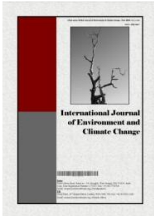

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Predictive Performance Analysis of PDF – IDF Model Types Using Rainfall Observations from Fourteen Gauged Stations

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Authors' contributions

This work was carried out in collaboration among all authors. Author ILN designed the study, wrote the protocol and supervised the study. Author MGS wrote the first draft of the manuscript, managed the analyses of nine stations, literature searches and documentation. Author AOD collected and managed data analyses of five western stations. All authors read and approved the final manuscript.

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ABSTRACT

The design of structures for flood mitigation depends on the adequate estimation of rainfall intensity over a given catchment which is achieved by the rainfall intensity duration frequency modelling. In this study, an extensive comparative analyses were carried out on the predictive performance of three PDF – IDF model types, namely: Gumbel Extreme Value Type 1 (GEVT – 1), Log-Pearson Type 3 (LPT – 3) and Normal Distribution (ND) in 14 selected cities in Southern Nigeria. This is to rank the order of best performance. The principle of general model development was adopted in which rainfall intensities at different durations and specified return periods were used as input data set. This is not same as return period specific model that involves rainfall intensities for various durations and a given return period. The predicted rainfall intensity values with the PDF – IDF model types indicate high goodness of fit (R^2) and Mean Squared Errors (MSE)

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ranging from: (a) R^2 = 0.875 – 0.992; MSE = 33.17 – 224.6 for GEVT – 1; (b) R^2 = 0.849 – 0.990; MSE = 65.34 – 405.5 for LPT – 3 and (c) R^2 = 0.839 – 0.992; MSE = 29.23 – 200.2 for ND. The comparative analysis of all the 42 general models (14 locations versus 3 model types) considered showed that the order of best performance is LPT – 3 1st, GEVT - 1 2nd and ND 3rd for each return period (10, 50 and 100 years). The Kruskal Wallis test of significance indicates that no significant difference exists in the predictive performance of the three General models across the board. This may be due to the fact that the fourteen locations of the study area are bordering with the Atlantic Ocean and seems to have similar climatology. These developed General models are recommended for the computation of intensities in the fourteen locations for the design of flood control structures; and the order of preference should be LPT – 3 > GEVT – 1 > ND.

Keywords: Probability distribution function; GEVT – 1; normal; Log-Pearson Type 3; Kruskal Waliis test of significance.

1. INTRODUCTION

Rainfall Intensity-Duration-Frequency (IDF) model is a basic instrument that is required for deriving rainfall characteristics in any catchment area. A long time rainfall records are needed from weather station to enable IDF model development which are lacking in most developing Countries like Nigeria. IDF models are represented in graphical forms as curves or in empirical equation forms either used for assessment of extreme rainfall event crucial in hydrologic risk analysis and design [1]. Most IDF models developed are site specific and are reliable tools for estimation of runoff by Rational method for urban drainage sizing, design and operation of flood control projects including estimation of ground water recharge [2,3].

The wide application of IDF model has aroused attention from researchers the world over to achieve accurate estimation of the relationship expressed in IDF models. Several types of IDF equations have been proposed by different scholars to define this relationship accurately. [4,5] were among early scholars who introduced the concept of relating intensity, duration and frequency of rainfall. [6] proposed and amended Bernard equation by adding a representative term of rainfall characteristics aimed at improving IDF curve fitting. [7] used efficient parameterization to propose a new approach for constructing IDF curves. [8] derived IDF using maple language for Basrah, Iraq. [9] presented the use of Artificial Neural Network (ANN) for predicting future extreme rainfall event in the form of IDF curves for Alabama, USA. They concluded that the ANN was superior to the stochastic method on the rainfall prediction. [10] studied the application of optimization technique to estimate IDF parameters and concluded that IDF curves resulting from optimization method were more accurate as compared to the conventional method of multiple regression.

In recent times some studies have been done on IDF models developed for different cities in Nigeria [11-16]. Regression and correlation analyses were used with most of the results presented as equations and in graphical form. Further studies by some scholars [17-20] examined the predictive ability of different types of IDF models for Abakaliki and selected cities in Southern Nigeria, respectively. They concluded that simple two parameter quotient IDF type model predicted higher intensity at duration equal or less than 30 minutes and also predicted lowest relative error.

1.1 Rainfall IDF Model Relationships

Forehlich [21] gave a summary of a number of equation types used in fitting IDF models as in Table 1. The equations in the table are all empirical and indicate that rainfall intensity decreases with increase in duration considering any given return period. The parameters of the equations represent how climatic and regional features influence the watershed or catchment area.

IDF models find expression in any of the equation forms shown in Table 1. The modified Sherman Equation (6) type pre-supposes that the constant *b* is zero. This assures achievement of maximum value for duration *t*. Apparently, it reduces the physiographic constants into three and thus, makes the equation dimensionless.

Table 1. Rainfall IDF Equation Types ±

S/No.	Equation Form	Equation type
1	I = $aT_d{}^b$	2-parameter power equation
2	I = $\frac{a_1}{(b_1+T_d)}$	2-parameter quotient linear equation
3	I = $\frac{a_2}{T_d{}^{C_2}}$	2-parameter quotient power equation
4	I = $\frac{a_3}{(b_3+T_d)^{C_3}}$	3-parameter quotient non-linear equation
5	I = $\frac{a_4}{(b_4+T_d{}^{C_4})}$	3-parameter quotient power equation
6	I = $\frac{cT^m}{(b+T_d)^n}$	4-parameter quotient non-linear equation

±Source: [21]

The Sherman's modified 3-parameter quotient non-linear equation was applied by different authors [2,7,22-26] in the form given as Equation (1):

$$\text{Intensity, } I = \frac{cT_r{}^m}{T_d{}^a} \qquad (1)$$

Where*;* I = rainfall intensities (mm/hr); T_r = return period in years*;* T_d = duration of rainfall in minutes; and c, a, and m = physiographic constants. Equation (1) depicts probabilities that are conditional of rainfall-intensities averages over duration typical of storm intervals.

The present study focuses on development of IDF equations and curves using Generalized Reduced Gradient (GRG) Solver, an optimization technique for the estimation of IDF equation parameters for more accurate and efficient prediction of rainfall intensities for both short and long term return periods deploying probability distribution functions (PDF) in frequency analysis of the rainfall data. This will help to map out the spatial distribution and variation of rainfall intensities and their predictions in the study area. Thus, determine which of the PDF-IDF model types will be preferable for design considerations in their order of rainfall intensity predictive performance. Interestingly, earlier studies by [26] had shown that the PDF-IDF models exhibited superiority over the non-Probability Distribution Function (nPDF)-IDF models because their return periods were limited by the number of years of data collection. Meanwhile, literature revealed the extent of coverage of PDF- IDF modeling in Nigeria (see Table 2), that is percentage coverage in all the Southern States is about 40% while the Northern States is less than 30%.

Table 2. PDF- IDF coverage in Nigeria

South – East Cities	South – Central	South – West	North - East	North – Central	North - West
Owerri (P)	Port Harcourt (Y) (P)	Ikeja (Y)	Maiduguri (N)	Ilorin (N)	Sokoto (N)
Umuahia (P)	Yenagoa (N)	Abeokuta (Y)	Gombe (N)	Minna (Y)	Kano (Y)
Enugu (P)	Warri (P)	Ibadan (Y)	Bauchi (Y)	Lafia (N)	Kaduna (N)
Abakaliki (P)	Asaba (N)	Osogbo (Y)	Jalingo (N)	Lokoja (Y)	Birnin Kebbi (N)
Onitsha (P)	Uyo (Y) (P)	Akure (Y)	Dutse (N)	Makurdi (Y)	Gusua (N)
nAwka (N)	Calabar (P)	Ekiti (N)	Yola (N)	Jos (N)	Katsina (N)
P (83.33%)	Y (33.33%)	Y (66.67%)	Y (16.67%)	Y (50.00%)	Y (16.67%)
N (16.67%) Y (None)	N (66.66%)	N (33.33%)	N (83.33%)	N (50.00%)	N (83.33%)

NB: Y = Probability Distribution Function (PDF) IDF models in existence, P = Non-probability Distribution Function (nPDF) IDF models, while N = there is none

2. MATERIALS AND METHODS

2.1 Study Area

The study area lies within the boundary of longitude 4°15’N - 6°30’N and latitude 5°32’E - 8°22’E of the lower Niger River in Nigeria. Fig. 1 shows location map of the study area. Fourteen locations were chosen for the study, fully identified by GIS and region of Nigeria (see Table 3). The Atlantic Ocean borders the area in the South and the Northern boundary is close to the Savanah which stretches to the Sahara desert. These two geographic features influence the rainfall pattern of the study area.

Table 3. Geographical location of study area

S/N	Site description	Region in Nigeria	Geographic Location
1	Abakaliki	South-East	6°14’N and 8°13’E
2	Enugu	South-East	6°28’N and 7°33’E
3	Onitsha	South-East	6°08’N and 6°47’E
4	Owerri	South-East	5°30’N and 7°0’E
5	Port Harcourt	South-South	4°46’N and 7°02’E
6	Calabar	South-South	4°58’N and 8°20’E
7	Benin City	South-South	6°20’N and 5°37’E
8	Uyo	South-South	5°03’N and 7°55’E
9	Warri	South-South	5°33’N and 5°45’E
10	Ikeja	South-West	6°27’N and 3°24’E
11	Akure	South-West	7°15’N and 5°12’E
12	Ibadan	South-West	7°23’N and 3°54’E
13	Abeokuta	South-West	7°09’N and 3°21’E
14	Osogbo	South-West	7°48′ N and 4°24′ E

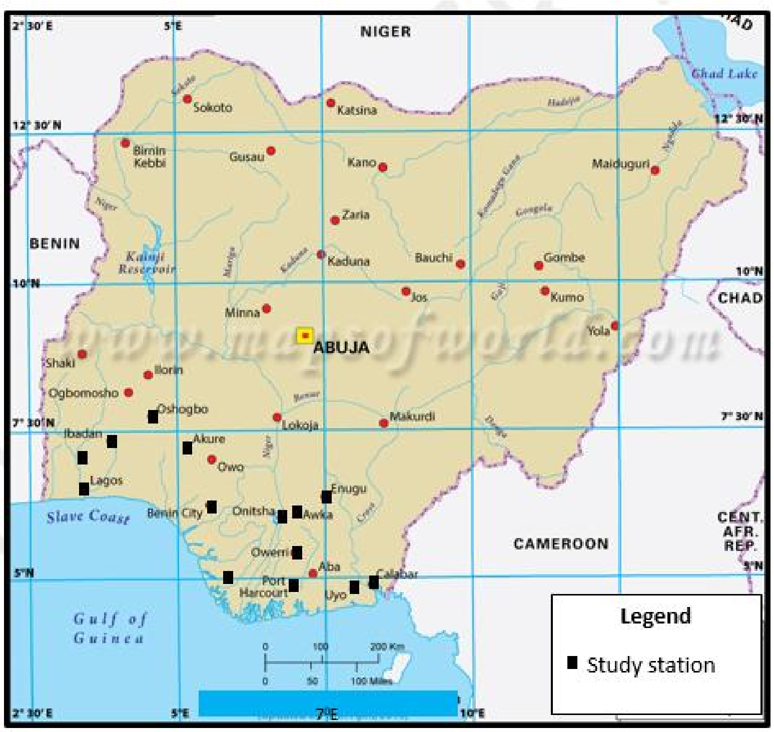


Fig. 1. Location map showing study area in Southern Nigeria

2.2 Data Collection

The daily rainfall amount and duration for the fourteen locations being studied were extracted from autographic rain gauge records obtained from the Nigerian Meteorological Agency (NIMET), Oshodi, Lagos, Nigeria. The daily rainfall amount and duration were sorted and converted into ranked observed annual rainfall intensities for different durations in decreasing order of their magnitude. The ranked observed annual rainfall intensities for different durations and their log-equivalent are as shown in Table 4 for Benin City were the basic input data used for the study.

2.3 Data Analysis

The data analysis for this study required the development of Probability Distribution Function (PDF) rainfall Intensity-Duration-Frequency (IDF) models. In an earlier study by [26] a detailed comparative study of PDF and non-Probability Distribution Function (nPDF) IDF models, established that the Log Pearson Type-3 (LPT-3) was the best-fit PDF model, ideal for data fitting, while the Gumbel Extreme Value Type-1 (GEVT-1) PDF model predicted highest rainfall intensities. The Normal PDF-IDF model exhibited the best mean squared error (MSE). These three type of PDF models were thus, adopted for the rainfall frequency analysis for this study.

The purpose for the rainfall frequency analysis is basically to relate the magnitude of rainfall event to their frequency of occurrence through the use of appropriate probability distribution function. The approximation of the magnitude of rainfall intensity, X_T (mm/hr) for each duration (minutes) for a specified return period T (years) is given by the following equation [27]:

$$X_T = \bar{x} + K_T S \qquad (2)$$

Where $\bar{x}$ = sample mean, S = sample standard deviation and K_T = frequency factor. The standard deviation and frequency factor are functions of the return period, T and PDF type. Therefore, to generate the K_T for the GEVT-1, Equation (3) is applied.

$$K_T = \frac{-\sqrt{6}}{\pi}\left\{0.5772 + In\left[In\left(\frac{T}{T-1}\right)\right]\right\} \qquad (3)$$

Furthermore, Equation (2) can be re-written in the log-form as Equation (4) for the approximation of the rainfall intensity value applying the Log Pearson Type-3 (LPT-3) PDF method to generate K_T values.

$$\text{Log } X_T = \text{Log } \bar{X} + K_T \times Log\ S \qquad (4)$$

The Log-equivalent of the ranked observed annual rainfall intensity values including their corresponding statistical parameters are used. The generation of the K_T values can be obtained from standard frequency factor table published in [2] or using [28] approximate method of Equation (5).

$$K_T = Z + (Z^2 - 1)K + \frac{1}{3}(Z^3 - 6)K^2 - (Z^2 - 1K3 + ZK4 + 13K5 \qquad (5)$$

Where $K = \frac{C_s}{6}$ for $C_s \neq 0$, but at $C_s = 0$, K_T = Z, normal variate. While C_s is the coeficient of skewness of sample.

For the Normal probability distribution function (PDF), the K_T values were equally generated by the use of Equation (6)

$$K_T = z = w - \frac{2.515517 + 0.802853w + 0.010328w^2}{1 + 1.432788w + 0.189269w^2 + 0.001308w^3} \qquad (6)$$

Where $w = \left[In\left(\frac{1}{p^2}\right)\right]^{1/2}$ for ($0 < p \leq 0.5$) (7)

And, z = standard normal variate; while probability function, $p = \frac{1}{T}$

Equation (1) was the IDF equation adopted for the study. Thus, the calibration including the computation of the coefficient of determination (R^2) and the Mean Squared Error (MSE) followed the optimization technique or non-linear regression method as reported by Zakwan [10].

3. RESULTS AND DISCUSSION

3.1 Results

3.1.1 PDF and rainfall intensity transformation

The observed daily rainfall amounts and durations for the fourteen locations were transformed into the logarithmic values (see Table 4, a case of Benin city). Likewise, the frequency factors for GEVT – 1, LPT – 3 and Normal distributions were obtained by the evaluation of Equations (3), (5) and (6) respectively, as shown in Tables 5, 6 and 7. Furthermore, the original intensity values (not logarithmic) were used alongside with the frequency factor values in Table 4 to generate GEVT – 1 intensity values in Table 8. In like manner, the original intensity values combined with Normal frequency factor (Table 7) were both used as input data to evaluate Equation (3) to yield intensity values as shown in Table 10. Also, the intensity values (logarithmic equivalent in Table 4) were combined with the LPT – 3 frequency factor of Table 5 and substituted in Equation 5 to yield intensity values in Table 9.

Table 4. Observed Annual Rainfall Intensities (in Logarithmic) of Different Durations for Benin City

Year	Duration (minutes)							
	10	**20**	**30**	**40**	**50**	**60**	**90**	**120**
1	1.8820$^{\pm}$	1.9571	1.6355	1.6415	1.9123	1.7513	1.7614	1.6637
2	1.9823	1.8055	1.7007	1.5977	1.3845	1.7649	1.5424	1.4914
3	1.4857	1.8407	1.1072	1.4191	1.4701	1.6484	1.4790	1.3802
4	1.7126	1.8463	1.4771	1.5359	1.7295	1.5977	1.4867	1.5883
5	1.8274$^{\pm}$	1.7582	1.8426	1.9196	1.6306	1.6425	1.7087	1.3345
6	1.8537	1.6444	1.7466	1.5809	1.4136	1.8363	1.3991	1.5617
7	1.8351	1.6618	1.7059	1.8293	1.5977	1.6721	1.6902	1.3802
8	1.4857	1.4857	1.9096	1.6946	1.6170	1.8363	1.5627	1.6628
9	1.6590	1.5340	1.5977	1.8473	1.5911	1.7332	1.5674	1.6064
10	1.5775	1.5416	1.4014	1.5302	1.4975	1.6191	1.3039	1.5472
Mean	1.7301	1.7075	1.6124	1.6596	1.5844	1.7102	1.5502	1.5216
Standard Deviation	0.1734	0.1579	0.2344	0.1609	0.1570	0.0868	0.1424	0.1202
Skewness	-0.2558	0.0210	-1.0615	0.3471	0.8384	0.3141	-0.1360	-0.4463

$^{\pm}$ *Rainfall intensity value in log-form*

Table 5. Computed Gumbel EVT-1 Distribution Frequency Factor for Benin City

Return period	2	5	10	25	50	100
K_T values	-0.16425	0.719	1.304	2.044	2.592	3.1363

Table 6. Computed normal distribution frequency factor for Benin City

Return period	2	5	10	25	50	100
P	0.5	0.2	0.1	0.04	0.02	0.01
W	1.17741	1.794123	2.145966	2.537272	2.79715	3.034854
K_T values	-1E-07	0.841457	1.281729	1.751077	2.054189	2.326785

Table 7. Computed LPT-3 distribution frequency factors for Benin City

Duration (mins)	C_s	Return period					
		2	**5**	**10**	**25**	**50**	**100**
10	-0.25584	0.042562	0.851187	1.250988	1.660023	1.91494	2.138098
20	0.021031	-0.00351	0.840415	1.283959	1.758298	2.065459	2.342251
30	-1.06148	0.171318	0.8458	1.114497	1.34503	1.466212	1.558339
40	0.34706	-0.05765	0.819669	1.312574	1.864487	2.235747	2.579753
50	0.838404	-0.13699	0.772814	1.333958	2.001417	2.472303	2.923548
60	0.314056	-0.0522	0.822157	1.310193	1.854256	2.21893	2.555961
90	-0.13595	0.022647	0.847307	1.266215	1.703403	1.980647	2.226591

Table 8. Computed rainfall intensity values for Gumbel EVT-1 for Benin City

Duration (mins)	Return period (Years)					
	2	5	10	25	50	100
10	53.97	73.17	85.88	101.95	113.86	125.69
20	50.89	68.11	79.51	93.91	104.59	115.20
30	42.48	60.57	72.54	87.68	98.90	110.05
40	45.57	62.11	73.06	86.90	97.17	107.36
50	38.15	53.05	62.91	75.38	84.62	93.80
60	50.50	59.91	66.15	74.02	79.86	85.66
90	35.25	45.84	52.85	61.71	68.28	74.80
120	32.88	40.80	46.05	52.67	57.59	62.47

Table 9. Computed rainfall intensity values for Log-Pearson Type-3 distribution for Benin City

Duration (mins)	Return period (Years)					
	2	5	10	25	50	100
10	54.63	75.45	88.50	104.20	115.36	126.11
20	50.93	69.22	81.33	96.64	108.06	119.50
30	44.94	64.67	74.77	84.67	90.40	95.01
40	44.70	61.87	74.27	91.13	104.57	118.78
50	36.55	50.78	62.20	79.17	93.85	110.48
60	50.77	60.47	66.67	74.33	79.95	85.53
90	35.76	46.86	53.76	62.04	67.94	73.65
120	33.92	42.11	46.63	51.58	54.83	57.77

3.1.2 Sherman's model Calibration for general models

In IDF model development, we have both return period specific (RPS) and non-return period specific (NRPS) models. The non- return period specific is herein called the general models. The RPS requires smaller input data because it involves a given return period for specified durations. However, the general models are more involved input wise in that the various durations multiplied by number of specified return periods constitute the number of input data required. For example, in Table 8, the total number of return periods is 6 while number of specified duration is 8 giving rise to 48 sets of input data. These set of input data from Table 8 are used to calibrate Equation (1) resulting to the general model for Benin city and the same approach was repeated for the remaining thirteen locations for GEVT – 1 models (see Table 11). In like manner, the approach for generating GEVT – 1 model calibration was repeated for LPT – 3 and Normal distribution PDF - IDF models for the fourteen locations (see Tables 12 and 13 respectively).

Table 10. Computed rainfall intensity values for normal distribution for Benin City

Duration (mins)	Return period (Years)					
	2	5	10	25	50	100
10	57.54	75.82	85.39	95.59	102.18	108.10
20	54.09	70.49	79.06	88.21	94.11	99.43
30	45.84	63.07	72.08	81.69	87.89	93.48
40	48.64	64.40	72.64	81.43	87.10	92.20
50	40.92	55.11	62.53	70.44	75.55	80.15
60	52.25	61.21	65.90	70.91	74.13	77.04
90	37.22	47.30	52.58	58.20	61.83	65.10
120	34.35	41.90	45.84	50.05	52.77	55.21

Table 11. Summary of Gumbel EVT-1 PDF general rainfall IDF models for various locations

S/N	Locations	IDF models	R^2	MSE
1	Abakaliki	$I = \frac{816.88T_r^{0.1782}}{T_d^{0.7506}}$	0.890	216.1
2	Benin	$I = \frac{95.973T_r^{0.1875}}{T_d^{0.2633}}$	0.933	35.82
3	Calabar	$I = \frac{34081.74T_r^{0.1802}}{T_d^{1.7502}}$	0.992	213.7
4	Enugu	$I = \frac{150T_r^{0.2016}}{T_d^{0.3295}}$	0.896	114.3
5	Onitsha	$I = \frac{270.39T_r^{0.2225}}{T_d^{0.5455}}$	0.875	224.6
6	Owerri	$I = \frac{288.08T_r^{0.2213}}{T_d^{0.4809}}$	0.928	181.1
7	Port Harcourt	$I = \frac{416.54T_r^{0.2412}}{T_d^{0.5613}}$	0.975	109.39
8	Uyo	$I = \frac{323.03T_r^{0.1966}}{T_d^{0.4110}}$	0.919	221.5
9	Warri	$I = \frac{449.23T_r^{0.2285}}{T_d^{0.665}}$	0.901	302.3
10	Abeokuta	$I = \frac{551.809T_r^{0.188}}{T_d^{0.596}}$	0.987	147.70
11	Akure	$I = \frac{407.886T_r^{0.175}}{T_d^{0.525}}$	0.982	125.70
12	Ibadan	$I = \frac{394.69T_r^{0.175}}{T_d^{0.554}}$	0.990	57.05
13	Ikeja	$I = \frac{527.802T_r^{0.181}}{T_d^{0.581}}$	0.990	95.27
14	Osogbo	$I = \frac{305.106T_r^{0.188}}{T_d^{0.603}}$	0.990	33.17

Table 12. Summary of LPT-3 PDF general rainfall IDF models for various locations

S/N	Locations	IDF models	R^2	MSE
1	Abakaliki	$I = \frac{706.92T_r^{0.2112}}{T_d^{0.7081}}$	0.895	218.0
2	Benin	$I = \frac{98.299T_r^{0.1876}}{T_d^{0.2290}}$	0.889	65.34
3	Calabar	$I = \frac{28153.233T_r^{0.1635}}{T_d^{1.6637}}$	0.987	341.8
4	Enugu	$I = \frac{162.557T_r^{0.1835}}{T_d^{0.3333}}$	0.899	106.1
5	Onitsha	$I = \frac{326.70T_r^{0.2926}}{T_d^{0.6492}}$	0.870	405.5
6	Owerri	$I = \frac{311.11T_r^{0.2291}}{T_d^{0.5059}}$	0.9133	248.2
7	Port Harcourt	$I = \frac{481.679T_r^{0.30}}{T_d^{0.654}}$	0.961	256.84
8	Uyo	$I = \frac{427.32T_r^{0.1036}}{T_d^{0.4328}}$	0.871	260.5
9	Warri	$I = \frac{628.06T_r^{0.3011}}{T_d^{0.8342}}$	0.849	796.7
10	Abeokuta	$I = \frac{561.676T_r^{0.211}}{T_d^{0.622}}$	0.987	162.31
11	Akure	$I = \frac{402.607T_r^{0.201}}{T_d^{0.540}}$	0.984	127.47
12	Ibadan	$I = \frac{375.689T_r^{0.210}}{T_d^{0.564}}$	0.987	84.95

S/N	Locations	IDF models	R^2	MSE
13	Ikeja	$I = \frac{519.214T_r^{0.204}}{T_d^{0.592}}$	0.990	106.86
14	Osogbo	$I = \frac{326.687T_r^{0.140}}{T_d^{0.582}}$	0.968	91.86

Table 13. Summary of normal PDF general rainfall IDF models for various locations

S/N	Locations	IDF Models	R^2	MSE
1	Abakaliki	$I = \frac{931.19T_r^{0.1275}}{T_d^{0.7649}}$	0.880	200.2
2	Benin	$I = \frac{104.54T_r^{0.1346}}{T_d^{0.2205}}$	0.904	32.07
3	Calabar	$I = \frac{37955.94T_r^{0.1293}}{T_d^{1.7524}}$	0.992	194.7
4	Enugu	$I = \frac{156.20T_r^{0.1518}}{T_d^{0.3196}}$	0.839	123.6
5	Onitsha	$I = \frac{286.89T_r^{0.1608}}{T_d^{0.5242}}$	0.861	178.8
6	Owerri	$I = \frac{311..06T_r^{0.1601}}{T_d^{0.4660}}$	0.903	173.1
7	Port Harcourt	$I = \frac{443.67T_r^{0.175}}{T_d^{0.538}}$	0.970	92.71
8	Uyo	$I = \frac{358.00T_r^{0.1415}}{T_d^{0.4090}}$	0.906	194.1
9	Warri	$I = \frac{469.30T_r^{0.1652}}{T_d^{0.6362}}$	0.89	244.6
10	Abeokuta	$I = \frac{603.235T_r^{0.135}}{T_d^{0.589}}$	0.984	149.53
11	Akure	$I = \frac{448.634T_r^{0.125}}{T_d^{0.523}}$	0.980	116.87
12	Ibadan	$I = \frac{435.316T_r^{0.125}}{T_d^{0.553}}$	0.990	50.50
13	Ikeja	$I = \frac{579.532T_r^{0.130}}{T_d^{0.577}}$	0.989	95.29
14	Osogbo	$I = \frac{330.983T_r^{0.135}}{T_d^{0.592}}$	0.989	29.23

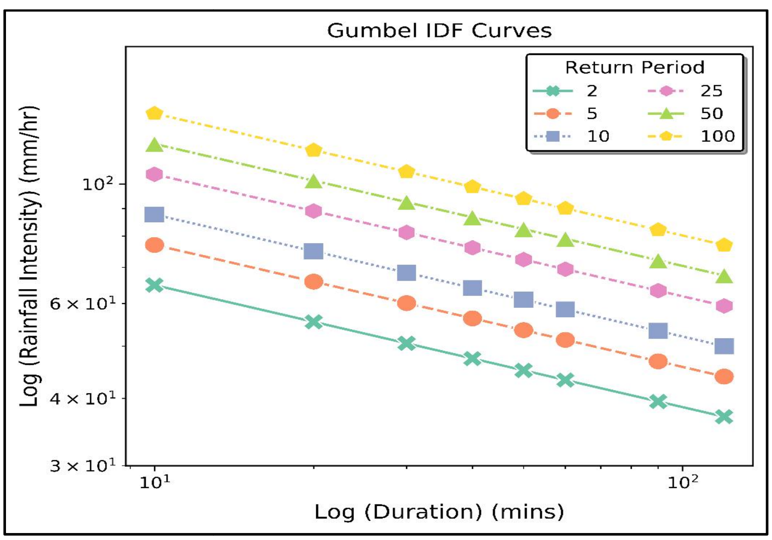


Fig. 2. IDF curves for intensities of rainfall predicted from Gumbel EVT-1 general model on a Log-Log scale for Benin City

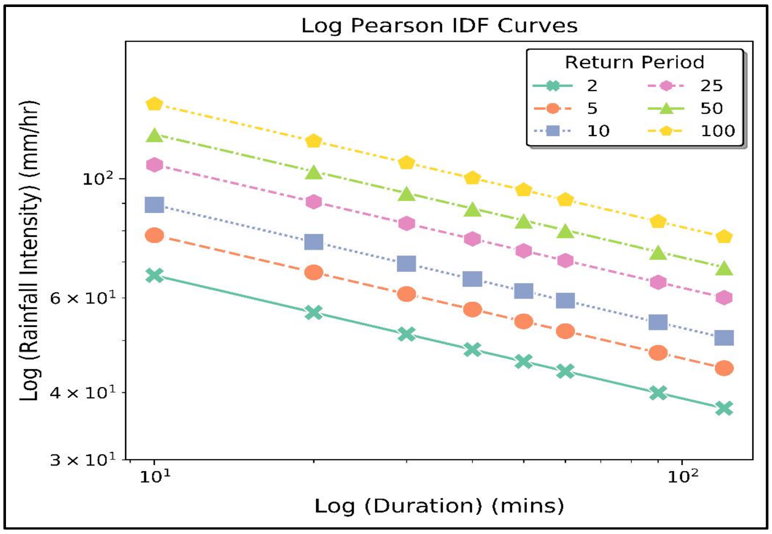


Fig. 3. IDF curves for intensities of rainfall predicted from LPT-3 general model on a Log-Log scale for Benin City

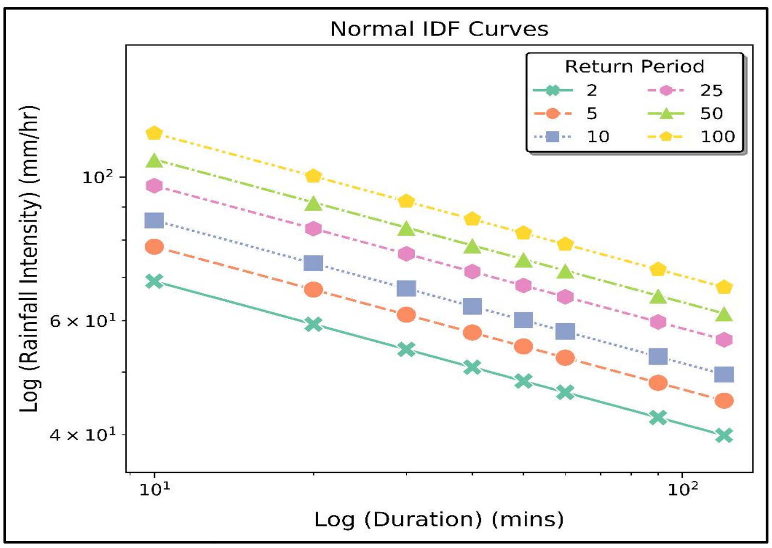


Fig. 4. IDF curves for intensities of rainfall predicted from normal general model on a Log-Log scale for Benin City

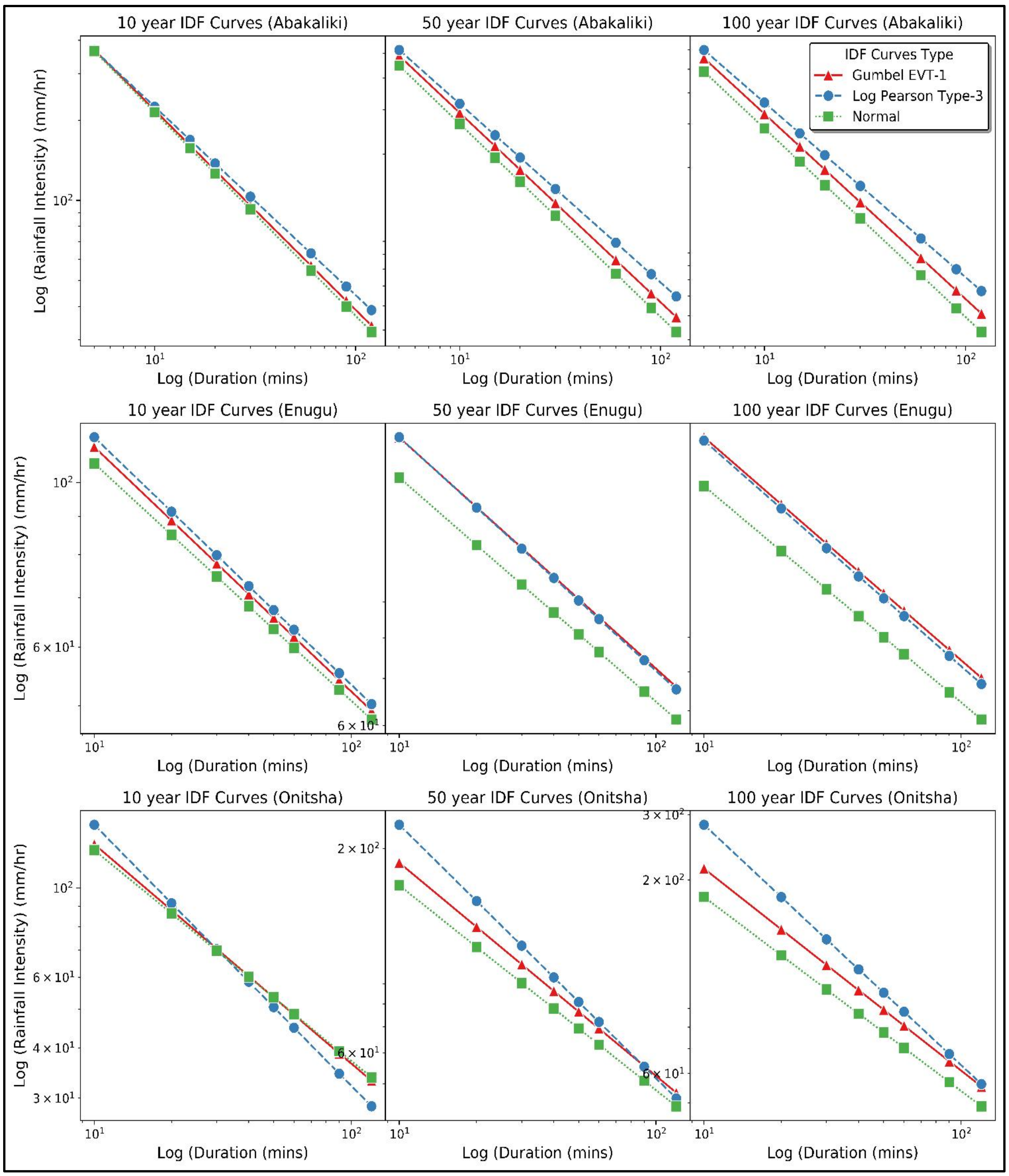
3.1.3 Rainfall intensity distribution curves

For purpose of illustration, the rainfall PDF - IDF curves for Benin city were generated for GEVT – 1, LPT – 3 and Normal distributions respectively (See Figs. 2 to 4). For each case, a family of curves for return periods of 2, 5, 10, 25, 50 and 100 years for various durations were plotted on logarithmic scale (log – log plot).

3.1.4 Comparative analysis of GEVT – 1, LPT – 3 and normal PDF - IDF general models

The performance evaluation of the three PDF – IDF model types were carried out for GEVT – 1, LPT – 3 and Normal models, first through plotting of predicted intensity values for 10, 50 and 100 year return periods at specified durations. The first four sets of plots represent the South East, second set of five plots represent South – South and the third five sets stand for South – West (see Figs. 5, 6 and 7) respectively. Sequel to the figures, second evaluation was made in form of ranking of the three PDF – IDF model types using the plots in order of 1st, 2nd and 3rd positions respectively (see Table 14).

The computation of the skewness of the observed logarithmic rainfall intensities was carried out as form of normality test. Given the result of the skewness test (see Table 15), it is obvious that a non – parametric statistic (Kruskal Wallis test) was appropriate for test of significance at 5% level.



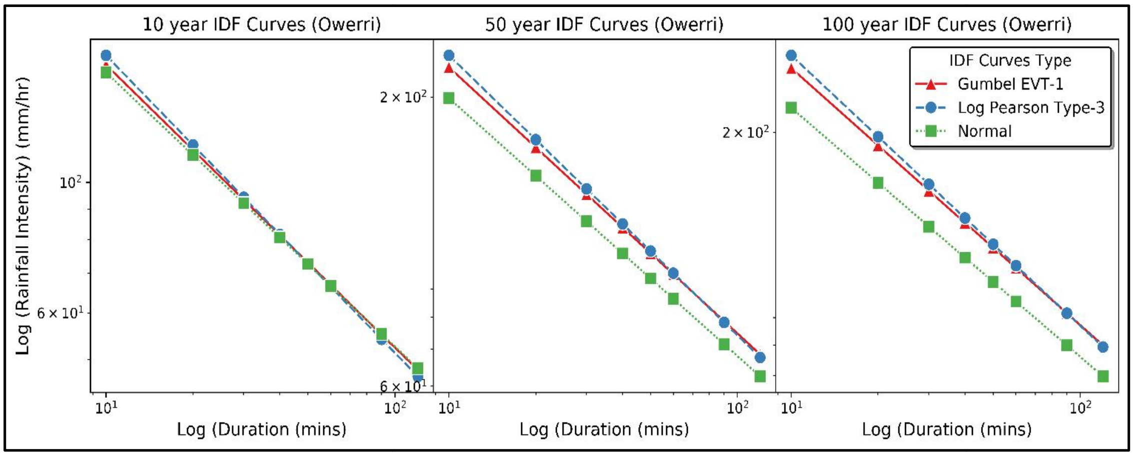
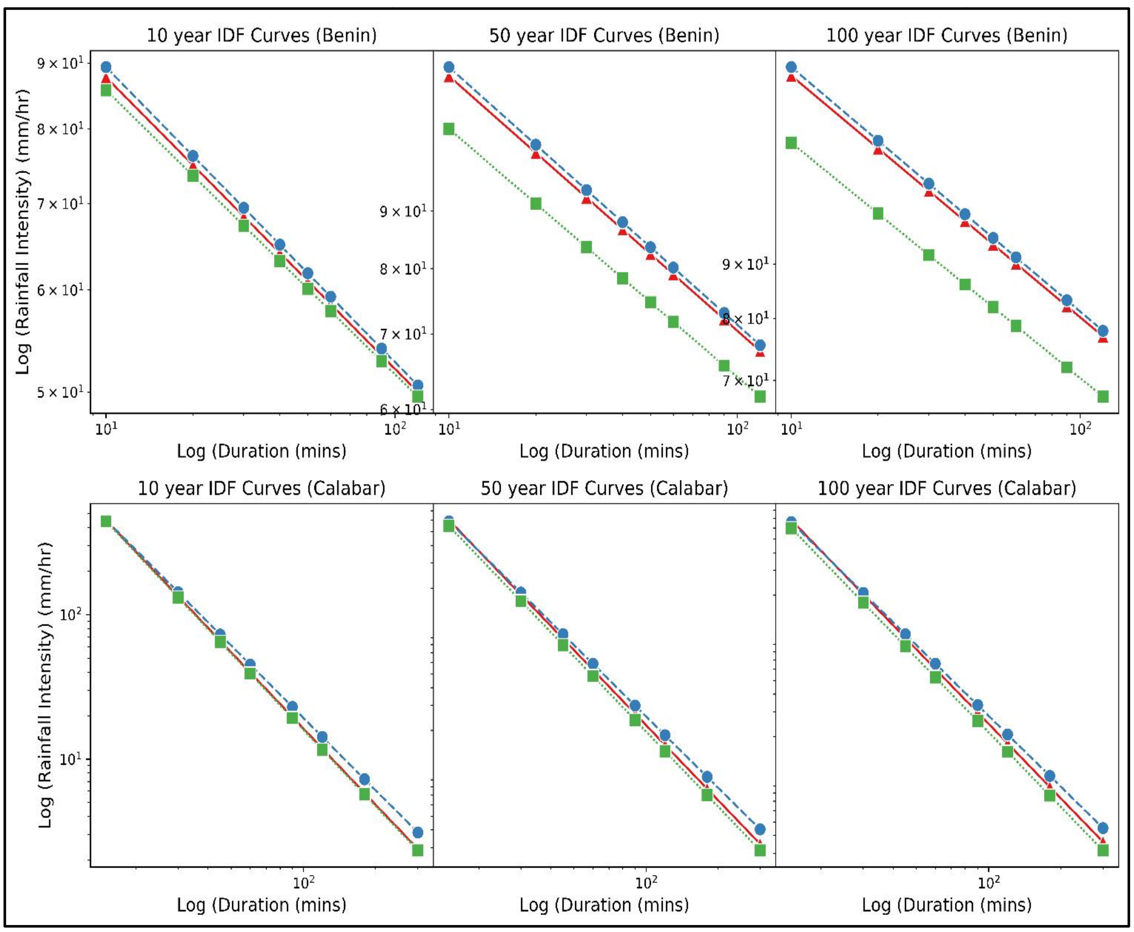


Fig. 5. Comparison of IDF type curves distribution for 10, 50 and 100 years return periods for South – East locations



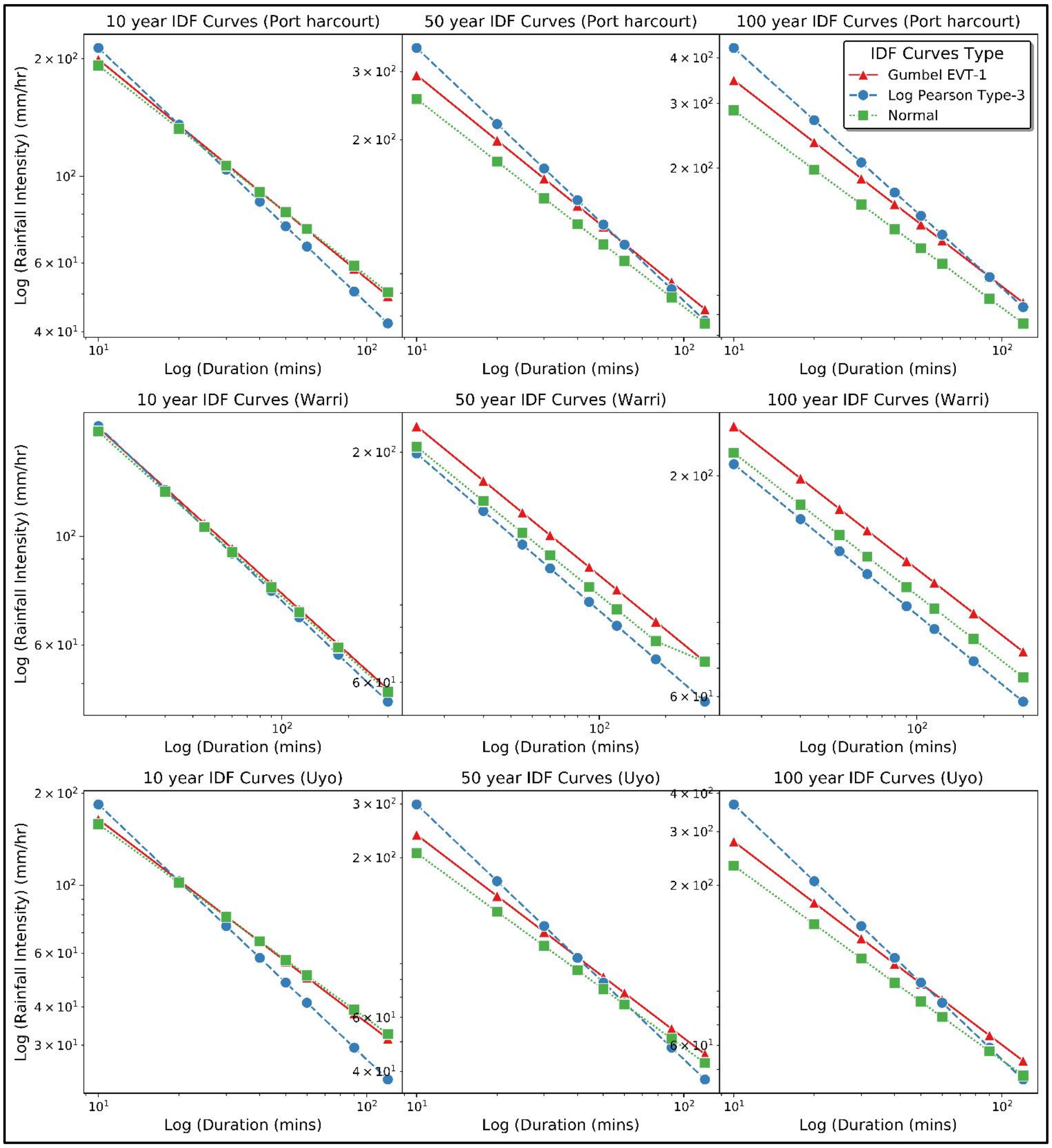
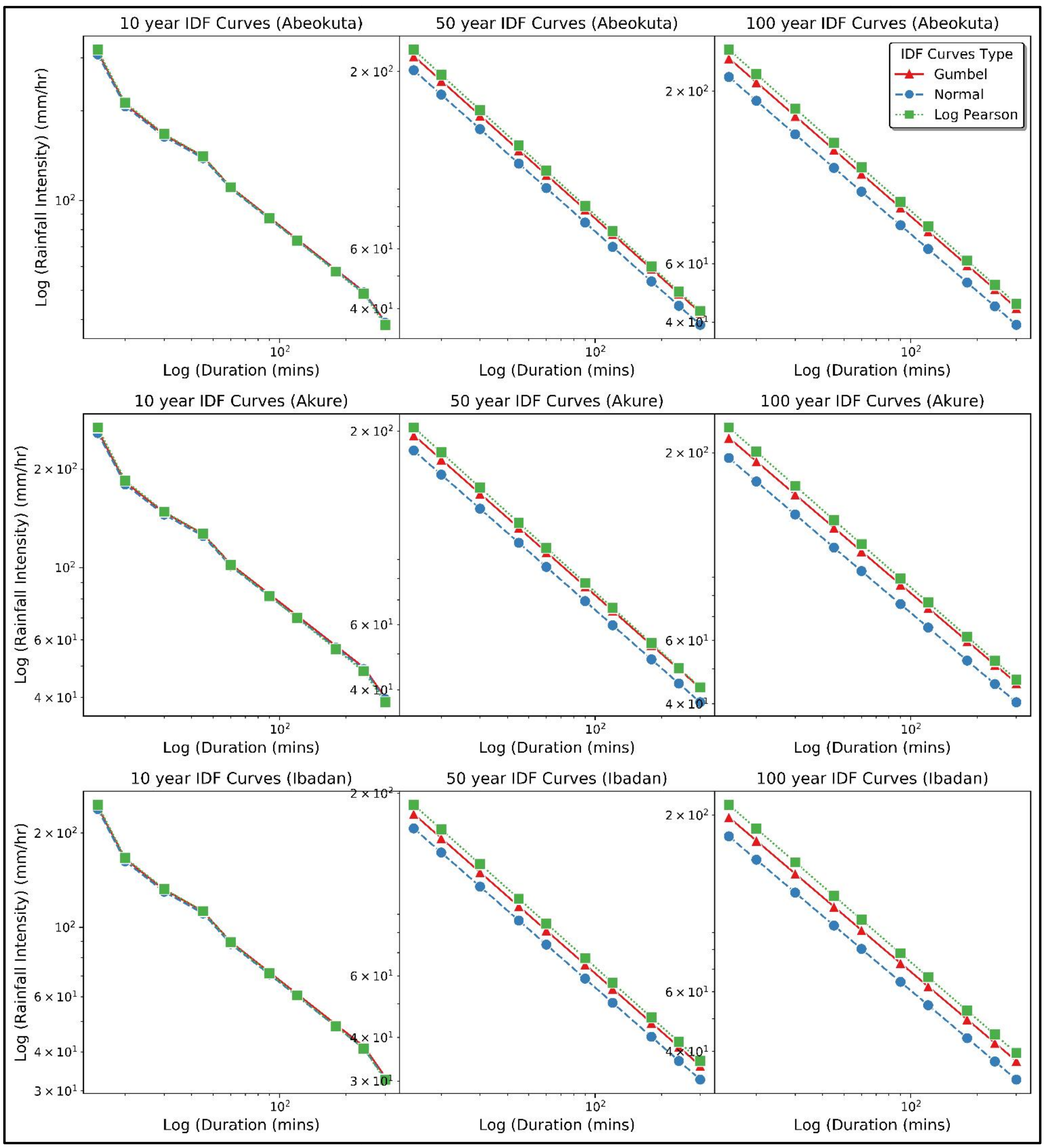


Fig. 6. Comparison of IDF type curves distribution for 10, 50 and 100 years return periods for South - South locations

Table 14. Graphical comparison for best model in predicting intensity for 10, 50 and 100 year return periods

Location	10 year return period			50 year return period			100 year return period		
	GEVT-1	LPT-3	Normal	GEVT-1	LPT-3	Normal	GEVT-1	LPT-3	Normal
Abakaliki	2nd	1st	3rd	2nd	1st	3rd	2nd	1st	3rd
Benin	2nd	1st	3rd	2nd	1st	3rd	2nd	1st	3rd
Calabar	2nd	1st	3rd	2nd	1st	3rd	2nd	1st	3rd
Enugu	2nd	1st	3rd	1st	2nd	3rd	1st	2nd	3rd

Location	10 year return period			50 year return period			100 year return period		
	GEVT-1	LPT-3	Normal	GEVT-1	LPT-3	Normal	GEVT-1	LPT-3	Normal
Onitsha	1st	3rd	2nd	2nd	1st	3rd	2nd	1st	3rd
Owerri	2nd	1st	3rd	2nd	1st	3rd	2nd	1st	3rd
Port Harcourt	1st	3rd	2nd	2nd	1st	3rd	2nd	1st	3rd
Uyo	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd
Warri	2nd	3rd	1st	2nd	1st	3rd	2nd	1st	3rd
Abeokuta	2nd	1st	3rd	2nd	1st	3rd	2nd	1st	3rd
Akure	2nd	1st	3rd	2nd	1st	3rd	2nd	1st	3rd
Ibadan	2nd	1st	3rd	2nd	1st	3rd	2nd	1st	3rd
Ikeja	2nd	1st	3rd	2nd	1st	3rd	2nd	1st	3rd
Osogbo	2nd	1st	3rd	1st	2nd	3rd	1st	2nd	3rd



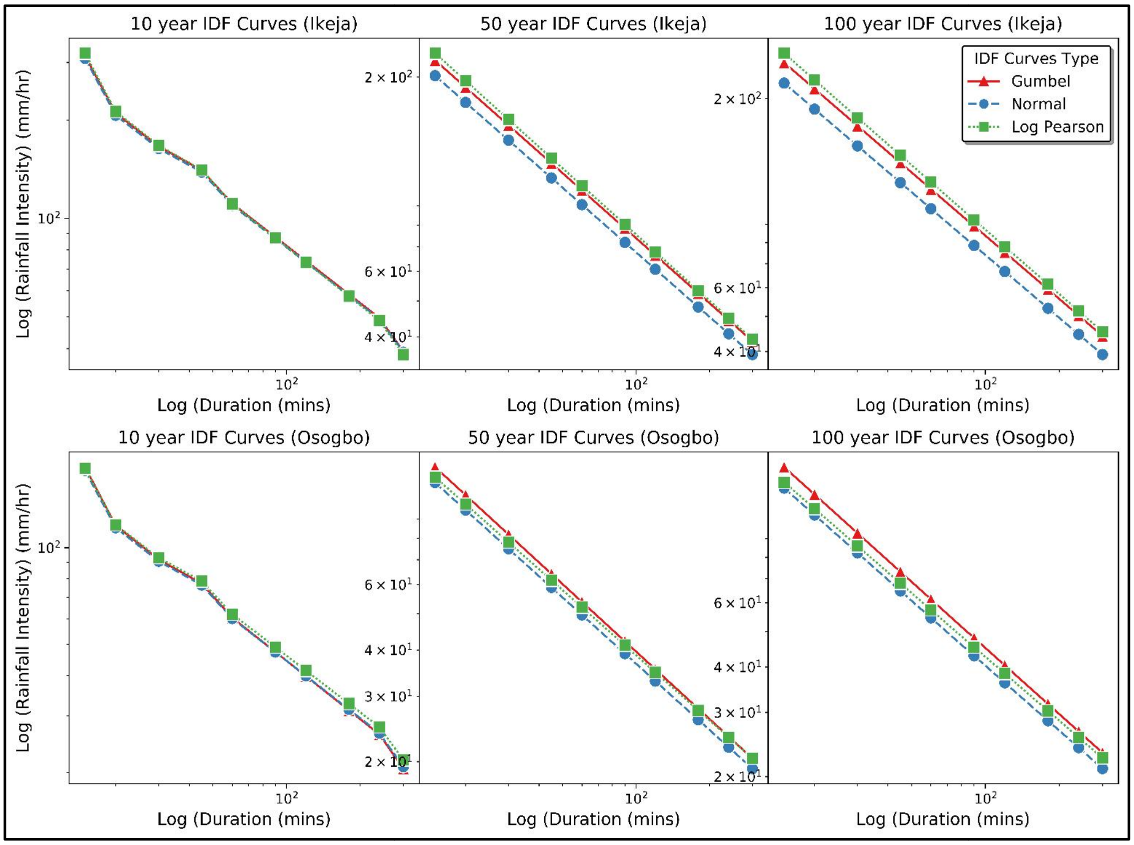


Fig. 7. Comparison of IDF type curves distribution for 10, 50 and 100 years return periods for South - West locations

Table 15. Coefficient of Skewness for observed logarithmic annual rainfall intensities for different locations

Location	Durations (mins)							
	10	**20**	**30**	**40**	**50**	**60**	**90**	**120**
Abakaliki	0.88	0.88	0.95	0.64	0.51	1.32	1.02	1.05
Benin	0.26	0.52	0.16	0.81	1.73	0.53	0.45	-0.16
Calabar	-0.09	0.78	2.22	0.72	-0.18	1.53	0.50	-0.86
Enugu	-0.18	0.27	-0.35	0.43	-0.15	0.07	0.57	0.73
Onitsha	0.77	1.23	0.79	0.87	1.21	0.03	0.34	0.89
Owerri	0.74	1.11	1.52	0.23	1.41	0.12	-0.17	-0.05
Port Harcourt	0.26	0.23	-0.26	-0.28	-0.56	-0.25	-0.29	-0.18
Uyo	-0.78	-0.64	-1.08	-0.61	-1.38	-1.50	-1.85	-1.30
Warri	1.39	-0.06	-0.53	1.32	0.87	0.39	-0.47	0.10
Abeokuta	0.16	0.25	0.64	0.26	0.51	0.53	0.95	1.32
Akure	0.85	0.72	0.62	0.29	0.33	0.43	0.06	2.36
Ibadan	1.09	0.74	0.62	0.36	0.17	-0.24	-0.23	0.27
Ikeja	0.85	0.72	0.62	0.29	0.33	0.43	0.06	2.36
Osogbo	0.67	0.51	0.16	0.40	0.11	0.59	-0.37	-0.46

The graphical abstract is shown in Fig. 8 below:

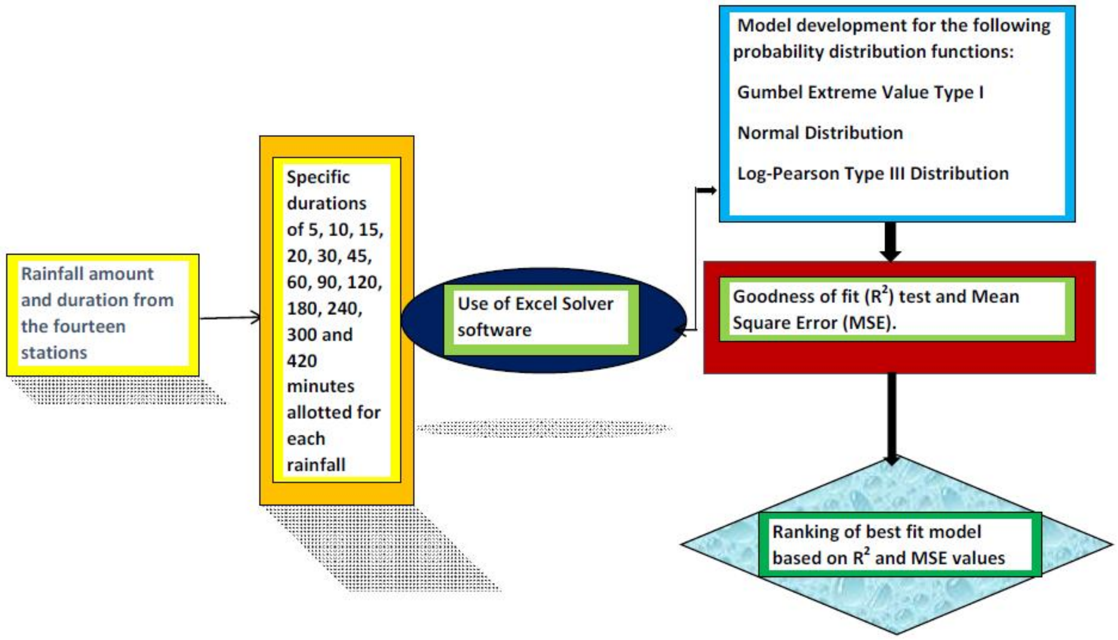


Fig. 8. Model calibration graphical abstract

3.2 Discussion

3.2.1 PDF & rainfall intensity transformation

The logarithmic form of the rainfall intensities, that is, observed rainfall amount over duration with descriptive statistics that gives information on mean, standard deviation and skewness of rainfall intensities for different durations is as shown in Table 4. The computed standard deviation values were found useful in the transformation of the rainfall intensities using GEVT – 1, LPT – 3 and Normal distribution functions. Also, the coefficient of skewness substitutes for normality test for rainfall intensity values for a given duration; if its value is zero then it is normally distributed otherwise it is skewed. The benefit of this approach is to adopt the non – parametric statistics in further data analysis where normal distribution does not exist which is the case for all the rainfall intensity values at the fourteen study locations. The rainfall intensity transformation was actualized using Equation (2) that yielded GEVT -1, LPT – 3 and Normal distributions herein taken as PDF - IDF values. The use of Equation (2) to transform rainfall intensities yielded equivalent PDF intensity values. Further use of the PDF intensity values to calibrate Sherman's empirical rainfall (Equation (1)) gives rise to what is called PDF – IDF models [18,19,26,30].

3.2.2 Sherman's model Calibration for general models

The general PDF - IDF models for GEVT – 1, LPT - 3 and Normal distributions are as presented in Tables 5, 6 and 7 respectively. They are of high correlation values with goodness of fit (R^2) and Mean Squared Errors (MSE) ranging from: (a) R^2 = 0.875 – 0.992; MSE = 33.17 – 224.6 for GEVT – 1; (b) R^2 = 0.849 – 0.990; MSE = 65.34 – 405.5 for LPT – 3; and (c) R^2 = 0.839 – 0.992; MSE = 29.23 – 200.2 for Normal distribution. These results are in agreement with previous studies in Nigeria [18,19,26,29,30]. The general models have advantage of predicting rainfall intensities at various durations and specified return periods over other empirical models with serial numbers 1 – 5 in Table 1 which do not have inbuilt return period parameter as against serial number 6 equation, modified as Equation (1).

3.2.3 Rainfall intensity distribution curves

As a typical example, the predicted rainfall intensities from the general model for GEVT – 1, LPT – 3 and Normal distributions were plotted in log – log graph paper (Figs. 2, 3 and 4). The remaining thirteen locations show similar features. In all the three PDF-IDF models the values of intensity decrease with increasing duration. For Figs. 2, 3 and 4, each contains a total of six plots for different return periods (2, 5, 10, 25, 50 and 100 years). Among the common features of the IDF curves observed in the plots are:

(i) Intensity decreases with increase in duration;
(ii) Intensity increases with increase in return period for a given duration; and
(iii) Maximum intensity occurs at short duration for a given return period and also higher intensity at longer return period for a given duration and these are in keeping with results found in literature [2,23,31].

3.2.4 Comparative analysis of GEVT– 1, LPT– 3 and ND PDF - IDF general models

The logarithmic plots of the rainfall intensities versus durations for the fourteen locations as presented in Figs. 5 to 7 indicate visible differences in the predictive performance of the three model types. Sequel to this, performance ranking was carried out for the three PDF – IDF model types for return periods of 10, 50 and 100 years (see Table 14). The outcome of the ranking in the order of 1st, 2nd & 3rd positions are as follows: (a) for 10 years return period, 1st: LPT – 3 = 71.4%, GEVT – 1 = 21.4% & ND = 7.1%, for 2nd position GEVT – 1 = 78.6%, ND = 14.3% & LPT – 3 = 7.1% and for 3rd position, ND = 78.6%, LPT – 3 = 21.4% & GEVT – 1 = 0.0%; (b) for 50 year return period, 1st position: LPT – 3 = 78.6%, GEVT – 1 = 21.4% & ND = 0.0%; for 2nd position: GEVT – 1 = 78.6%, LPT – 3 = 21.4% & ND = 0.0% and for 3rd position, ND = 100% & others = 0.0%; (c) In the case of 100 years return period, 1st position we have LPT - 3 = 78.6%, GEVT – 1 = 21.4% & ND = 0.0%; for 2nd position, GEVT -1 = 78.6%, LPT – 3 = 21.4% and ND = 0.0% and for 3rd position, ND = 100%, others = 0.0%.

From the trend of events, it is obvious that the order of best performance is LPT – 3 1st, GEVT - 1 2nd and ND 3rd for each of the return period (10, 50 and 100 year). These observations are in agreement with those in literature [26,32].

The Kruskal Wallis test of significance as a non – parametric statistic was carried out on the predicted rainfall intensities for 10, 50 and 100 years return periods same as those used to plot Figs. 5, 6 and 7. The results indicate as follows: for 10 years return period, the computed p-values range from 0.8521 to 0.9827; 50 years return period 0.3526 to 0.9254 and for 100 years return period 0.2423 to 0.8789. Given that the computed p-values are greater than the significant level of 0.05% that implies no significant difference across board. It is interesting to note that the same intensity values as per Figs. 5 to 7 show visible differences yet these differences are not significant. That is to say that the fourteen locations are from the same population; in-order words the fourteen locations in Southern Nigeria (see Fig. 1) are within the same climatic region, all bordering the Atlantic Ocean.

4. CONCLUSION

The developed PDF – IDF model types for GEVT – 1, ND and LPT - 3 distributions are in agreement with PDF theory which shows higher intensity occurrence at shorter durations and lower intensity values at longer durations. The prediction of rainfall intensity with the Probability Distribution Functions (PDF) are of high goodness of fit (R^2) and low Mean Squared Errors (MSE) ranging from: (a) R^2 = 0.875 – 0.992; MSE = 33.17 – 224.6 for GEVT – 1; (b) R^2 = 0.849 – 0.990; MSE = 65.34 – 405.5 for LPT – 3 and (c) R^2 = 0.839 – 0.992; MSE = 29.23 – 200.2 for Normal distribution. The comparative analysis of all the general PDF-IDF model types considered showed that the order of best performance is 1st position = LPT – 3, 2nd = GEVT - 1 and 3rd = ND for the study area. The Kruskal Wallis test of significance indicates as follows: for 10 year return period, the computed p-values range from 0.8521 to 0.9827; 50 year return period the values range from 0.3526 to 0.9254 and for 100 year return period the values range from 0.2423 to 0.8789. Given that the computed p-values are greater than the significant level of 0.05%, that implies no significant difference exists across board. Therefore, these developed general PDF-IDF models are recommended for the computation of intensities in the fourteen locations for the design of flood control structures.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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