

## The Effects of Omitted Variable on Multicollinearity in Hierarchical Linear Modelling

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### Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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## Abstract

This study investigates the impact of violation of the assumption of the hierarchical linear model where covariate of level – 1 collinear with the correct functional and omitted variable model. This was carried out via Monte Carlo simulation. In an attempt to achieve this omitted variable bias was introduced. The study considers the multicollinearity effects when the models are in the correct form and when they are not in the correct form. Also, multicollinearity test was carried out on the data set to find out whether there is presence of multicollinearity among the data set using Variance Inflation Factor (VIF). At the end of the study, the result shows that, omitted variable has tremendous impact on hierarchical linear model.

*Keywords: Hierarchical; monte carlo; multicollinearity; omitted variable; linear model.*

## 1 Hierarchical Linear Model

Hierarchical Linear Modelling (HLM) is a statistical technique that is used for analyzing data in a clustered or “nested” structure, in which lower-level units of analysis are nested within higher-level units of analysis. Some sublevels are nested within another sublevel which is equally nested in the main levels. The wide spread of the

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application of hierarchical levels data was due to the works of (Smith 1973) [1, 2]. Hierarchical linear modelling has been applied in various fields like education, social works, health, in conjunction with approaches such as covariance component modelling, multilevels modelling, fixed and random effect modelling, and mixed level modelling [2].

Most researchers in the fields such as psychology, sociology, health and education are frequently confronted with data that are hierarchical in nature. In longitudinal research, repeated observations are nested within individuals (i.e.units) and these individuals are nested within groups. The pervasiveness of hierarchical data has led to the development of many statistical methods which are hierarchical linear modelling (HLM) [3], multi-level modelling, mixed linear modelling, or growth curve modelling. Various methodologies for hierarchical linear modelling have been proposed by Aitkin and Longford [4], DeLeeuw and Kreft [5], Goldstein [6], Mason, Wong and Entwisle [7], Raudenbush [8], and Raudenbush and Bryk [9]. All their proposed methods were for estimating effects within hierarchical linear modelling. These techniques have helped researchers to model hierarchical data at several levels of aggregation thereby addressing issues of aggregation bias, efficient estimation of effects, and individual by setting interactions [8].

The complexity of these hierarchical methods leads to misuse and confusion, which stands as barriers to applied researchers. The assumptions underlying the hierarchical linear models are similar to the assumptions underlying ordinary least squares regression estimation such as: linear relationships, homoscedasticity and normal distribution of the disturbance error. In ordinary least squares regression, it is established that violations of these assumptions lead to highly inaccurate parameter estimates or large standard errors particularly when the sample size is too small, but it can only be efficient when the sample size is large; that is when the law of large number is applied [10].

### 1.1 Limitation of the Study

Owing to financial constraints and time, simulated data was used in this study.

## 2 Methodology

### 2.1 The Omitted Model (the O-model) (mis-specified)

The omitted variable model, the O-model, derived by omitting one of the level-1 predictors (i.e. $X_3$ ) of the T-model. This model is as follows:

$$\begin{aligned} \text{Level-1: } Y_{ij} &= \beta_{0j} + \beta_{1j}X_{1ij} + \beta_{2j}X_{2ij} + r_{ij} \\ \text{Level-2: } \beta_{0j} &= \gamma_{00} + \gamma_{01}W_{1j} + \gamma_{02}W_{2j} + u_{0j} \\ \beta_{1j} &= \gamma_{10} + \gamma_{11}W_{1j} + \gamma_{12}W_{2j} + u_{1j} \\ \beta_{2j} &= \gamma_{20} + \gamma_{21}W_{1j} + \gamma_{22}W_{2j} + u_{2j} \dots\dots\dots 1 \end{aligned}$$

### 2.2 Data generative process

1. The magnitude of the collinearity amongst the level-1 predictors are (r=0.0, 0.2).
2. The magnitude of the collinearity between the omitted level-1 predictors and level-2 predictors are (r=-0.2, -0.6).
3. The level-1 sample size (n<sub>j</sub>= average of 3, 5,10)
4. The level-2 sample size (N<sub>j</sub>= 10, 30, 50).
5. The magnitude of the intraclass correlation (ICC).

The first two conditions above define the intercorrelations among predictors modelled in the T-models. The omitted level-1 predictors,  $X_3$ , was assumed to be highly correlated with  $X_2$  (r=0.99), while the predictors was assumed to be weakly and uncorrelated with  $X_1$  ( r=0.2) respectively. The group means of  $X_3$  were also assumed to be highly correlated with one of the level-2 predictors,  $W_2$  (r=-0.6), but weakly correlated with  $W_1$  (r=-0.2).

### 3 Analysis

#### 3.1 Presentation of results obtained using simulated data sets

In Table 1, considering  $\beta_0$  when N=10, the study reveals that Penalised Quasi-likelihood (PQL) outperformed other estimators. Considering  $\beta_1$ , SGD (Glm) outperformed other estimators. SGD (moment) outperformed other estimators while considering  $\beta_2$ . Considering  $\beta_3$ , Penalized Quasi-likelihood (PQL) outperformed other estimators.

**Table 1. Showing the correct functional form of hierarchical linear model with different estimators (bias and MSE) asymptotically for  $n = 3, \rho_1 = 0.0, \rho_2 = -0.2$**

$n = 3, \rho_1 = 0.0, \rho_2 = -0.2$		$\beta_0$	$\beta_1$	$\beta_2$	$\beta_3$
N	trueV	1.0000	0.3000	0.1000	1.0000
10	PQL	<b>0.1148(0.0165)</b>	0.1226(0.0236)	0.0934(0.0116)	<b>0.9822(0.9711)</b>
	SGD(Glm)	0.5404(0.2920)	<b>0.0698(0.0049)</b>	0.1144(0.0131)	1.0927(1.1940)
	SGD(linear)	0.5404(0.2920)	0.0698(0.0049)	0.1144(0.0131)	1.0927(1.1940)
	SGD (moment)	0.1201(0.1084)	0.1226(0.0151)	<b>0.0063(0.1144)</b>	0.9865(0.9735)
30	PQL	<b>0.4666(0.2609)</b>	<b>0.1168(0.0292)</b>	<b>0.0074(0.0016)</b>	<b>0.6698(0.4529)</b>
	SGD(Glm)	0.5634(0.3175)	0.2718(0.0739)	0.0113(0.0168)	1.0334(1.0680)
	SGD(linear)	0.5634(0.3175)	0.2718(0.0739)	0.1296(0.0168)	1.0334(1.0680)
	SGD (moment)	0.5136(0.2657)	0.3490(0.1222)	0.1296(0.0169)	1.0017(1.0033)
50	PQL	<b>0.1525(0.0652)</b>	0.4339(0.2055)	<b>0.0348(0.0026)</b>	<b>1.0614(1.1320)</b>
	SGD(Glm)	0.5909(0.3492)	<b>0.0439(0.0019)</b>	0.0751(0.0056)	1.1782(1.3881)
	SGD(linear)	0.5909(0.3492)	0.0439(0.0019)	0.0751(0.0056)	1.1782(1.3881)
	SGD (moment)	0.4049(0.1654)	0.0760( <b>0.0066</b> )	0.0407( <b>0.0017</b> )	1.1639(1.3551)

Source: author's computation

Considering  $\beta_0$  when N=30, the study reveals that Penalised Quasi-likelihood (PQL) outperformed other estimators. Considering  $\beta_1$ , Penalised Quasi-likelihood (PQL) also outperformed other estimators. Penalised Quasi-likelihood (PQL) outperformed other estimators while considering  $\beta_2$ . Considering  $\beta_3$ , Penalized Quasi-likelihood (PQL) outperformed other estimators.

Considering  $\beta_0$  when N=50, the study reveals that Penalised Quasi-likelihood (PQL) outperformed other estimators. Considering  $\beta_1$ , SGD (Glm) is the most consistent estimator while SGD (moment) is the most efficient estimator while considering  $\beta_2$ . Considering  $\beta_3$ , Penalized Quasi-likelihood (PQL) outperformed other estimators.

**Table 2 showing the correct functional form of hierarchical linear model with different estimators (bias and MSE) asymptotically for  $n = 5, \rho_1 = 0.0, \rho_2 = -0.6$**

$n = 5, \rho_1 = 0.0, \rho_2 = -0.6$		$\beta_0$	$\beta_1$	$\beta_2$	$\beta_3$
N	trueV	1.0000	0.3000	0.1000	1.0000
10	PQL	<b>0.2519(0.1242)</b>	0.1985(0.0410)	0.0276(0.0029)	0.7015(0.4993)
	SGD(Glm)	0.7135(0.5091)	0.0416(0.0017)	0.0681(0.0046)	0.6902(0.4764)
	SGD(linear)	0.7135(0.5091)	<b>0.0416(0.0017)</b>	0.0681(0.0046)	0.6902(0.4764)
	SGD (moment)	0.3391( <b>0.1163</b> )	0.1698(0.0289)	<b>0.0181(0.0004)</b>	<b>0.5661(0.0032)</b>
30	PQL	1.1872(1.4815)	0.2444(0.0721)	0.1635(0.0293)	<b>0.6993(0.4945)</b>
	SGD(Glm)	<b>0.1259(0.0159)</b>	0.2147(0.0462)	0.0195(0.0005)	0.7802(0.6086)
	SGD(linear)	0.1259(0.0159)	<b>0.2146(0.0461)</b>	<b>0.0194(0.0004)</b>	0.7802(0.6086)
	SGD (moment)	0.5410(0.2948)	0.3423(0.1174)	0.1482(0.0221)	0.7425(0.5513)
50	PQL	0.2530(0.1016)	0.5476(0.3177)	0.1337(0.0190)	<b>0.2252(0.0597)</b>
	SGD(Glm)	0.2072(0.0432)	<b>0.1926(0.0370)</b>	<b>0.0042(0.0002)</b>	0.8710(0.7587)
	SGD(linear)	0.2073(0.0433)	0.1927(0.0371)	0.0043(0.0003)	0.8710(0.7587)
	SGD (moment)	<b>0.1074(0.0430)</b>	0.2986(0.0372)	0.0588(0.0004)	0.8440(0.7587)

Source: author's computation

In Table 2, considering  $\beta_0$ , with N=10, Penalized Quasi-likelihood (PQL) is the most consistent estimator while SGD (moment) is the most efficient estimator. Considering  $\beta_1$ , SGD (linear) outperformed other estimators. SGD (moment) outperformed other estimators while considering  $\beta_2$ . Considering  $\beta_3$ , SGD (moment) also outperformed other estimators. Overall, SGD (moment) is the best estimators.

Considering  $\beta_0$ , with N=30, SGD (Glm) outperformed other estimators. Considering  $\beta_1$ , SGD (linear) outperformed other estimators. SGD (linear) also outperformed other estimators while considering  $\beta_2$ . Considering  $\beta_3$ , Penalized Quasi-likelihood (PQL) outperformed other estimators. Overall, SGD (linear) is the best estimators.

Considering  $\beta_0$ , with N=50, SGD (moment) outperformed other estimators. Considering  $\beta_1$  and  $\beta_2$ , SGD (Glm) outperformed other estimators. Considering  $\beta_3$ , PQL outperformed other estimator. Overall, SGD (Glm) is the best estimator.

**Table 3. Showing the correct functional form of hierarchical linear model with different estimators (bias and MSE) asymptotically for  $n = 10, \rho_1 = 0.2, \rho_2 = -0.2$**

$n = 10, \rho_1 = 0.2, \rho_2 = -0.2$		$\beta_0$	$\beta_1$	$\beta_2$	$\beta_3$
N	trueV	<b>1.0000</b>	<b>0.3000</b>	<b>0.1000</b>	<b>1.0000</b>
10	PQL	<b>0.5243(0.3233)</b>	<b>0.1190(0.0286)</b>	<b>0.0694(0.0066)</b>	<b>0.4525(0.2095)</b>
	SGD(Glm)	0.9265(0.8058)	0.6735(0.4536)	0.1871(0.0350)	0.5648(0.3190)
	SGD(linear)	0.9265(0.8058)	0.6735(0.4536)	0.1871(0.0350)	0.5648(0.3190)
	SGD (moment)	1.2785(1.6372)	0.8814(0.7771)	0.2742(0.0753)	0.5938(0.3526)
30	PQL	<b>0.2519(0.1242)</b>	0.1985(0.0491)	0.0276(0.0029)	0.7015(0.4993)
	SGD(Glm)	0.7135(0.5091)	0.0416(0.0017)	0.0681(0.0046)	0.6902(0.4764)
	SGD(linear)	0.7135(0.5091)	<b>0.0416(0.0017)</b>	0.0681(0.0046)	0.6902(0.4764)
	SGD (moment)	0.3391( <b>0.1163</b> )	0.1698(0.0289)	<b>0.0181(0.0004)</b>	<b>0.5661(0.3205)</b>
50	PQL	1.1872(1.4815)	0.2444(0.0721)	0.1635(0.0293)	<b>0.6993(0.4945)</b>
	SGD(Glm)	<b>0.1259(0.0159)</b>	0.2146(0.0461)	0.0194(0.0004)	0.7802(0.6086)
	SGD(linear)	0.1259(0.0159)	<b>0.2146(0.0461)</b>	<b>0.0194(0.0004)</b>	0.7802(0.6086)
	SGD (moment)	0.5410(0.2948)	0.3423(0.1174)	0.1482(0.0221)	0.7425(0.5513)

Source: author's computation

In Table 3, when N=10, considering  $\beta_0, \beta_1, \beta_2,$  and  $\beta_3$  Penalized Quasi-likelihood (PQL) outperformed other estimators.

Considering  $\beta_0$ , when N=30, Penalized Quasi-likelihood (PQL) has a minimum bias while SGD (moment) has a minimum mean square error (MSE). Considering  $\beta_1$ , SGD (linear) outperformed other estimators. Considering  $\beta_2,$  and  $\beta_3,$  SGD (moment) outperformed other estimators.

Considering  $\beta_0$ , when N=50, SGD (Glm) outperformed other estimators. Considering  $\beta_1$ , SGD (linear) outperformed other estimators. SGD (linear) also outperformed other estimator while considering  $\beta_2$ . Considering  $\beta_3,$  Penalized Quasi-likelihood (PQL) outperformed other estimators.

In Table 4, considering  $\beta_0$  when N=10, the study reveals that SGD (moment) outperformed other estimator. Considering  $\beta_1,$  PQL outperformed other estimators. PQL and SGD (GLM) outperformed other estimators while considering  $\beta_2.$  It was observed that overall, PQL estimator is the best.

Considering  $\beta_0$  when N=30, the study reveals that SGD (moment) outperformed other estimators. Considering  $\beta_1$ , Penalized Quasi-likelihood (PQL) also outperformed other estimators. Penalized Quasi-likelihood (PQL) and SGD (GLM) outperformed other estimators while considering  $\beta_2.$  It was observed that overall, PQL estimator is the best.

Considering  $\beta_0$  when N=50, Considering  $\beta_0$  when N=50, the study reveals that SGD (moment) outperformed other estimators. Considering  $\beta_1$ , Penalized Quasi-likelihood (PQL) also outperformed other estimators.

Penalized Quasi-likelihood (PQL) and SGD (GLM) outperformed other estimators while considering  $\beta_2$ . It was observed that overall, PQL estimator is the best.

**Table 4. Showing the incorrect functional form (omitted variable) of hierarchical linear model with different estimators (bias and MSE) asymptotically for  $n = 3, \rho_1 = 0.0, \rho_2 = -0.2$**

$n = 3, \rho_1 = 0.0, \rho_2 = -0.2$		$\beta_0$	$\beta_1$	$\beta_2$
N	trueV	<b>1.0000</b>	<b>0.3000</b>	<b>0.1000</b>
10	PQL	0.1733(0.0331)	<b>0.0563(0.0045)</b>	<b>0.0135(0.0012)</b>
	SGD(Glm)	0.1357(0.0184)	0.1652(0.0273)	0.0219( <b>0.0005</b> )
	SGD(linear)	0.1357(0.0184)	0.1652(0.0273)	0.0219(0.0006)
	SGD (moment)	<b>0.1218(0.0148)</b>	0.1794(0.0322)	0.0310(0.0009)
30	PQL	0.1733(0.0331)	<b>0.0563(0.0045)</b>	<b>0.0135(0.0012)</b>
	SGD(Glm)	0.1357(0.0184)	0.1652(0.0273)	0.0219( <b>0.0005</b> )
	SGD(linear)	0.1357(0.0184)	0.1652(0.0273)	0.0219(0.0006)
	SGD (moment)	<b>0.1218(0.0148)</b>	0.1794(0.0323)	0.0301(0.0009)
50	PQL	0.1733(0.0331)	<b>0.0563(0.0045)</b>	<b>0.0135(0.0012)</b>
	SGD(Glm)	0.1357(0.0184)	0.1652(0.0273)	0.0219( <b>0.0005</b> )
	SGD(linear)	0.1357(0.0184)	0.1652(0.0273)	0.0219(0.0006)
	SGD (moment)	<b>0.1218(0.0148)</b>	0.1794(0.0273)	0.0301(0.0009)

Source: author's computation

**Table 5. Showing the incorrect functional form (omitted variable) of hierarchical linear model with different estimators (bias and MSE) asymptotically for  $n = 5, \rho_1 = 0.0, \rho_2 = -0.6$**

$n = 5, \rho_1 = 0.0, \rho_2 = -0.6$		$\beta_0$	$\beta_1$	$\beta_2$
N	trueV	<b>1.0000</b>	<b>0.3000</b>	<b>0.1000</b>
10	PQL	0.4302(0.2101)	1.7320(3.0036)	1.8832(3.5495)
	SGD(Glm)	0.3810(0.1451)	1.1234(1.2621)	<b>0.6010(0.3612)</b>
	SGD(linear)	0.3810(0.1451)	1.1234(1.2621)	<b>0.6010(0.3612)</b>
	SGD (moment)	<b>0.2592(0.0672)</b>	<b>0.9296(0.8641)</b>	0.9883(0.9767)
30	PQL	0.4302(0.2101)	1.7320(3.0036)	1.8832(3.5495)
	SGD(Glm)	0.3810(0.1451)	1.1234(1.2621)	<b>0.6010(0.3612)</b>
	SGD(linear)	0.3810(0.1451)	1.1234(1.2621)	<b>0.6010(0.3612)</b>
	SGD (moment)	<b>0.2592(0.0672)</b>	<b>0.9296(0.8641)</b>	0.9883(0.9767)
50	PQL	0.4302(0.2101)	1.7320(3.0036)	1.8832(3.5495)
	SGD(Glm)	0.3810(0.1451)	1.1234(1.2621)	<b>0.6010(0.3612)</b>
	SGD(linear)	0.3810(0.1451)	1.1234(1.2621)	<b>0.6010(0.3612)</b>
	SGD (moment)	<b>0.2592(0.0672)</b>	<b>0.9296(0.8641)</b>	0.9883(0.9767)

Source: author's computation

Considering  $\beta_0$  when N=10, the study reveals that SGD (moment) outperformed other estimators. Considering  $\beta_1$ , SGD (moment) also outperformed other estimators. SGD (GLM) and SGD (linear) outperformed other estimators while considering  $\beta_2$ . It was observed that overall, SGD (moment) outperformed all other estimators.

Considering  $\beta_0$  when N=30, the study reveals that SGD (moment) outperformed other estimators. Considering  $\beta_1$ , SGD (moment) also outperformed other estimators. SGD (GLM) and SGD (linear) outperformed other estimators while considering  $\beta_2$ . It was observed that overall, SGD (moment) outperformed all other estimators.

Considering  $\beta_0$  when N=50, the study reveals that SGD (moment) outperformed other estimators. Considering  $\beta_1$ , SGD (moment) also outperformed other estimators. SGD (GLM) and SGD (linear)

outperformed other estimators while considering  $\beta_2$ . It was observed that overall, SGD (moment) outperformed all other estimators.

## 4 Multicollinearity Tests On Bias

### 4.1 Using the multiple linear regression

lm(formula = y ~ x1 + x2 + x3, data = vinc\_bias1)

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	1.172e-01	3.633e-06	32259	1.97e-05 ***
x1	-1.236e+00	4.636e-07	-2665762	2.39e-07 ***
x2	2.536e+00	3.984e-06	636556	1.00e-06 ***
x3	NA	NA	NA	NA

Residual standard error: 1.611e-07 on 1 degrees of freedom

Multiple R-squared: 1, Adjusted R-squared: 1

F-statistic: 1.652e+14 on 2 and 1 DF, p-value: 5.502e-08

**Comment:** The R-square value of 1 is not practically obtainable in real life data. This is an indication of multicollinearity

### 4.2 Overall multicollinearity diagnostics

MC Results detection		
Determinant  X'X :	0.000000e+00	1
Farrar Chi-Square:	4.222040e+01	1
Red Indicator:	9.652000e-01	1
Sum of Lambda Inverse:	1.310614e+15	1
Theil's Method:	1.000000e+00	1
Condition Number:	1.583578e+08	1

1 --> COLLINEARITY is detected by the test

0 --> COLLINEARITY is not detected by the test

Using all the methods considered above, multicollinearity is confirmed present

### 4.3 Correlation matrix

	x1	x2	x3
x1	1.0000000	0.9937156	-0.9676217
x2	0.9937156	1.0000000	-0.9332880
x3	-0.9676217	-0.9332880	1.0000000

The correlation matrix above shows that there is high correlation among the independent variables

#### 4.4 Correlation matrix visualized

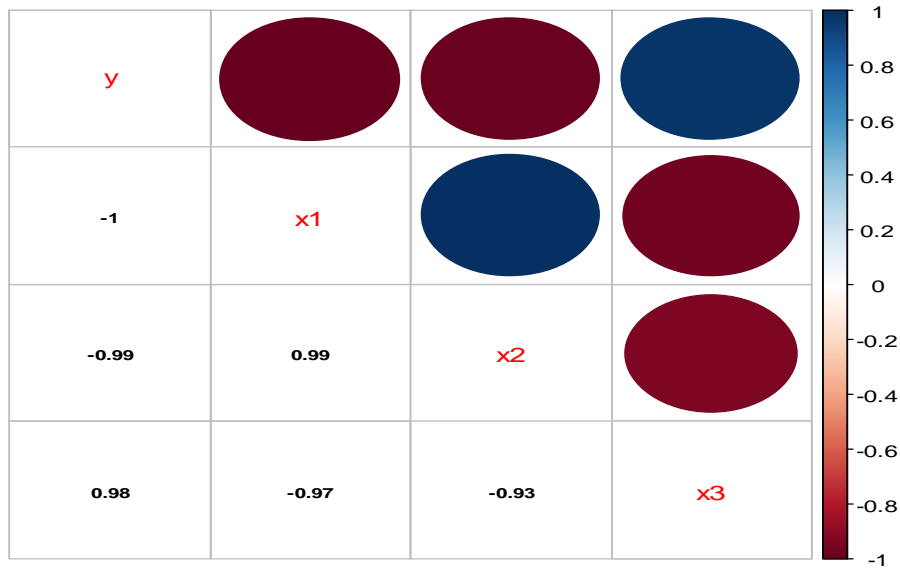
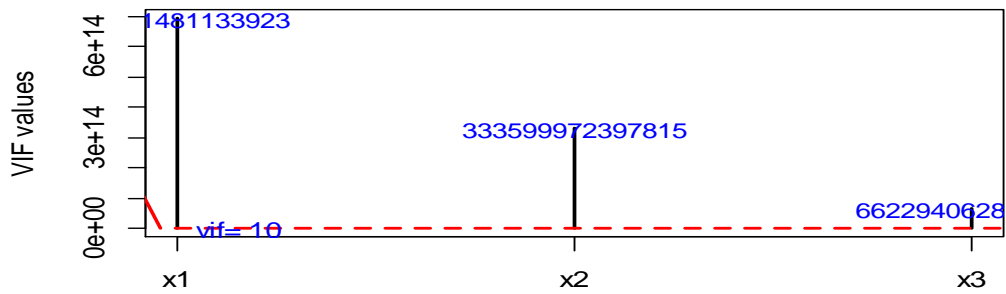


Fig. 1

#### VIF Plot



#### Eigenvalues Plot

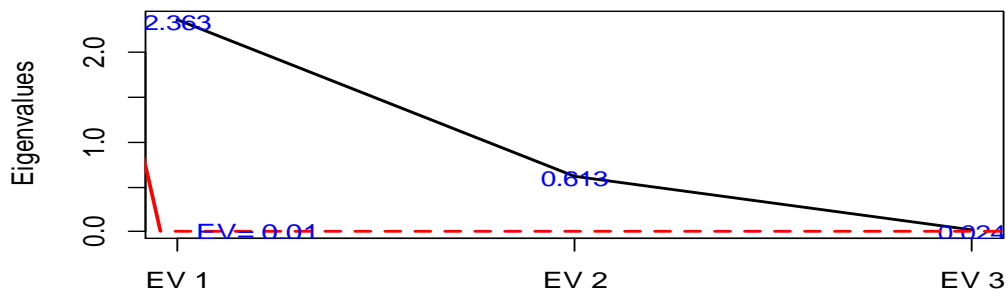


Fig. 2

**Comment:** As a rule of thumb, any variable having a variance inflation factor (VIF) or Eigen value higher than the boundary line is collinear with another variable. It can be visualized from the plots above that the variables x1, x2, and x3 have crossed the boundary, we can say that the variables are collinear.

## 5 Multicollinearity Tests on MSE

### 5.1 Using multiple linear regression

```
lm(formula = y ~ x1 + x2, data = vinc_mse1)
      Estimate Std. Error t value Pr(>|t|)
(Intercept) 1.520e+01 1.680e-06 9044928 7.04e-08 ***
x1          -9.732e-02 2.396e-07 -406190 1.57e-06 ***
x2          -8.886e+00 2.407e-06 -3692190 1.72e-07 ***
Residual standard error: 3.861e-07 on 1 degrees of freedom
Multiple R-squared: 1, Adjusted R-squared: 1
F-statistic: 1.853e+14 on 2 and 1 DF, p-value: 5.195e-08
```

### 5.2 Correlation matrix

```
      x1      x2      x3
x1 1.000000 0.9771642 0.9999683
x2 0.9771642 1.0000000 0.9788258
x3 0.9999683 0.9788258 1.0000000
```

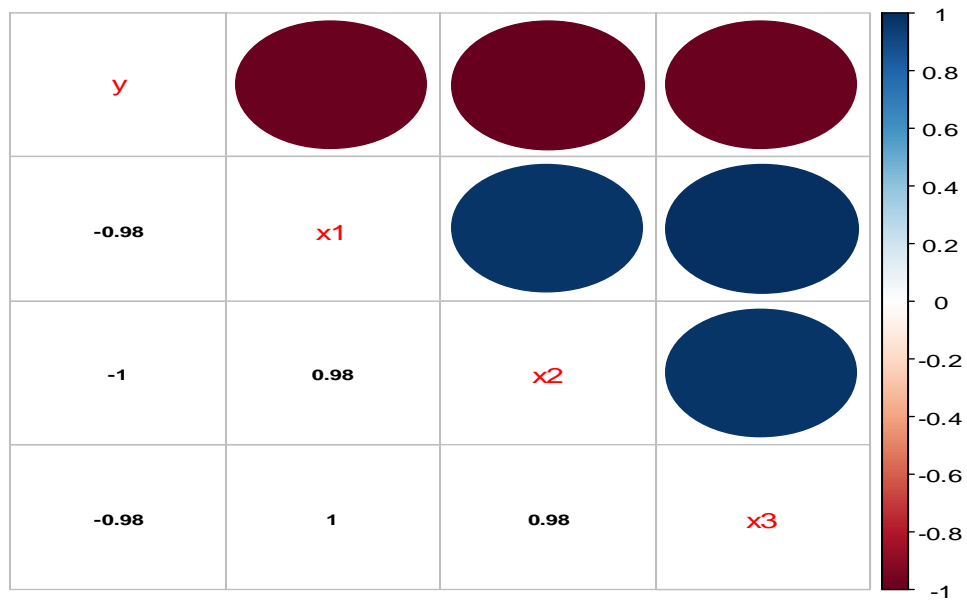


Fig. 3

### 5.3 Overall multicollinearity diagnostics

```
MC Results detection
Determinant [X'X]: 0.000000e+00 1
Farrar Chi-Square: 4.665360e+01 1
Red Indicator: 9.854000e-01 1
Sum of Lambda Inverse: 2.801226e+17 1
Theil's Method: 1.000000e+00 1
Condition Number: NaN NA
```

1 --> COLLINEARITY is detected by the test  
 0 --> COLLINEARITY is not detected by the test



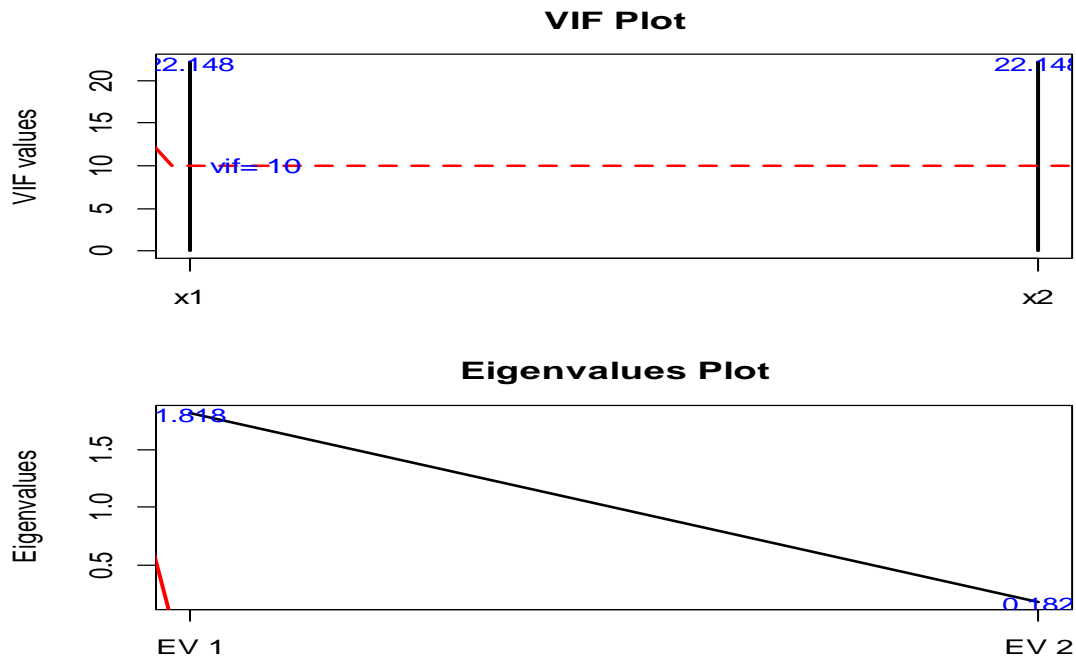


Fig. 4

**Comment:** Fig. 4, indicates the Variance Inflation Factor (VIF) and Eigen values plot. It can be visualized from the plots above that the variables x1, x2, and x3 have crossed the boundary which indicates the variables are collinear.

#### 5.4 Test of autocorrelation and stationarity on True Model Bias and Mse

Fig. 5 indicates the plot of autocorrelation of bias and MSE data. The plot indicates that there is stationarity in the data.

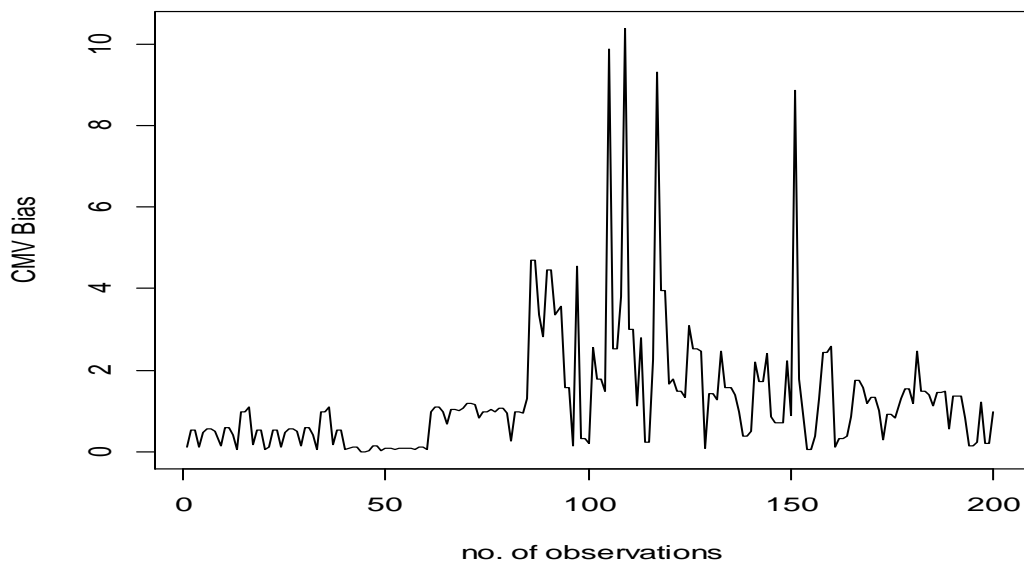


Fig. 5. Time plot for CMV Bias

### ACF for CMV Bias Series

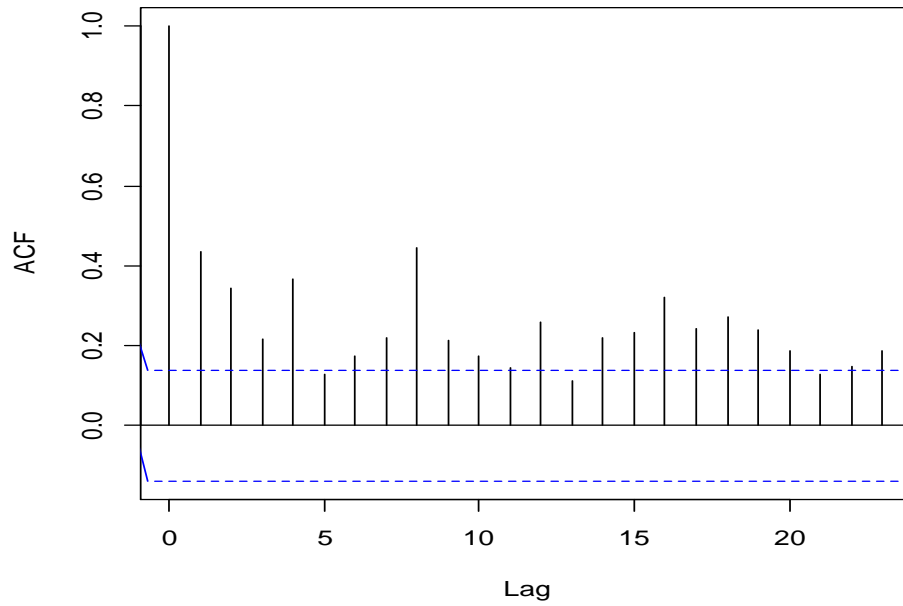


Fig. 6. ACF for CMV Bias

Fig. 6 indicates the plot of Autocorrelation function (ACF) of the complete model value (CMV) for the bias data. The plot indicates that there is stationarity in the data

### PACF for CMV Bias Series

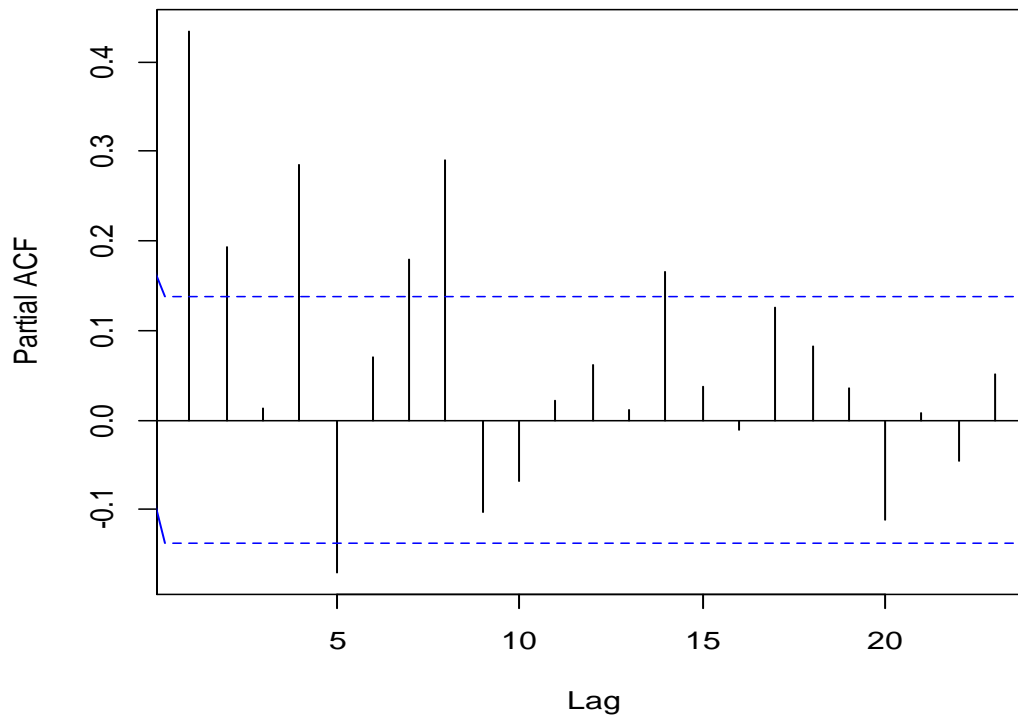


Fig. 7. PACF for True Model Bias

Fig. 7 indicates the plot of Partial Autocorrelation Function (PACF) of the complete model value (CMV) for the bias data. The plot indicates that there is stationarity in the data

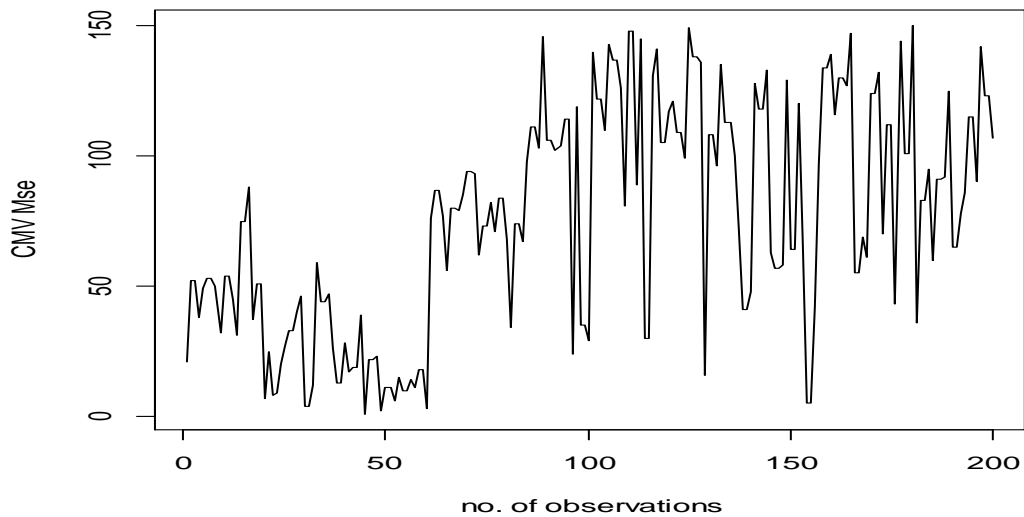
### 5.5 Augmented Dickey-Fuller test

**data: Bias\_cmV**

**Dickey-Fuller = -3.9566, Lag order = 5, p-value = 0.01249**

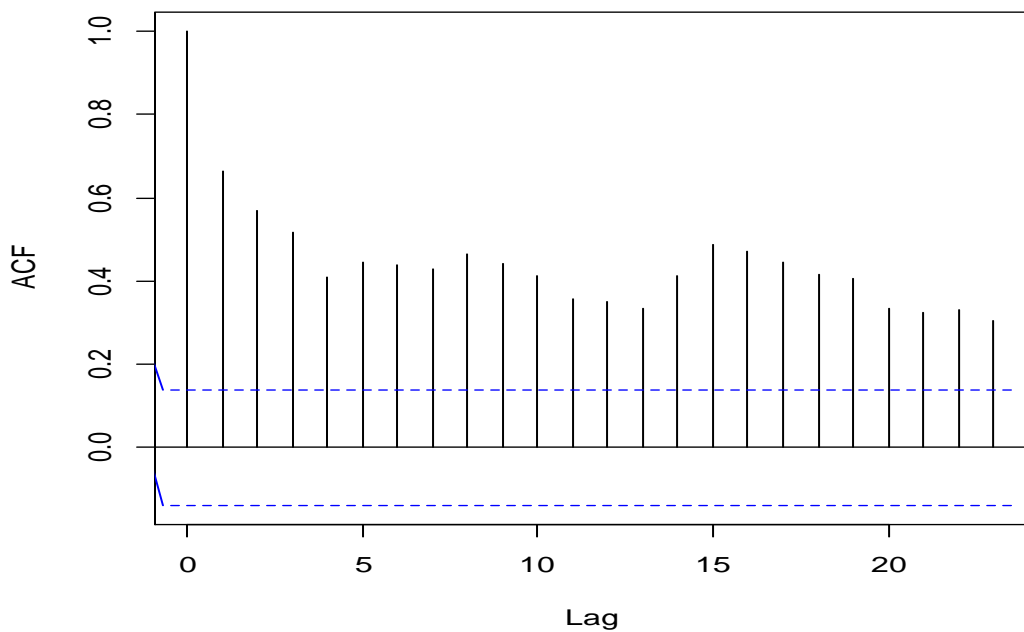
**alternative hypothesis: stationary**

Conclusion: The hypothesis of non-stationarity is rejected, therefore, the series is stationary



**Fig. 8. Time plot for CMV Bias**

### ACF for CMV mse Series



**Fig. 9. ACF for CMV MSE**

Figs. 8 and 9 are plot of autocorrelation for complete model values. They both indicates stationarityof the data.

### PACF for CMV mse Series

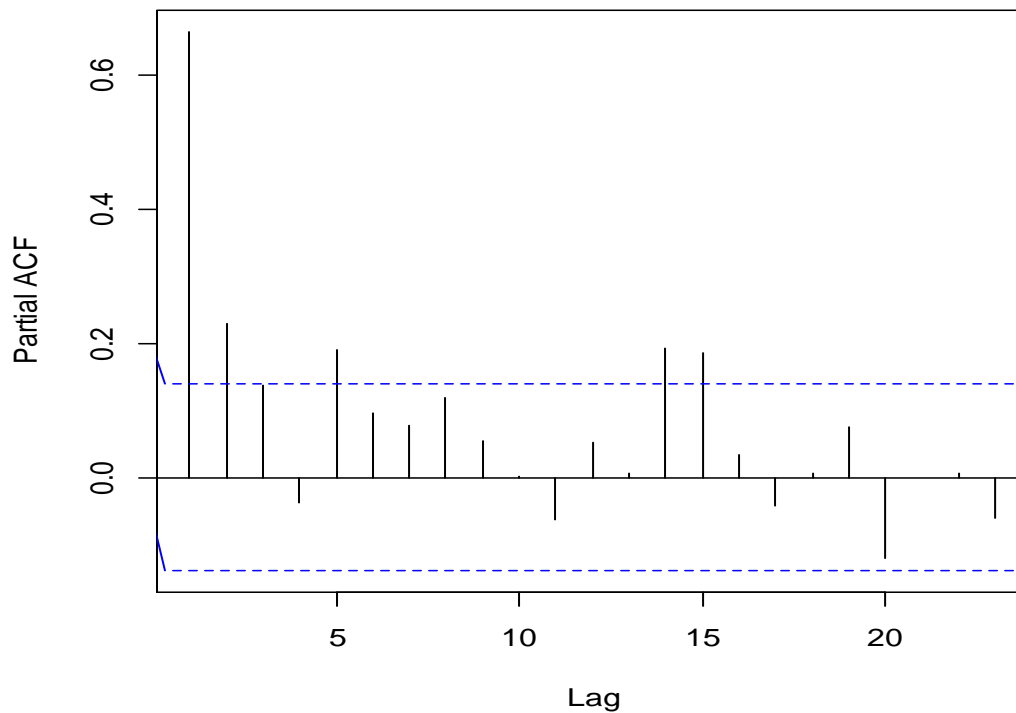


Fig. 10. PACF for CMV Mse

### 5.6 Augmented Dickey-Fuller test

data: mse\_cmV

Dickey-Fuller = -3.5945, Lag order = 5, p-value = 0.03523

alternative hypothesis: stationary

Conclusion: The hypothesis of non-stationarity is rejected, therefore, the series is stationary

### 6 Conclusion

The study reveals that under multicollinearity, the estimators outperformed themselves at different points when consideration is given to  $\beta_0$ ,  $\beta_1$  and  $\beta_2$  at varying  $n$ ,  $N$ ,  $\rho_1$  and  $\rho_2$ . More so, as the data point increases the estimators are asymptotically consistent and efficient. Simulation shows that the omission of variables yields bias in both regression coefficients and variance components. It also suggests that omitted effects at lower levels may cause more severe bias than at higher levels. Important factors resulting in bias were found to be the level of an omitted variable, its effect size, and sample size.

### Competing Interests

Author has declared that no competing interests exist.

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