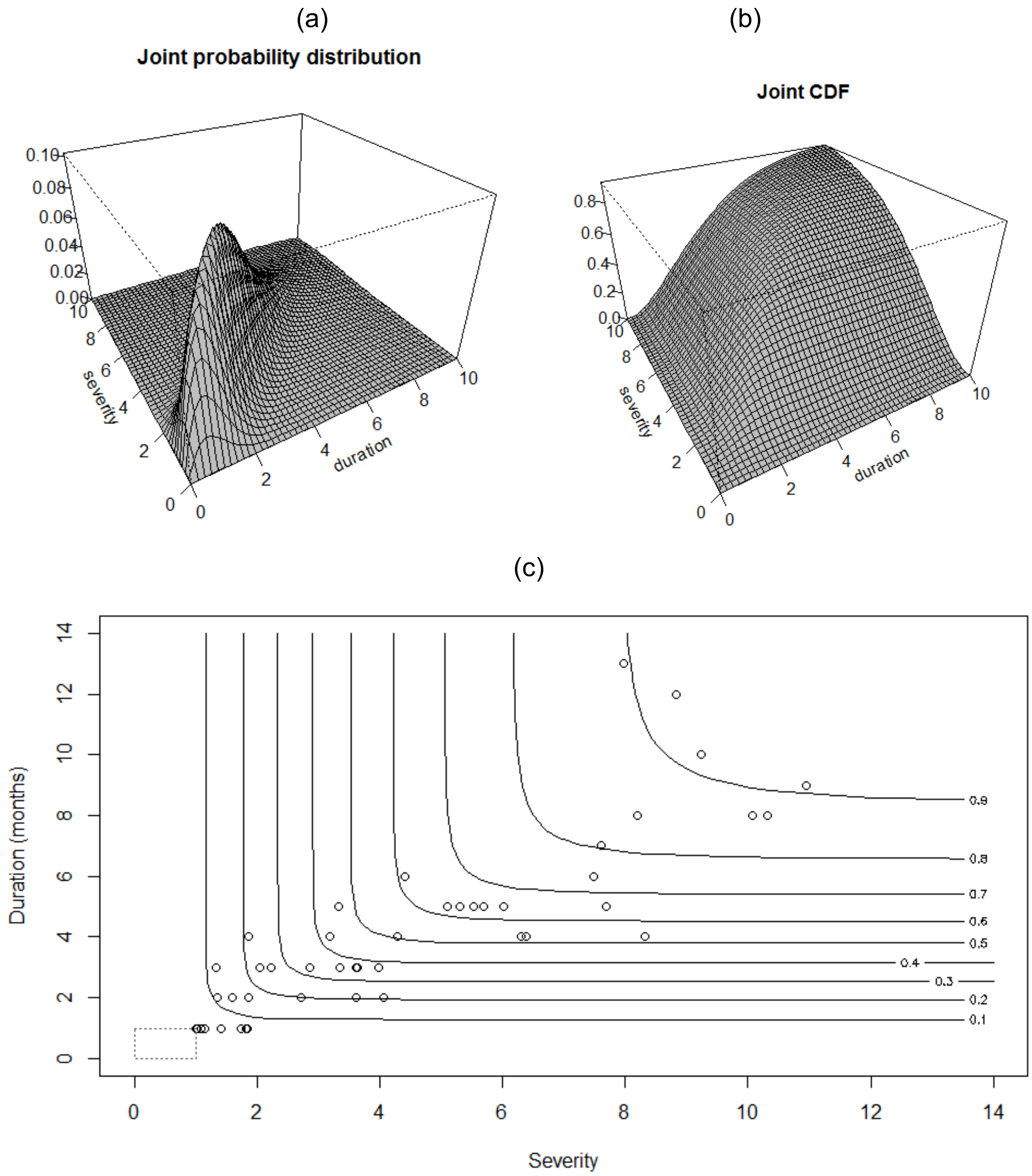

Fig. 7. (a) Joint PDF (b) Joint CDF of drought severity and dration (c) Contour plot of joint CDF superimposed on historical drought events

Table 6. Univariate return periods (years) for each drought variable

Return period	Duration (months)	Severity
2	4.1	4.4
5	6.1	7.0
10	8.4	8.9
20	10.0	10.6
100	12.2	12.9

Generally, T_{DS} (Eq. 6) has smaller return periods than that of $T_{DS}^{'}$ (Eq. 7) for given severity, duration and Table 7 confirmed it. Drought duration and severity in 1974 drought event are 9 and 10.95 respectively. The univariate return periods for this drought duration and drought severity 12.9 years and 22.8 years respectively; whereas estimated T_{DS} and $T_{DS}^{'}$ are 10.1 years and 44.2 years.

Table 7. Joint return periods (years) of the drought events for given return levels in Table 6

T_{DS} (years)	$T_{DS}^{'}$ (years)
1.8	2.3
3.6	6.3
6.8	18.6
12.2	55.6
27.5	275.2

4. CONCLUSION

In this study, we calculated 3-month SPI series from the monthly rainfall data from 1951 to 2007. The most intensive droughts were indentified in 1974 and 1976 from the SPI series. Drought characteristics, drought duration and severity were derived using SPI series. Occurrences of 46 major drought events were indentified based on drought severity and drought duration. Anuradhapura region has experienced the longest drought durations in 1968, 1986 and 1992 while the most severe droughts in 1955, 1974, 1986, 1992 and 1995. Altogether, mainly 4 droughts are identified in year 1955-1956, 1974, 1986 and 1992.

It was found that marginal distributions of drought duration and drought severity fit to a gamma distribution quite well among other distributions based on AIC and BIC statistics. For further confirmation KS test was used.

Gaussian copula and Frank Copula found to be the best copulas based on tail dependency plot and AIC, BIC, values. Frank Copula was selected as the best copula since it had the minimum AIC and BIC values as well as it fits quite well according to Cramer-von Mises S_n statistic.

The joint distribution was constructed by combining the indentified gamma marginal distributions using the Frank copula. The univariate return periods were calculated for drought duration and severity and using copula based joint distribution multivariate return periods (T_{DS} and $T_{DS}^{'}$) were calculated. The univariate return periods in 1974 drought event was identified. The calculated univariate return period for drought duration of 9 was 12.9 years and for drought severity of 10.95 was 22.8 years. A drought can be expected if drought duration ≥9 or severity ≥10.95 in once in 10.1 years. Similar drought event as occurred in 1974 can be expected in another 44.2 years. Thus based on those results we can minimize the drought risks by pre-planning and make decision against adverse affects of droughts.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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APPENDIX

Copulas

U and V are the uniformly distributed random variables varying from 0 to 1, τ is the Kendall's tau rank correlation coefficient of the correlation between X and Y.

U=F_x (X) = Cumulative Distribution Function of X

V=Fy (Y) = Cumulative Distribution Function of Y

Gumbel- Haugaard Copula

$C(u,v) = \exp\left(-[(-\log(u))^{\theta} + (-\log(v))^{\theta}]^{\frac{1}{\theta}}\right)$
; Where $\theta \in [1, \infty]$
H(x, y) =C (F_x (X), F_y(Y))
$= \exp\left(-[(-\log(F(X)))^{\theta} + (-\log(F(Y)))^{\theta}]^{\frac{1}{\theta}}\right)$
$\tau = \frac{\theta}{\theta+1}$.
; Where θ is copula parameter and τ is Kendall's tau.

Gumbel family covers only positive dependency.*

Frank Copula

The Frank Copula function is given by;

$C(u,v) = -\frac{1}{\theta}\ln(1 + \frac{(e^{-\theta u}-1)(e^{-\theta v})-1}{e^{-\theta}-1})$
; Where$\theta \in (0, \infty)$, $\theta \neq 0$
H(X, Y) = C ((F_x(X), F_y(Y))
$= -\frac{1}{\theta}\ln\,(1 + \frac{(e^{-\theta \mathrm{Fx(X)}}-1)(e^{-\theta \mathrm{Fy(Y)}})-1}{e^{-\theta}-1})$
$\tau = 1 - \frac{4[D_1(-\theta)-1]}{\theta}$; Where D_1 is the first order Debye function D_k which is defined as
$D_k(\theta) = D_k(\theta)\frac{k}{X^k}\int_0^{\theta}\frac{t^k}{e^t-1}dt$; $\theta > 0$and the Debye function D_k with the negative argument can be expressed as $D_k(-\theta) = D_k(\theta) + \frac{k\theta}{k+1}$

Where θ is copula parameter and τ is Kendall's tau.

Frank family covers the positive and the negative dependency.

Clayton Copula

The Clayton copula function is,

$C(u,v) = max[(u^{-\theta} + v^{-\theta} - 1)^{\frac{-1}{\theta}}, 0]$
; Where $\theta \in [-1, \infty]\backslash\{0\}$
$$\tau = \frac{\theta}{\theta + 2}$$

Where θ is copula parameter and τ is Kendall's tau.

Clayton family covers only the positive dependency.

Joe Copula

The Joe Copula function is given by;

$$C(u,v) = 1 - ((1-u)^{\theta} + (1-v)^{\theta} - (1-u)^{\theta}(1-v)^{\theta})^{\frac{1}{\theta}}$$
; Where $1 \leq \theta < \infty$

$$\tau = 1 + \frac{4}{\theta^2}\int_0^1 tlog(t)(1-t)^{\frac{2(1-\theta)}{\theta}}dt$$

Where θ is copula parameter and τ is Kendall's tau.

Joe copula covers only the positive dependency.

Gaussian Copula

The Gaussian Copula function is given by;

$$C(u,v) = \int_{-\infty}^{\emptyset^{-1}(u)}\int_{-\infty}^{\emptyset^{-1}(v)} \frac{1}{2\pi(1-\theta^2)^{1/2}} exp\left\{-\frac{x^2 - 2xy\theta + y^2}{2(1-\theta^2)}\right\} dxdy$$

Where θ is the parameter of the copula, and $\emptyset^{-1}$ $(\cdot)$ is the inverse of the standard univariate Gaussian distribution function

$$\theta = \sin\left(\frac{\pi}{2}\tau\right)$$

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