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Review

Critical thinking: Essence for teaching mathematics and mathematics problem solving skills

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Critical thinking is a learned skill that requires instruction and practice. Mathematics education instructors at both the secondary and post-secondary levels can enhance students' critical thinking skills by (i) using instructional strategies that actively engage students in the learning process rather than relying on lecture and note memorization, (ii) focusing instruction on the process of learning rather than solely on the content, and (iii) using assessment techniques that provide students with an intellectual challenge rather than memory recall. Several barriers can impede critical thinking instruction. Lack of training, limited resources, biased preconceptions and time constraints conspire to negate learning environments that promote critical thinking. However, actively engaging students in project-based or collaborative activities can encourage students' critical thinking development if instructors model the thinking process, use effective questioning techniques and guide students' critical thinking processes. The examples provided challenge instructors to think of students as users of information rather than receivers of information. 'It is possible to store the mind with a million facts and still be entirely uneducated.''

Key words: Mathematics, problem solving skills, instructional strategies, critical thinking.

INTRODUCTION

What is critical thinking and why is it so important? The critical thinking community defined critical thinking as "the intellectually disciplined process of actively and skillfully conceptualizing, applying, analyzing, synthesizing, and/or evaluating information gathered from, or generated by observation, experience, reflection, reasoning, or communication as a guide to belief and action" (Scriven and Paul, 2007). Critical thinking has also been referred to as met cognition (Tempelaar, 2006) or the process of "thinking about thinking" as defined and originally purposed by Flavell (1979). Critical thinking skills are important because they enable students ''to deal effectively with social, scientific, and practical problems'' (Shakirova, 2007). Simply put, students who are able to think critically are able to solve problems effectively. Merely having knowledge or information is not enough. To be effective in the workplace (and in their personal lives), students must be able to solve problems to make effective decisions, they must be able to think critically.

Yet many teachers continually struggle to engage students in critical thinking activities (Templeaar, 2006), and students seldom use critical thinking skills to solve complex, real-world problems (Bartlett, 2002; Rippin et al., 2002). Why?

The answer may be in our instructional methods. Two quotes that are often cited together reflect this supposition (Schafersman, 1991). First, Clement (1979) stated that ''we should be teaching students how to think. Instead, we are teaching them what to think''. Secondly, Norman (1981) noted that ''it is strange that we expect students to learn, yet seldom teach them anything about learning.'' Although content is important, the process of how students learn the material is equally important.

Objective

The main objective of this article is to analyze and synthesize secondary research to provide best practices for incorporating critical thinking instructional methods into mathematics education classrooms at both the

secondary and post-secondary levels. The underlisted areas will be discussed in this article:

1) Relation of critical thinking to Bloom et al.'s taxonomy of the cognitive domain;
2) Relation of critical thinking to instructional design;
3) Modeling critical thinking skill;
4) Questioning techniques;
5) Guiding student critical thinking
6) Barriers to critical thinking.

RELATION OF CRITICAL THINKING TO BLOOM ET AL.'S TAXONOMY OF THE COGNITIVE DOMAIN

Bloom and his colleagues in 1956 produced one of the most often cited documents in establishing educational outcomes - The taxonomy of the cognitive domain. They proposed that knowing is actually composed of six successive levels arranged in a hierarchy: knowledge, comprehension, application, analysis, synthesis and evaluation. Research over the past 40 years has generally confirmed that the first four levels are indeed a true hierarchy. That is, knowing at the knowledge level is easier than and subsumed number, the level of comprehension and so forth up to the level of analysis. However, research is mixed on the relationship of synthesis and evaluation. It is possible that these two are reversal or they could be two separate, though equally difficult activities. Synthesis and evaluation are two types of thinking that have much in common (the first four levels of Bloom's taxonomy, but are quite different in thinking) requires an individual to look at parts and well purpose. Evaluation, which might be considered equivalent to critical thinking as used in this paper focuses on making an assessment or judgment based on an analysis of a statement or proposition. Synthesis, which might be considered more equivalent to creative thinking requires an individual to look at parts and relationships (analysis) and then to put these together in a new and original way.

There is some evidence to suggest that this equivalent but different relationship between critical/ evaluative and creative synthesis thinking is appropriate. Huitt (1992) classified techniques used in problem solving and decision making into two groups roughly corresponding to the critical/creative dichotomy. One set of techniques tended to be more linear and serial, more structured, more rational and analytical and more goal-oriented, these techniques are often taught as part of critical thinking exercises. The second set of techniques tended to be more holistic and parallel, more emotional and intuitive, more creative, more visual and more tactual/ kinesthetic, these techniques are more often taught as part of creative thinking exercises. This distinction also corresponds to what is sometimes referred to as left brain thinking analytic, serial, logical, objective as compared to right brain thinking global, parallel, emotional and subjective.

RELATION OF CRITICAL THINKING TO INSTRUCTIONAL DESIGN

Those who have the ability to hear, do not always actively listen. Similarly, those who have the ability to know, do not always critically think. The premise that critical thinking is to knowing as listening is to hearing implies that critical thinking is a learned skill that must be developed, practiced, and continually integrated into the curriculum to engage students in active learning. To support this premise, focused attention needs to be placed on the application of content, the process of learning, and methods of assessment.

In terms of the application of content, teaching techniques that promote memorization offer temporary knowledge do not support critical thinking. Although some content, such as vocabulary definitions, do require memory, it is the application of the content that stimulates thinking. Instruction that supports critical thinking uses questioning techniques that require students to analyze, synthesize, and evaluate information to solve problems and make decisions (think) rather than merely to repeat information (memorize). Because critical thinking is a mental habit that requires students to think about their thinking and about improving the process, it requires students to use higher-order thinking skills, not memorize data or accept what they read or are told without critically thinking about it (Scriven and Paul, 2007; Schafersman, 1991; Templeaar, 2006). Therefore, critical thinking is a product of education, training, and practice.

To link critical thinking skills to content, the instructional focus should be on the process of learning. How will the students get the information? Research supports the premise that lecture and memorization do not lead to long term knowledge or the ability to apply that knowledge to new situations (Celuch and Slama, 1999; Daz-Lefebvre, 2004; Kang and Howren, 2004). Traditional instructional methods use too many facts and not enough conceptualization; too much memorizing and not enough thinking. Therefore, lecture and note memorization do not promote critical thinking. Instructional strategies that employ students' higher order thinking skills lead to improved critical thinking skills (Duplass and Ziedler, 2002; Hemming, 2000; Wong, 2007).

Additionally, assessments should emphasize thinking rather than facts (Ennis, 1993). Graded assignments, quizzes, or tests should become intellectual challenges rather than memory recall (Schafersman, 1991). Subjective tools such as essay questions and case studies require students to apply their knowledge to new situations and are better indicators of understanding than

objective true/false or standardized multiple choice assessments. However, instructors can create multiple choice questions that require critical thinking. For example, a question that asks students to identify the example that best applies a specific concept requires more critical thinking and analysis than a question that asks students to identify the correct term for a given definition. Ennis (1993) stated that although they are more labour intensive to create than equally effective open ended critical thinking assessments, multiple choice tests are easier to grade. To enhance students' processing skills, it is important to review test questions and explain correct answers by modeling the critical thinking process (Brown and Kelly, 1986; Duplass and Ziedler, 2002; Schafersman,1991).

MODELING CRITICAL THINKING SKILLS

Although mathematics education students perceive critical thinking as an important skill (Davis et al., 2003), they typically do not know how to think critically. Students are not born with the ability to think critically, and their prior learning experiences often do not require them to think critically. Therefore, instructors who wish to integrate this skill in their classroom experiences must first model the behavior (Hemming, 2000). Students must learn how to think critically before they can apply the skill to content scenarios. Modeling can be demonstrated in a discussion setting by asking a question and ''walking students through'' the process of critically thinking.

Further, critical thinking activities should be based on a structure that includes four elements; ''ill-structured problems, criteria for assessing thinking, student assessment of thinking and improvement of thinking'' (Broadbear, 2003). Ill-structured problems are questions, case studies, or scenarios that do not have a definite right or wrong answer, they include debatable issues that require reflective judgment. For example, asking students to evaluate comparable websites, such as Wal-Mart and Target, requires them to think about the content of the websites, their format and their usability. Right and wrong answers do not exist as long as the student's choice is supported by logical reasoning. The second element, criteria for assessing thinking, provides students with a framework for thinking about their thinking. Why do you think Target's navigational menus are easier to use than Wal-Mart's? Why do you like one's colour scheme over the other? What is your perspective based upon? Providing students with individualized feedback based on their responses allows them to address specific criteria upon which they can assess their thinking, which is the third element. If instructors model the criteria for assessing thinking and provide a framework, students will eventually apply these techniques on their own (Lundquist, 1999).

Finally, the process concludes with improvement of thinking. By creating a culture of inquiry where students can think about their thinking processes and practice logical constructs, students will become more willing to reconsider and revise their thinking (Duplass and Ziedler, 2002).

QUESTIONING TECHNIQUES

In their research, Haynes and Bailey (2003) emphasized the importance of asking the right questions to stimulate students' critical thinking skills. Other researchers (Brown and Kelley, 1996; Hemming, 2000) also focused on integrating questioning techniques into class discussions to support an educational environment where students can demonstrate and practice critical thinking skills. Brown and Kelley's book, Asking the Right Questions: A guide to critical thinking, documented the premise that students' critical thinking is best supported when instructors use critical questioning techniques to engage students actively in the learning process. Sample questions from all these studies include the following:

1) What do you think about this?
2) Why do you think that?
3) What is your knowledge based upon?
4) What does it imply and presuppose?
5) How are you viewing it?
6) Should it be viewed differently?

These questions require students to evaluate the clarity and accuracy of their thinking as well as the depth and breadth of their thinking. Have they considered all the alternatives? Do they know why they think the way they do? Students need to determine whether the content they are using is relevant and if their thinking process is logical. By questioning their thought process, students can begin thinking about their thinking. Research on questioning methodology also suggests that instructors should wait for student responses (Brown and Kelley, 1986; Hemming, 2000). Too often, the students' silence is filled by the instructor rewording the question or asking a different student for a response. However, most students need at least 8 to 12 s to process and formulate their response, especially in critical thinking situations (Schafersman, 1991). If a question is based on note memory recall, speed may be relevant. However, thinking requires time and patience. Give students the time they need to think critically.

Research also provides strategies for using questioning methods in online learning environments (Astleitner, 2002; MacKnight 2000). Discussion boards, virtual chat rooms and instant messages provide forums for questioning and critical thinking. In synchronous environments, instructors can question students as they construct their responses. Although this is not possible

with asynchronous communication, instructors can model the critical thinking process and assign activities that utilize questioning techniques and critical thinking responses. In all learning environments, it is important to guide students through the critical thinking process.

GUIDING STUDENTS' CRITICAL THINKING

When students are accustomed to being passive learners by merely memorizing and recalling information, it may be difficult at first to engage them in active learning situations that require critical thinking skills (Brown and Kelley, 1986). Instructors should be aware of students' initial resistance and guide them through the process to create a learning environment where students feel comfortable thinking through an answer rather than simply having an answer. For example, peer coaching techniques can engage students in active learning and critical thinking opportunities (Ladyshewsky, 2006). Assign students to two-person teams; one student is the problem solver, and the other is the peer coach. Using the six steps to effective thinking and problem solving, or ''IDEALS'' (Facione, 2007), the problem solver works through a case study or activity by responding to questions from the peer coach. The IDEALS are to identify, define, enumerate, analyze, list and self correct:

I – Identify the problem: What is the real question we are facing?
D – Define the context: What are the facts that frame this problem?
E – Enumerate the choices: What are plausible options?
A – Analyze options: What is the best course of action?
L – List reasons explicitly: Why is this the best course of action?
S – Self correct: look at it again, what did we miss?

This problem solving technique guides students through the critical thinking process and utilizes learner collaboration. Similar strategies include integrating project based learning activities that require students to apply their knowledge by constructing a real world product. As a final guide to students' practice, use peer assessments to facilitate students' critical thinking and meta cognitive skills (Hou et al., 2007).

BARRIERS TO CRITICAL THINKING

Several researchers (Landsman and Gorski, 2007; Sandholtz et al., 2004; Sheldon and Biddle, 1998; Wong, 2007) suggest that the current educational trend to standardize curricula and focus on test scores undermines instructors' ability to address critical thinking in the classroom. The emphasis on ''teaching to the test'' distracts the learning process from student centered instruction and places the emphasis on the content. If the focus is on learning, students should be given the freedom (and responsibility) to explore content, analyze resources, and apply information.

Unfortunately, students are not typically taught to think or learn independently, and they rarely pick up these skills on their own (Ladsman and Gorski, 2007; Lundquist, 1999; Rippen et al., 2002). Critical thinking is not an innate ability. Although some students may be naturally inquisitive, they require training to become systematically analytical, fair and open-minded in their pursuit of knowledge. With these skills, students can become confident in their reasoning and apply their critical thinking ability to any content area or discipline (Lundquist, 1999). Critical thinking is often compared to the scientific method, it is a systematic and procedural approach to the process of thinking (Scriven and Paul, 2007). Just as students learn the process of the scientific method, they must also learn the process of critical thinking.

Four barriers often impede the integration of critical thinking in education: (i) lack of training, (ii) lack of information, (iii) preconceptions and (iv) time constraints. First, teachers often are not trained in critical thinking methodology (Broadbear, 2003). Elementary and secondary teachers know their content and receive training in the methods of instruction, but little if any of their training is devoted specifically on how to teach critical thinking skills. Post secondary instructors pursue additional content based instruction during graduate school, but often have no formal methodological training, much less skill based instructions. Secondly, few instructional materials provide critical thinking resources (Scriven and Paul, 2007). Some textbooks provide chapter based critical thinking discussion questions, but instructional materials often lack additional critical thinking resources. Thirdly, both teachers and students have preconceptions about the content that blocks their ability to think critically about the material. Preconceptions such as personal bias partiality prohibit critical thinking because they obviate analytical skills such as being fair, open minded and inquisitive about a topic (Kang and Howren, 2004). For example, many mathematics educators still continue using two spaces after ending punctuation even though typeface experts have documented that today's proportional fonts are designed for one space (American Psychological Association, 2001: Chicago Manual of Style Online, 2007). A critical analysis of the information provided on this typesetting topic would support the use of a single space. However, strong biases for two spaces preclude many mathematics teachers predominantly typing teachers from changing their opinion and adopting the acceptable procedure.

Finally, time constraints are barriers to integrating critical thinking skills in the classroom. Instructors often have a great deal of content to cover within a short time

period. When the focus is on content rather than student learning, shortcuts such as lectures and objective tests become the norm. Lecturing is faster and easier than integrating project based learning opportunities. Objective tests are faster to take and grade than subjective assessments. However, research indicates that lecturing is not the best method of instruction and objective tests are not the best method of assessment (Broadbear, 2003; Brodie and Irving, 2007).

CONCLUSION

The goal for mathematics educators who want to instill critical thinking skills in their classrooms is to think of their students not as receivers of information, but as users of information. Learning environments that actively engage students in the investigation of information and the application of knowledge will promote students' critical thinking skills. However, as with any skill, critical thinking requires training, practice, and patience. Students may initially resist instructional questioning techniques if they previously have been required only to remember information and not think about what they know. They may struggle with assessment questions that are not taken verbatim from the book. However, by encouraging students throughout the process and modeling thinking behaviors, students' critical thinking skills can improve. The effort is worth the reward; students who can critically think for themselves and solve real world problems.

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