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# Modeling of Passenger Delays at a Multimodal Transport Access Point of a University Campus

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### Author's contribution

HMA designed the study, performed the statistical analysis, wrote the protocol and the first draft of the manuscript. He managed the analyses of the study and the literature searches. The author has read and approved the final manuscript.

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## ABSTRACT

This study models the egress behaviour of passengers at a transportation access point with multimodal service with the objective of evaluating the delays associated with their departure from the access area. Passengers arriving at this access area have the taxi mode, minibus mode and the bus mode to choose from. A discrete choice model with the utility function based on mode fare and transit time was used to describe the egress behaviour of passengers. The resulting queues at the access area for the three modes were transient in character with time-varying arrival  $\lambda(t)$ , time varying service rates  $\mu(t)$ , a truncated bulk service single channel with first-in-first out queue discipline. The queuing systems were described by M/M<sup>k</sup>/K/FIFO with random clearing of all queued taxi and minibus passengers. A computer simulation package using Runge-Kutta numerical methods was used to solve the set of linear differential equations developed for the three transport modes serving the area. The simulation models were tested with field data collected from the access area on all trips made during the study period and were analysed at fifteen minute time intervals. The probabilities of delay predicted by the simulation models agreed closely with field observations. The chi-squared test indicated that there were no significant differences between the models and the field values at the 95% significant level. Passengers at the access area experienced an average delay of 16.4minutes, 12.4minutes and 23.5minutes for the taxi, minibus and bus modes respectively. The methodology would need to be tested with data from other areas with similar demand characteristics to build confidence into it.

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## DEFINITIONS

$\Pi_T$  = Expected full price for taxi mode;  $\Pi_{MB}$  = Expected full price for minibus mode;  $\Pi_B$  = Expected full price for bus mode;  $W_T$  = Taxi fare;  $W_{MB}$  = Minibus fare;  $W_B$  = Bus fare;  $V_T$  = Value of time of taxi passengers;  $V_{MB}$  = Value of time of minibus passengers;  $V_B$  = Value of time of bus passengers;  $E[T_T]$  = Expected taxi transit time including waiting for taxi, assuming no modal switch;  $E[T_{MB}]$  = Expected minibus transit time including waiting for minibus, assuming no modal switch;  $E[T_B]$  = Expected bus transit time including waiting for bus;  $\Pi_T^*$  = Conditional expected full price for taxi mode;  $\Pi_{MB}^*$  = Conditional expected full price for minibus mode;  $\Pi_B^*$  = Conditional expected full price for bus mode;  $\lambda_T(t)$  = Mean taxi passenger arrival rate, at time t;  $\lambda_{MB}(t)$  = Mean minibus passenger arrival rate, at time t;  $\lambda_B(t)$  = Mean bus arrival rate, at time t;  $1/\bar{\tau}_T$  = Mean round trip time per taxi;  $1/\bar{\tau}_{MB}$  = Mean round trip time per minibus;  $1/\bar{Y}_B$  = One way mean bus trip time for switching taxi passengers;  $1/\bar{Y}_{MB}$  = One way mean minibus trip time for switching taxi passengers;  $C_T \sim T$  = expected taxi service rates;  $C_{MB} \sim MB$  = expected minibus service rates;  $P_n(t)$  = Probability of n passengers in the system at time t.

## 1. INTRODUCTION

Transportation access offers considerable challenges to airport, railway and university campuses operators. Often passengers need to transfer to and from surface ground facilities to be in time to connect to other destinations or to attend scheduled meetings and lectures. In particular where multimodal choices are available transport service providers compete with each other for patronage. Facility planners and managers would be interested in passenger mode choice preferences as well as the demand for each mode. Passengers need not only choose between modes but would worry about the cost vis-à-vis the delay associated with each mode. The managers of the facility will again need to ensure that queues arising from the demand for each mode are serviced quickly with a view to raising the level of service.

This study relates to a multimodal transportation service point in a university campus in Northern Nigeria that operates two campuses and needs to link them with ground transportation services. Prospective passengers arriving at the access area either join the mode of their choice or wait for service if it is not immediately available. Thus a queuing situation arises. The interesting feature about the access area is that passengers arriving at the access area opt for the taxi and minibus modes in view of their lower loading and turn-around times. The surge of passenger demand for these modes often overwhelms the capacities of the two modes combined. Whereas passengers of the bus mode patiently await scheduled bus arrival times, passengers not serviced by their preferred modes (taxi and

minibus) switch to the bus mode upon the arrival of a bus. Thus the long queues from the taxi and minibus passengers are cleared at random times depending upon the arrival of a bus. It is therefore intended to investigate the modal access behaviour of passengers at the access area and its associated delays to passengers. Access behaviour at airports and rail stations have been studied and reported but that of universities with two campuses located many kilometers apart and with strong intercampus activities have yet to be reported. This paper therefore focuses on transportation access between university campuses with a view to improving transportation accessibility between them. We shall therefore seek to evaluate the delay of passengers served at the access area and generate policy options on effective solution to the problem of transporting passengers between the two campuses of the university. The rest of the paper is organised as follows: the next section reviews literature on the subject. In section 3, we present the data collection procedure used and in section 4 we present the modeling effort. Model calibration and validation will be presented in section 5 while the conclusions are in section 6.

## 2. LITERATURE REVIEW

Queuing systems are described by the arriving pattern of passengers, the service pattern of the facility and a queue discipline. There are various configurations for the arrival pattern, service patterns and the rules of behavior that characterize queuing systems. These ranges from stationary to non-stationary arrival and service patterns and a queue discipline that includes First-in-First-out, (FIFO), Last-in-First-Out (LIFO) and a host of others such as pre-emptive and non-pre-emptive queues Gross and Harris (1985).

Queuing systems are in essence, arrival (birth) and departure (death) processes in which arriving passengers demand for service at a facility and depart upon being served. They form a special type of Markovian models in which the conditional probability of finding the system in state  $j$  at time  $(t + \Delta t)$ , given that it occupied state  $i$  at time  $t$  depends only on the starting state and the transition time  $\Delta t$ . Queuing situations in which there are time-varying arrivals and departures are different in character from queues in which the arrival and service rates are constant. In the former, it may be vital to know how the system responds to sudden changes in arrivals or a breakdown of servicing capability. These present daunting problems in real time problems where queues form. In the case of the latter, the arrival and departure rates are postulated as in equations 1 and 2 below.

$$\lambda_n(t) = \lambda_n \text{ for all } t \quad (1)$$

$$\mu_n(t) = \mu_n \text{ for all } t \quad (2)$$

Where  $\lambda_n$  and the  $\mu_n$  are arrival and service rates and remain constant for all values of  $t$ . It is of interest to evaluate the state of the system to see if service rates are commiserate with the demand. (i.e. arrival rates) and for a stationary system, the state will be zero if no passengers arrive or if a single passenger arrives and is served in  $(t, t+\Delta t)$ . The probability of these changes is

$$P_{00}(\Delta t) = (1 - \lambda_0 \Delta t) + \lambda_0 \Delta t \mu_0 \Delta t \quad (3)$$

Also the system will move from state zero to state one if a single passenger arrives and none are served as follows:

$$P_{01}(\Delta t) = \lambda_0 \Delta t (1 - \mu_0 \Delta t) \quad (4)$$

Multiple arrivals will take the system from zero to any state  $j > 1$ . But in stationary systems the probability of change in arrival rates for small  $\Delta t$  is zero.

Thus

$$P_{0j}(\Delta t) = 0 \quad j > 1 \tag{5}$$

It is normal to find higher order states during the course of a day at a ground access area. Passenger balking, renegeing and switches between parallel queues make the number waiting for services to increase or decrease according to prevailing passenger behaviour. Thus the system can remain in state  $i$  with no arrivals or departures, move from  $i$  to  $i-1$  with a departure, or move from  $i$  to  $i+1$  when an arrival occurs in the interval  $(t, t+\Delta t)$ . These are stated as in equations 6 to 8.

$$P_{i,i-1}(\Delta t) = \lambda_i \Delta t (1 - \mu_i \Delta t) \tag{6}$$

$$P_{ii}(\Delta t) = \lambda_i \Delta t \mu_i \Delta t + (1 - \lambda_i \Delta t)(1 - \mu_i \Delta t) \tag{7}$$

$$P_{i,i+1}(\Delta t) = \lambda_i \Delta t (1 - \mu_i \Delta t) \tag{8}$$

As is the case with stationary systems multiple arrival or departures are again not admissible, hence

$$P_{ij}(\Delta t) = 0 \tag{9}$$

The complete the transition matrix for initial higher order states can be written as

$$\begin{pmatrix} \lambda_0 \mu_0 \Delta t^2 + (1 - \lambda_0 \Delta t) & \lambda_0 \Delta t (1 - \mu_0 \Delta t) & \dots & \dots \\ \lambda_1 \Delta t (1 - \mu_1 \Delta t) & \lambda_1 \mu_1 \Delta t^2 + (1 - \lambda_1 \Delta t)(1 - \mu_1 \Delta t) & \lambda_1 \Delta t (1 - \mu_1 \Delta t) & \dots \\ 0 & \lambda_2 \Delta t (1 - \mu_2 \Delta t) & \lambda_2 \mu_2 \Delta t^2 + (1 - \lambda_2 \Delta t)(1 - \mu_2 \Delta t) & \dots \end{pmatrix} \tag{10}$$

In vector notation,

$$P(t + \Delta t) = P(t)T \tag{11}$$

Where T is the transition matrix and multiplication leads to the following set of equations (12) and (13).

$$P_0(t + \Delta t) = P_0(t) [\lambda_0 \mu_0 \Delta t^2 + (1 - \lambda_0 \Delta t)] + P_1(t) [\lambda_1 \Delta t (1 - \mu_1 \Delta t)] \tag{12}$$

$$P_n(t + \Delta t) = P_{n-1}(t) [\lambda_{n-1} \Delta t (1 - \mu_{n-1} \Delta t)] + P_n(t) [\lambda_n \mu_n \Delta t^2 + (1 - \lambda_n \Delta t)(1 - \mu_n \Delta t)] + P_{n+1}(t) [\lambda_{n+1} \Delta t (1 - \mu_{n+1} \Delta t)] \tag{13}$$

By transferring  $P_n(t)$  to the left side, dividing by  $\Delta t$  and taking the limit as  $\Delta t$  tends to zero, we are led to the set of differential difference equations:

$$\frac{dP_0(t)}{dt} = -\lambda_0 P_0(t) + \mu_1 P_1(t) \tag{14}$$

$$\frac{dP_n(t)}{dt} = -(\lambda_n + \mu_n) P_n(t) + \lambda_{n-1} P_{n-1}(t) - \mu_{n+1} P_{n+1}(t) \tag{15}$$

For n > 1

As we shall see in subsequent sections of this paper, the arrival and service rates are time-dependent. Incorporating these into equations 14 and 15 lead us to the non-homogenous set.

$$\frac{dP_n(t)}{dt} = -\lambda_0(t) P_0(t) + \mu_1(t) P_1(t) \tag{16}$$

$$\frac{dP_n(t)}{dt} = -[\lambda_n(t) + \mu_n(t)] P_n(t) + \lambda_{n-1}(t) P_{n-1}(t) \tag{17}$$

$$+ \mu_{n+1}(t) P_{n+1}(t)$$

For n > 1

Equations 16 and 17 enable us to obtain the state of the system at any time (t) and to evaluate the performance of the system at the access area. It is realistic to assume that bulk queues could form when passengers mainly students are discharged from lecture sessions or large academic gatherings. On the service side, queued passengers can also be served in groups not larger than the capacity of the available modes of transport. Three situations therefore arise from the limitation of the vehicle sizes. Passengers less than the capacity of the vehicles could be served, passengers up to the capacity of the vehicles and passengers more than the capacity of the vehicles. The resulting equations for the three conditions are stated in equations 18 to 20.

$$\frac{dP_0(t)}{dt} = -\lambda(t) P_0(t) + \mu_1(t) P_1(t) \tag{18}$$

$$\frac{dP_n(t)}{dt} = \lambda_{n-1}(t) P_{n-1}(t) - [\lambda_n(t) + \mu_n(t)] P_n(t) + \mu_{n+1}(t) P_{n+1}(t) \tag{19}$$

0 < n < k

$$\frac{dP_k(t)}{dt} = \lambda_{k-1}(t) P_{k-1}(t) - \mu_k(t) P_k(t) \tag{20}$$

n = k

Where long term behavior of the queuing system is of interest steady-state solutions are invoked, in which case, the derivatives of P<sub>n</sub>(t) with respect to time must be zero. However our interest lies in the short term perturbations caused by the time-varying arrivals and short interruptions in service due to unavailability of vehicle units.

Ground transportation access is an important component of a comprehensive transport system. They provide the linkage between intercity modes such as air travel and rail. Indeed the service levels of these ground access areas influence the access and egress modes of passengers for the major travel modes Chieh-Hua et al. (2012). Even though surface access to airports and rail stations may rely heavily on the private car, the modal choice of passengers often encourage operators to allow multimode service to these facilities. Where multimodal ground access points are integrated, passengers may shift from one mode to another particularly if the waiting time for one mode becomes excessive. Also the level of service offered by each mode should be a balance of the passenger demand and the available service units. In any case a queue may ensue if service is not immediately available.

Queuing systems abound in practice and are applied in diverse human spheres where demand for service exceeds the available number of service units. In transportation, queuing systems have been applied to congestion studies Hai and Qiang (1998), at toll booths Zhang and Shi (2009), gap acceptance studies, queue behaviour of passenger departures at an airport Curry et al. (1972). On traffic signal control Andy and Hong (2007) have applied queuing methods in sensitivity analysis of signal control with delay derivatives. Delay estimates of mixed traffic flow at signalized intersections using queuing theory have been investigated in China Su et al. (2009). Traffic studies at bottleneck areas particularly at on and off ramps and their metering have also been analysed using queuing theory as in Zhang and Wei (2008) and the interaction of units in a queuing system such as traffic flow Dirk, Martin and Arne (2006) etc. Traffic flow disturbances are common occurrences on road networks where traffic incidents cause prolonged delays to motorists due to sections of highways being temporarily taken out of service. Baykal-Gursoy and Xiao (2004) applied the M/M/ system to study incident disturbances to traffic flow. Similarly, Baykal-Gursoy and Duan (2006) employed M/M/C queue systems to examine traffic states subjected to highway incidents. On-street parking behaviour of drivers affects throughput of vehicles and cause considerable interruptions to flow. This has been studied by Ibeas et al. (2009) by using the M/M/ queue model. Whereas the interest, in the use of the aforementioned queuing models have been in analyzing steady-state and stationary queuing systems, in this paper we examine the transient behaviour of queuing systems from a multimodal access point for university inter-campus transfer.

### **3. DATA COLLECTION**

Two stages of data collection were designed to capture the data required to conduct this study. Stage one involved the transport modes plying the two campuses. In this phase automatic detectors were installed to record the vehicle data such as arrival and departure times and the class of the vehicle. Stage two involved making observations about passenger arrivals to the access area; this facilitated the assessment of the passenger demand. The other component of passenger observation is the number of passengers disembarking at the access area. This facilitated the estimation of the delay of passengers carried by each mode. The manner of passenger arrival and service at the area are presented in the next section. The data collection exercise lasted for eight weeks and the step-wise arrival rates and service rates at 5-minute intervals were extracted as inputs to the queuing models for the three modes.

## 4. MODEL FORMULATION

Modeling of queuing systems generally involves two steps. These are selection of a model to describe the observed data and the specification of the parameters of the model. To begin however, it is necessary to decide whether one can use a steady-state model or go to a time varying representation of a queuing system, i.e. to test whether the arrival rate and service rate fluctuate over time. To do this, one could employ the test for homogeneity of variance such as Bartlett's test. The test results for the three modes at the 95% significant level lead to the rejection of the hypothesis that the arrival rate is constant over the groups of passengers at the access area. Thus for the three modes a stationary queuing model will be inappropriate as such only transient solutions would have meaning. However, even if the arrival rate  $\lambda(t)$  and the service rate  $\mu(t)$  are smooth well behaved functions it may be impossible to find a compact expression for  $P_n(t)$ . Again when these rate functions are described by curves developed from empirical data, a complete formal solution is totally out of reach. Koopman (1972) suggests that one approximates the time-varying arrival and service rate functions by a series of step functions. If these functions are changing very slowly one can tolerate a fairly long step size but if they change rapidly, shorter steps are necessary to achieve a reasonable approximation. Within the time period covered by a single step, the arrival and service rates are assumed constant. The finite set of differential equations with constant coefficients are then developed and solved step-wise. A second alternative is to find the solution within each time interval using numerical methods with constant for solving systems of linear differential equations with constant coefficients. Two of such are the predictor-corrector method and the Runge-Kutta method. Hartman (1972) state that the former method requires 50 percent longer execution time than the Runge-Kutta method for the same problem. The Runge-Kutta method thus will be used in this work. Therefore we have a time-varying demand  $\lambda(t)$ , exponential service  $\mu(t)$  and a first-in first-out (FIFO) queue discipline.

### 4.1 Model Development

With a FIFO queuing system having a time-varying arrival rate  $\lambda(t)$  with exponential service distribution  $\mu(t)$ , the taxi and the minibus modes are described by the exponential bulk service system. In this system service takes place on groups which cannot be larger than  $c$ . If less than  $c$  customers are present all are served. The state of the system is defined by the number of customers in the queue and the number of taxis and minibuses available for service. Incorporating the buses into the system requires that the bulk service M/M/K system be extended to a model with random clearing of all the queued taxi and minibus passengers. Random clearing is a form of involuntary reneging which clears the entire queue as distinguished from the single passenger reneging attribute of customers.

It is assumed that passengers plan their mode of travel on the basis of the expected full price  $\Pi$  which is defined as follows.

$$\Pi_T = W_T + V_T E[T_T] \quad \text{Taxi mode} \quad (21)$$

$$\Pi_{MB} = W_{MB} + V_{MB} E[T_{MB}] \quad \text{Minibus mode} \quad (22)$$

$$\Pi_B = W_B + V_B E[T_B] \quad \text{Bus mode} \quad (23)$$

From field observation it is assumed that  $E[T_T] < E[T_{MB}] < E[T_B]$  and  $W_T > W_{MB} > W_B$ . Passengers who value time will give preference to the taxi and minibus modes respectively since

$$\begin{aligned} \Pi_T &\leq \Pi_{MB} \\ \Pi_T &\leq \Pi_B \\ \Pi_{MB} &\leq \Pi_B \end{aligned} \tag{24}$$

And the value of time will satisfy

$$v_1^* = \frac{W_T - W_B}{E[T_B] - E[T_T]} \tag{25}$$

$$v_2^* = \frac{W_{MB} - W_B}{E[T_B] - E[T_{MB}]} \tag{26}$$

$$v_3^* = \frac{W_T - W_{MB}}{E[T_{MB}] - E[T_T]} \tag{27}$$

To see if a queued taxi and minibus passenger will switch to a bus if one arrives before a taxi or minibus, we consider the conditional expected full price given that the passenger is the  $n^{\text{th}}$  passenger in either of the taxi or minibus queues, that is:

$$\Pi_T^* = W_T + V_T E[T_T / n] = W_T - V_T [1 / \sim_T + n / c_T \sim_T] \tag{28}$$

$$\Pi_{MB}^* = W_{MB} + V_{MB} E[T_{MB} / n] = W_{MB} - V_{MB} [1 / \sim_{MB} + n / c_{MB} \sim_{MB}] \tag{29}$$

It is also possible for a queued taxi passenger to switch to a minibus if one arrives before a taxi, when a bus is not available. If a bus is ready to depart, then the conditional full prices are

$$\Pi_B^* = W_B + V_T [1 / Y_B] \text{ for taxi passengers} \tag{30}$$

$$\Pi_{MB}^* = W_{MB} + V_T [1 / Y_{MB}] \text{ for taxi when a minibus is departing} \tag{31}$$

$$\Pi_B^* = W_B + V_{MB} [1 / Y_B] \text{ for minibus passengers} \tag{32}$$

The largest number  $n_T^*$  of passengers who would remain in the taxi queue, given that a bus is departing immediately is

$$n_T^* = \frac{[W_B - W_T + V_T / Y_B - V_T / \sim_T]}{V_T} c_T \sim_T \tag{33}$$

$$n_T^{**} = \frac{[W_{MB} - W_T + V_T / Y_{MB} - V_T / \sim_T]}{V_T} c_T \sim_T \tag{34}$$

Similarly for the minibus mode

$$n_T^* = \frac{[W_B - W_{MB} + V_{MB} / Y_B - V_{MB} / \sim_{MB}]}{V_{MB}} C_{MB} \sim_{MB} \quad (35)$$

It is then possible to define  $n_T$  and  $n_{MB}$ , the maximum number of taxi and minibus passengers who will remain in the queue given that a university bus is leaving as

$$n_T = \begin{cases} n_T^* + [n_T^{**}] & \text{for } \begin{cases} n_T^* + n_T^{**} \geq 0 \\ n_T^* < 0 \\ n_T^{**} < 0 \end{cases} \\ 0 & \end{cases} \quad (36)$$

$$n_{MB} = \begin{cases} [n_{MB}^*] & \text{for } \begin{cases} n_{MB}^* \geq 0 \\ n_T^* + n_T^{**} < 0 \end{cases} \\ 0 & \end{cases} \quad (37)$$

Where  $[n_T^*], [n_T^{**}]$  and  $[n_{MB}^*]$  denotes the largest integer less than or equal to  $n_T^*, n_T^{**}$  and  $n_{MB}^*$  respectively. In the subsequent analysis it is assumed that  $n_T = 0$ , which is implied whenever

$$W_T - W_{MB} \geq V_T [1 / Y_B - 1 / \sim_T] \quad (38)$$

$$W_T - W_{MB} \geq V_{MB} [1 / Y_{MB} - 1 / \sim_T] \quad (39)$$

and

$$W_{MB} - W_B \geq V_{MB} [1 / Y_B - 1 / \sim_{MB}] \quad (40)$$

Whenever all the three modes are immediately available,  $n_T$  and  $n_{MB}$  will be zero when the fare premium on the taxi exceeds the value of the time saved by taking the taxi instead of the bus, and the minibus. This is the case for the minibus also. This limiting case is interesting because it focuses attention on the convenience of taxis and minibuses versus the buses rather than their enroute speed.

## 4.2 Bulk Service Solution

The equations for the bulk service problem become:

$$\frac{dP_o(t)}{d(t)} = -\lambda(t) P_o(t) + \mu_1(t) P_1(t) \quad (41)$$

$$\frac{dP_n(t)}{d(t)} = \lambda_{n-1}(t) P_{n-1}(t) - [\lambda_n(t) + \mu_n(t)] P_n(t) + \mu_{n+1}(t) P_{n+1}(t) \text{ for } 0 < n < k \quad (42)$$

$$\frac{dP_k(t)}{d(t)} = \lambda_{k-1}(t) P_{k-1}(t) - \mu_k(t) P_k(t) \quad (43)$$

When the system is truncated at  $n = k$ .

Equation (41) arises when an empty system is considered. Equation (42) represents the system when there are  $n$  of the possible  $k$  passengers requiring service at the access area. Finally, equation (43) represents the system when all  $k$  passengers are present. The equations are too complex for analytical manipulations hence the resort to computer simulations. The simulation process as well as the decision criteria for each transport mode is shown in the flow chart of Fig. 1.

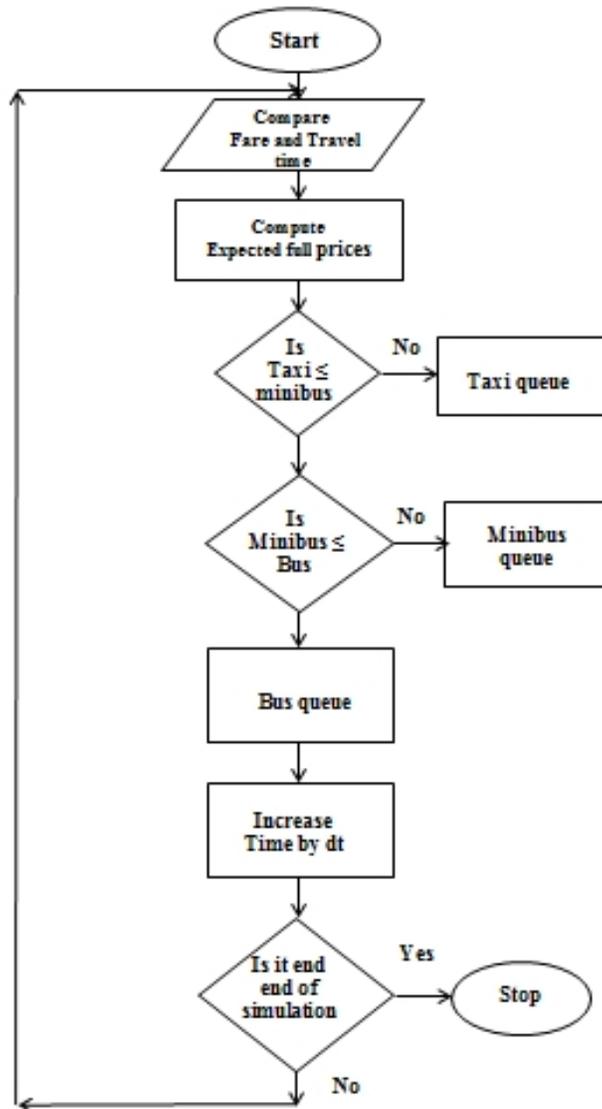


Fig. 1. Flow Chart for the Queuing Process at the Access Area

The queues arising from the three modes are analysed using the systems of equations 41 to 43. The equations are ordinary differential equations with variable coefficients. The fourth order Runge-Kutta method is used to solve the finite set of differential equations. The fourth order method requires four evaluations of the first derivative to obtain a Taylor's series

approximation through the terms of order  $h^4$ . The time-varying arrival rates and time-varying service rates at 5-minute intervals were used as inputs to the numerical evaluation of the queuing systems. The performance measures of the truncated time-varying queue are all related to the state probability distribution. From that distribution the program calculates the expected number in the queue, the expected total passenger minutes delay time in that interval, the expected number of rejects i.e. passengers who cannot gain access to the system because of saturation.

## 5. RESULTS AND DISCUSSION

The probabilities of delay as obtained from field observations for the three modes were compared with the model results and are shown in Figs. 2 to 4. The probabilities of delay for the taxi mode are highest from 12noon to 2.00pm. For the minibus mode the delays are highest from 10.30am to 12.30pm.

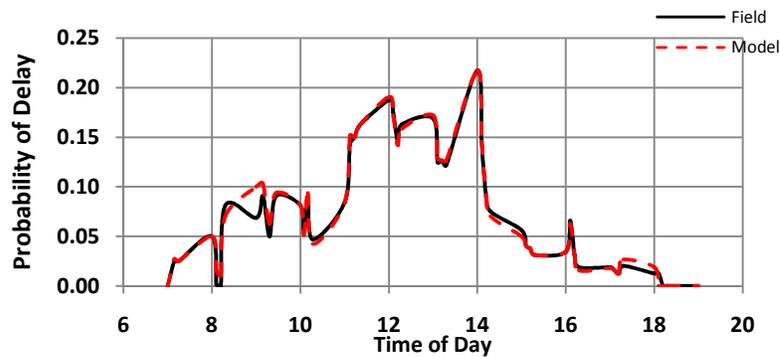


Fig. 2. Probabilities of Delay: Taxi mode

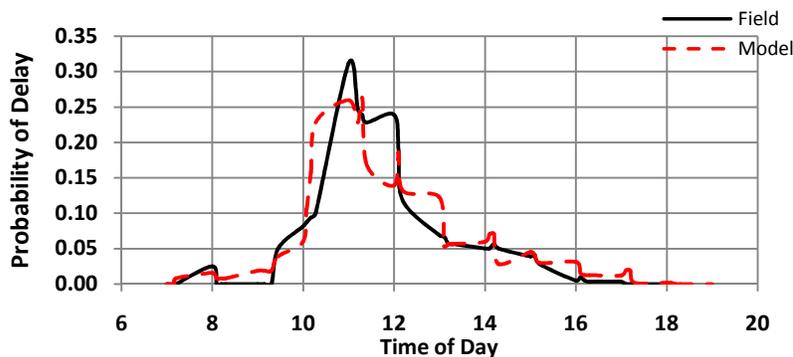
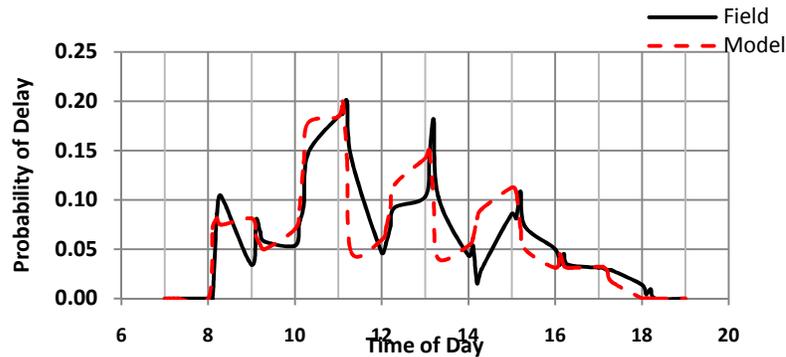


Fig. 3. Probabilities of Delay: Minibus Mode



**Fig. 4. Probabilities of Delay: Bus Mode**

There are three distinct delay periods for the bus made, from 10.30 to 11.45am, then from 12.30 to 1.30pm and from 2.30pm to 4.30pm. Departures for all passengers at the access area are associated with some delays for each mode. The taxi mode having the highest expected full price operated well until after 10am when passengers who value their time shift to the minibus mode when one arrives before a taxi or a bus. Up to 10am taxi passengers experience more delays than other modes thus encouraging switching from taxi to these modes. Minibus passenger delays are higher from 10 am and peaks at 11.00am. At this point, switching to the bus mode becomes visible. Minibus passenger delays reduce continually from then on until the demand and service rates afford a high level of service. The delay patterns for the bus mode show three peaks. This suggests the intermittency of the bus arrival times. The highest delay coincides with when taxi and minibus passenger switches are intense. If a bus arrives when all modes are available, passengers stick to their mode of choice and delays dropped until surges of passenger arrivals overwhelms the capacities of the taxi and minibus modes. Switching resumes again and the cycle continues. Comparison of the field and model results for all the parameters of delay are shown in Table 1.

**Table 1. Validation of simulation model**

Parameter	Mode	Field Value	Model Value	Differences (%)
Probability of Delay	Taxi	0.069	0.112	+4.3
	Minibus	0.073	0.069	-0.4
	Bus	0.064	0.088	+2.4
Average Delay(min)	Taxi	12.514	12.439	-7.5
	Minibus	16.337	16.401	+6.4
	Bus	23.518	23.462	-5.6
Cumulative Delay (min)	Taxi	660.278	662.345	+6.7
	Minibus	443.371	444.432	+6.1
	Bus	1132.569	1231.628	+5.9

The model results predicted the field values reasonably well. The chi-squared test indicated that there were no significant differences between the models and the field values at the 95% significant level. The probability of delay is highest for the bus, followed by the taxi and then the minibus mode. The average delays experienced by passengers are 12.514minutes

for the taxi mode, 16.337minutes for the minibus mode and 23.518minutes for the bus mode. Several reasons explain why the bus has the longest delay of its passengers. First, switching to the bus mode by other passengers add to the demand for the bus mode. Secondly, buses have the highest loading and disembarking times. This increases the turnaround time. Thirdly buses do not arrive to the access area empty in anticipation of queues of passengers; rather they wait to load passengers from the other campus. This increase their turnaround time considerably. Taxis and minibus modes arrive empty at peak periods to clear queues of passengers.

The minibus passengers had the lowest cumulative delay. Since the expected full price for the taxi mode is less than the taxi mode switches from the minibus mode to the taxi mode are uncommon. Minibus passengers switch only to the bus mode. The present level of demand for the three modes are 1467, 413, and 187 passengers. The number of trips required to meet the demand are 55 for the bus mode, 35 for the minibus mode and 137 for the taxi mode. In view of charter services offered by operators of the taxi mode fewer passengers were carried than would have been the case. This contributes to the switching behaviour of the taxi mode passengers

## **6. CONCLUSIONS**

The queuing system analysed in this paper is a time-varying demand, exponential bulk service and First-in First-out queue discipline.

The delays suffered by passengers for all the modes of transport are excessive. For passengers requiring catching scheduled lectures in the other campus, they will spend considerable time waiting for transportation. If one adds the trip times to the delays in waiting for transportation considerable lectures periods could be lost.

Delays are more pronounced from 11.am to 2.00pm. The delays for the taxi passengers are caused partly by operators offering charter services and thus not able to clear the taxi queues. Taxi charter services should be introduced to serve passengers who opt for them. This should improve taxi mode level of service at the access area.

The increased demand on the minibus mode is due to switching by the taxi passengers. More regular minibus schedules say every 15minutes will curtail switching of taxi passengers to the minibus mode as well as minibus passengers that will switch to the bus mode.

The buses offer the last resort to passengers who wish to depart the access area to catch lectures at the other campus. Failure of this mode to accommodate its own passengers and the switches from the taxi and the minibus passengers would mean that lectures will be missed altogether. The bus should be scheduled more frequently during peak times to clear the queues and minimize delays.

The model developed should be tested with data from other areas with similar demand characteristics to verify its transferability.

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## COMPETING INTERESTS

Author has declared that no competing interest exists.

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