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Metric on the Countable Soft Topological Space

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Authors' contributions

This work was carried out in collaboration between both authors. Author LF designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript. Authors LF and HF managed the analyses of the study. Author HF managed the literature searches. Both authors read and approved the final manuscript.

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Short Research Article

Abstract

The author, ATHAR KHARAL, defined the Euclidean distance using the symmetric difference of sets in soft space, however, we conclude that all the sets of ε-approximate elements among the soft sets can't calculate their symmetric difference in this way. To illustrate our point, in this paper, we define the finite(countable) soft topological space, and point out the Euclidean distance given by ATHAR KHARAL can just be applied to the countable soft space. Meanwhile, we give the general definition of the metric soft topological space, test the Euclidean distance being a metric in a countable soft topological space, and achieve a metric countable soft topology.

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As new mathematical approaches, soft set [1], [2] are widely applied in uncertainty reasoning, such as computer sciences, decision issues, financial analysis, and medical treatment, etc. The references [2]-[20] have shown the flourishing soft set researching achievements.

Systematically, Pual R.Halmos introduced the measure theory in [21] which is useful for its applications in modern analysis. Topology [22], as a well-defined mathematical discipline, which is discussed in many different fields by many authors who achieved lots of research results, is one of the greatest unifying ideas of mathematics.

There are influential researches about soft topology. In [23], the authors gave out the concept and idea of soft topology by means of soft operators, described its main properties, and obtained the structure of soft topological spaces. In [24], the researchers put forward the notion of soft topology discussing soft $T_i - spaces$. In [25], the authors gave the properties of soft topological space. In [26]-[33], the authors further studied all kinds of characters of soft topology, such as $\theta-$contiunity and connectedness, soft separation axioms, soft connectedness, soft compactness, and so on. And in [34], the author defined the Euclidean distance in the soft space, without discussing the metric soft topological space. However, all the previous research results have not considered the metric soft topology.

This paper will point out the defect of Euclidean distance in the soft space defined by ATHAR KHARAL in [34], define the finite(countable) soft topology, and further study metric finite(countable) soft topology. It consists of three sections. In section 1, we recall some basic knowledge including concepts and operations of soft sets and topology. In section 2, we discuss the metric on the countable soft topological space, and the conclusion is section 3.

1 Basic Knowledge

Definition 1.1. [1] Let A be a subset of parameters set E, and $\mathcal{P}(U)$ be the power set of universe U, then a pair (F, A) is a **soft set** over U, where F is the mapping from A to $\mathcal{P}(U)$.

Namely, the soft set is a parameterized family about the subsets of universe. For all $e \in E$, every set $F(e)$ may be considered as the set of e-approximate elements (e-elements) of the soft set (F, E).

Inspired by this idea, we can consider a soft set (F, E) as the class of approximations, i.e.

$$(F, E) = \{F(e) \mid e \in E\} = \{(F(e), e) \mid e \in E\}.$$

Definition 1.2. *molodsov* Let (F_1, B_1) and (F_2, B_2) be the soft sets over a common universe U, and E be an attributes set. And $B_1, B_2 \subseteq E$, $F_i : B_i \to \mathcal{P}(U)(i = 1, 2)$ be a set-value mapping over U.

(i) If $B_1 \subseteq B_2$, and $F_1(x) \subseteq F_2(x)$, for all $x \in B_1 \subseteq B_2$, then the soft set(F_1, B_1) is a **soft subset** of the soft set (F_2, B_2), and denoted as $(F_1, B_1)\widetilde{\subset}(F_2, B_2)$.

(ii) Two soft sets (F_1, B_1) and (F_2, B_2) are said **soft equal**, if $(F_1, B_1)\widetilde{\subset}(F_2, B_2)$, and $(F_2, B_2)\widetilde{\subset}(F_1, B_1)$. We denote as $(F_1, B_1) = (F_2, B_2)$.

(iii) The **union** of (F_1, B_1) and (F_2, B_2) is the soft set (H, C), where $C = B_1 \cup B_2$, and for all $e \in C$, denoted as $(F_1, B_1)\widetilde{\cup}(F_2, B_2) = (H, C) = (H, B_1 \cup B_2)$, where

$$H(e) = \begin{cases} F_1(e), & if \quad e \in B_1 - B_2 \\ F_2(e), & if \quad e \in B_2 - B_1 \\ F_1(e) \cup F_2(e), & if \quad e \in B_1 \cap B_2 \end{cases}$$

(iv) The **intersection** of (F_1, B_1) and (F_2, B_2) is the soft set (H, C) is denoted as $(F_1, B_1) \sqcap (F_2, B_2)$ and is defined as $(F_1, B_1) \sqcap (F_2, B_2) = (H, C)$, where $C = B_1 \cap B_2$, and for all $e \in C, H(e) = F_1(e) \cap F_2(e)$.

(v) The **relative complement** of (F, B) is denoted by $(F, B)^c$ and is defined by $(F, B)^c = (F^c, B)$, where $F^c : B \to \mathcal{P}(U)$, and $F^c(x) = U - F(x)$, for all $x \in B$.
Obviously, $((F, B)^c)^c = (F, B)$.

(vi) (F, B) is said to be a **relative null soft set**, denoted by $\mathcal{N}$, if for all $x \in B, F(x) = \emptyset$.

If $B = E$, then it is called **absolute null soft set**, denoted as $\widetilde{\emptyset}$.

(vii) (F, B) is a **relative whole soft set** (denoted by $\widetilde{U}$), if for all $x \in B, F(x) = U$.

For convenience to define the definition of distance of soft sets, we give the following soft operators:

Definition 1.3. [24] Let $\mathcal{T}$ be the collection of soft sets over X, in which X is an initial universe set and E is a set of parameters, then $\mathcal{T}$ is a **soft topology** on X if

(1) $\widetilde{\emptyset}$ and $\widetilde{X}$ belong to $\mathcal{T}$.
(2) If $\mathcal{T}_1 \subseteq \mathcal{T}$, then $\bigcup_{(F,B)\in\mathcal{T}_1}(F, B) \in \mathcal{T}$. We say the union is closed about any numbers of soft sets in $\mathcal{T}$.
(3) If $(F, B), (G, C) \in \mathcal{T}$, then $(F, B) \sqcap (G, C)$ is also in $\mathcal{T}$, that is, the intersection of any two soft sets is closed in $\mathcal{T}$.

The triplet $(X, \mathcal{T}, E)$ is called a **soft topological space** over X.

The members of τ are **soft open sets** in $\mathcal{T}$. The relative complement $(F, B)^c = (F^c, B)$ is said to be a **soft closed set** in $\mathcal{T}$ if $(F, B)^c \in \tau$.

If (F, B) is both soft open and soft closed, then (F, B) is a **soft clopen set**.

Notation 1.1. *To get the unity of symbols, we can let G be objects set, M be parameters set, $B \subseteq M, F : B \to \mathcal{P}(G)$ be a mapping, and the pair (F, B) is the soft set over G. In the subsequence, (G, τ, M) stands for a soft topological space, and M is a finite attributes set.*

2 The Properties of Finite Soft Metric Topological Space

First, we review the definition of metric between sets.

Definition 2.1. [21] Let X be a set, $\rho : X \times X \to \mathbb{R}$ be a mapping, if for all $x, y, z \in X$ having:

(1) ρ satisfies the **positivity**, that is, $\rho(x, y) \geq 0$, and $\rho(x, y) = 0$ when $x = y$;
(2) ρ is **symmetric**, that is, $\rho(x, y) = \rho(y, x)$;
(3) ρ satisfies the **triangle inequality**, i.e., $\rho(x, y) + \rho(y, z) \geq \rho(x, z)$.

Then ρ is called a **metric** over X, (X, ρ) is a **metric space**. and the real number $\rho(x, y)$ is the **distance** from point x to point y.

Next, we define the metric over the soft topology.

Definition 2.2. Let (G, τ, M) be a soft topological space, and (F, B) be a soft set in τ, $x \in G$.

(1) We say $x \in (F, B)$(read as x **belongs to** the soft set (F, B)), for all $e \in B$, if $x \in F(e)$, and x is called a **soft point**.

(2) The points belong to some soft set, we call these points as **soft points**.

(3) If there is a soft open set (F_1, B_1) over G, such that $x \in (F_1, B_1)\tilde{\subset}(F, B)$, then x is a **soft interior point** of (F, B) and (F, B) is the **soft neighborhood** of x.

(4) if (F, B) is a soft open (closed) set, then (F, B) is a soft open (closed) neighborhood of x.

Remark 2.1. If there exists a metric on some soft topological space, then this metric also can define the soft neighborhood of soft point.

Definition 2.3. Let (X, ρ) be a measure space, $x \in X$, for any $\varepsilon > 0$.

(1) The set $\{y \in x|\rho(x, y) < \varepsilon\}$ is called a **ε-ball** of x, or **ε-neighborhood** of x, denoted as $B(x, \varepsilon)$ or B_ε.

(2) Let $A \subseteq X$, for all $x \in A$, if there exists $B(x, \varepsilon)$ such that $B(x, \varepsilon) \subset A$, then A is called an **open set of** (X, ρ).

Definition 2.4. Let (G, τ, M) be a soft topological space, and $\tau = \{(F, B)|(F, B)$ is a soft open set over $G\}$ be a soft topology over G.

(1) If for all $(F, B) \in \tau$, for all $e \in B, F(e) \subset G, F(e)$ is a finite subset of G, then τ is called a **finite soft topology**, and (G, τ, M) is called a **finite soft topological space**.

(2) If for all $(F, B) \in \tau$, for all $e \in B, F(e) \subset G, F(e)$ is a countable subset of G, then τ is called a **countable soft topology**, and (G, τ, M) is called a **countable soft topological space**.

Clearly, the finite soft topology is a special case of the countable soft topology.

Remark 2.2. (1) If G is a finite universe, then τ must be a finite soft topology (defined over G).

(2) Suppose that (G, τ, M) is a soft discrete topological space, if G is an infinite universe, then τ is a countable infinite soft discrete topology (defined over G); if G is a finite universe set, then τ is a finite soft discrete topology .

Definition 2.5. Let (G, τ, M) be a finite soft topological space, $(F_1, B_1), (F_2, B_2) \in \tau$.

(i)) The **difference** between (F_1, B_1) and (F_2, B_2) is the soft set (H, C), where $C = B_1 \cap B_2$. We denote (H, C) as $(F_1, B_1) - (F_2, B_2) = (H, B_1 \cap B_2)$, where $H(e) = F_1(e) \cap F_2^c(e) = F_1(e) \cap (U - F_2(e)),\ e \in C$.

(ii) The **symmetric difference** between (F_1, B_1) and (F_2, B_2) is denoted as $(F_1, B_1)\triangle(F_2, B_2)$, and is defined as $((F_1, B_1)\tilde{\cup}(F_2, B_2)) - ((F_1, B_1) \sqcap (F_2, B_2))$.

Example 2.1. *Let (F_1, B_1) and (F_2, B_2) be the soft sets over a common universe $G = \{a, b, c\}$, and $M = \{e_1, e_2, e_3\}$ be an attributes set. $B_1 = \{e_1, e_3\} \subseteq E, B_2 = \{e_1, e_2, e_3\} = E$, and $(F_1, B_1) = \{(e_1, \{a, c\}), (e_3, \{b, c\})\}, (F_2, B_2) = \{(e_1, \{a, c\}), (e_2, \{c\}), (e_3, \{b\})\}$, then*

$$(F_2, B_2)^c = \{(e_1, \{b\}), (e_2, \{a, b\}), (e_3, \{a, c\})\};$$
$$(F_1, B_1) - (F_2, B_2) = \{(e_1, \emptyset), (e_3, \{c\})\};$$
$$(F_1, B_1)\tilde{\cup}(F_2, B_2) = \{(e_1, \{a, c\}), (e_2, \{c\}), (e_3, \{b, c\})\};$$
$$(F_1, B_1) \sqcap (F_2, B_2) = \{(e_1, \{a, c\}), (e_3, \{b\})\};$$
$$(F_1, B_1)\triangle(F_2, B_2) = \{(e_1, \emptyset), (e_3, \{c\})\}.$$

In [34], author defined the **Euclidcan distance** in the soft space as following:

Definition 2.6. [34] For two soft sets $(F, A), (G, B)$ in a soft space (X, E) , where A and B are not identically void, we define **Euclidean distance** as:

$$e((F, A), (G, B) = ||A\triangle B|| + \sqrt{\sum_{\varepsilon \in A\cap B} || F(\varepsilon)\triangle G(\varepsilon)||^2}$$

Normalized Euclidean distanceas:

$$q((F,A),(G,B)) = \frac{||A\triangle B||}{\sqrt{||A\cup B||}} + \sqrt{\sum_{\varepsilon\in A\cap B}\chi(\varepsilon)}$$

$$where\ \chi(\varepsilon) = \begin{cases} \frac{||F(\varepsilon)\triangle G(\varepsilon)||^2}{||F(\varepsilon)\cup G(\varepsilon)||}, & if\ F(\varepsilon)\cup G(\varepsilon)\neq\emptyset \\ 0, & otherwise \end{cases}$$

Lemma 2.1. *[34] For the soft sets $(F_\emptyset, E), (F_X, E)$ and an arbitrary soft set (F, A) in a soft space, we have:*

(1) $e((F,A),(F,A)^c) = 2||A||$*;*
(2) $q((F,A),(F,A)^c) = \sqrt{2||A||}$*;*
(3) $e((F_\emptyset,E),(F_X,E)) = \sqrt{||E||\cdot||X||}$*;*
(4) $q((F_\emptyset,E),(F_X,E)) = \sqrt{||E||}$.

Remark 2.3. (1) According to this definition, it is clear that if $(F,A) = (G,B)$, then $e((F,A),(G,B)) = 0$; conversely, if $e((F,A),(G,B)) = 0$, then:

$$\begin{aligned} & e((F,A),(G,B)) = 0 \\ \Rightarrow\ & ||A\triangle B||\ and \sum_{\varepsilon\in A\cap B} ||\ F(\varepsilon)\triangle G(\varepsilon)||^2 \\ \Rightarrow\ & A\triangle B = \emptyset\ and\ F(\varepsilon)\triangle G(\varepsilon) = \emptyset,\ for\ all\ \varepsilon\in A\cap B \\ \Rightarrow\ & A\cup B = A\cap B\ and\ F(\varepsilon)\cup G(\varepsilon) = F(\varepsilon)\cap G(\varepsilon), \\ & for\ all\ \varepsilon\in A\cap B \\ \Rightarrow\ & A = B\ and\ F(\varepsilon) = G(\varepsilon),\ for\ all\ \varepsilon\in A\cap B \\ \Rightarrow\ & (F,A) = (G,B) \end{aligned}$$

That is, $e((F,A),(G,B)) = 0$ if and only if $(F,A) = (G,B)$.

(2) There is a defect in this definition, which makes it only applied to the countable universe, i.e., when $F(\varepsilon)$ and $G(\varepsilon)$ are uncountable sets, it can not be applied to compute $||\ F(\varepsilon)\triangle G(\varepsilon)||$. For example:

Let (G,τ,M) be a soft real topological space, in which $G = \mathbb{R}, M = \{e_1,e_2,e_3\}$. (F_1,B_1) and (F_2,B_2) are soft set in τ over $\mathbb{R}$.

Suppose that $B_1 = \{e_1,e_2\}, B_2 = \{e_1,e_2,e_3\}$, and $(F_1,B_1) = \{(e_1,(1,3)),(e_2,(1,4))\},(F_2,B_2) = \{(e_1,(1,4)),(e_2,(0,3.5),(e_3,(2,4))\}$, then $B_1\cap B_2 = \{e_1,e_2\}$.

From the above definition, if we want to compute $e((F_1,B_1),(F_2,B_2))$, then we need to know the cardinality of $F_1(e_1)\triangle F_2(e_2)$. However, $F_1(e_1)\triangle F_2(e_1) = [3,4), F_1(e_2)\triangle F_2(e_2) = (0,1]\cup[3.5,4)$, so we can not calculate its cardinality.

(3) The results given by above Lemma are not correct which should be as follows:

(1) $e((F,A),(F,A)^c) = \sqrt{||A||\cdot||X||^2}$;
(2) $q((F,A),(F,A)^c) = \sqrt{||A||\cdot||X||}$;
(3) $e((F_\emptyset,E),(F_X,E)) = \sqrt{||E||\cdot||X||^2}$;
(4) $q((F_\emptyset,E),(F_X,E)) = \sqrt{||E||\cdot||X||}$.

In fact, by the definition of soft relative complement, we know

$$\begin{aligned}
&By \quad (F,A)^c = (F^c, A)\\
&\Rightarrow \quad F^c(\varepsilon) = (F(\varepsilon))^c = X - F^c(\varepsilon), \quad for\ all\ \varepsilon \in A\\
&\Rightarrow \quad F(\varepsilon) \triangle F^c(\varepsilon) = (F(\varepsilon) \cup F^c(\varepsilon)) - (F(\varepsilon) \cap F^c(\varepsilon)) = X\\
&\Rightarrow \quad ||\ F(\varepsilon) \triangle F^c(\varepsilon)|| = ||A|| \cdot ||X||^2\\
&and \quad \chi(\varepsilon) = \tfrac{||F(\varepsilon)\triangle F^c(\varepsilon)||^2}{||F(\varepsilon)\cup F^c(\varepsilon)||} = \tfrac{||X||^2}{||X||} = ||X||\\
&\Longrightarrow \begin{cases} e((F,A),(F,A)^c) = \sqrt{||A|| \cdot ||X||^2} \\ q((F,A),(F,A)^c) = \sqrt{||A|| \cdot ||X||} \end{cases}\\
&\qquad And \quad F_\emptyset(\varepsilon) \triangle F_X(\varepsilon) = \emptyset \triangle X = X\\
&\qquad and \quad \chi(\varepsilon) = \tfrac{||F_\emptyset(\varepsilon)\triangle F_X(\varepsilon)||^2}{||F_\emptyset(\varepsilon)\cup F_X(\varepsilon)||} = \tfrac{||X||^2}{||X||} = ||X||\\
&\qquad \Longrightarrow \begin{cases} e((F_\emptyset,E),(F_X,E)) = \sqrt{||E|| \cdot ||X||^2} \\ q((F_\emptyset,E),(F_X,E)) = \sqrt{||E|| \cdot ||X||}. \end{cases}
\end{aligned}$$

Based on the remark , we can modify the above definition as follows:

Definition 2.7. Let (G, τ, M) be a countable soft topological space , (F_1, B_1) and (F_2, B_2) be arbitrary soft sets in τ, where $B_1, B_2 \subseteq M$, B_1 and B_2 are not identically void, $e, q : \tau \times \tau \to \mathbb{R}^+$ are mappings. we denote **Euclidean distance** between (F_1, B_1) and (F_2, B_2) as $e((F_1, B_1), (F_2, B_2))$, and define as :

$$e((F_1, B_1), (F_2, B_2)) = ||B_1 \triangle B_2|| + \sqrt{\sum_{\varepsilon \in B_1 \cap B_2} ||\ F_1(\varepsilon) \triangle F_2(\varepsilon)||^2}$$

and we denote **Normalized Euclidean distance** between (F_1, B_1) and (F_2, B_2) as $q((F_1, B_1), (F_2, B_2))$, and define as:

$$q((F_1, B_1), (F_2, B_2)) = \frac{||B_1 \triangle B_2||}{\sqrt{||B_1 \cup B_2||}} + \sqrt{\sum_{\varepsilon \in B_1 \cap B_2} \chi(\varepsilon)}$$

$$where\ \chi(\varepsilon) = \begin{cases} \frac{||F_1(\varepsilon)\triangle F_2(\varepsilon)||^2}{||F_1(\varepsilon)\cup F_2(\varepsilon)||}, & if\ F_1(\varepsilon) \cup F_2(\varepsilon) \neq \emptyset \\ 0, & otherwise \end{cases}$$

Example 2.2. *Let (G, τ, M) be a finite soft topological space, in which $G = \{1, 2, 3, 4\}, M = \{e_1, e_2, e_3, e_4\}, B_i \subseteq M(i = 1, 2, 3)$. Take $B_1 = \{e_1, e_2, e_3\}, B_2 = \{e_2, e_3\}, B_3 = M$, and choose the following soft sets in τ:*

$(F_1, B_1) = \{(e_1, \{1, 3, 4\}), (e_2, \{2\}), (e_3, \{1, 2, 3\})\}$,
$(F_2, B_2) = \{(e_2, \{2, 3\}), (e_3, G\})$,
$(F_3, B_3) = \{(e_1, \{2, 4\}), (e_2, \{2, 3, 4\}), (e_3, \{1, 3\}), (e_4, G)\}$.

Then,

$B_1 \cap B_2 = \{e_2, e_3\},\ B_1 \triangle B_2 = \{e_1\}$
$B_1 \cap B_3 = \{e_1, e_2, e_3\},\ B_1 \triangle B_3 = \{e_4\}$
$B_2 \cap B_3 = \{e_2, e_3\},\ B_2 \triangle B_3 = \{e_1, e_4\}$

$F_1(e_2) \triangle F_2(e_2) = \{2\} \triangle \{2,3\} = \{3\}$
$F_1(e_3) \triangle F_2(e_3) = \{1,2,3\} \triangle \{1,2,3,4\} = \{4\}$
$F_1(e_1) \triangle F_3(e_1) = \{1,3,4\} \triangle \{2,4\} = \{1,2,3\}$
$F_1(e_2) \triangle F_3(e_2) = \{2\} \triangle \{2,3,4\} = \{3,4\}$
$F_1(e_3) \triangle F_3(e_3) = \{1,2,3\} \triangle \{1,3\} = \{2\}$
$F_2(e_2) \triangle F_3(e_2) = \{2,3\} \triangle \{2,3,4\} = \{4\}$
$F_2(e_3) \triangle F_3(e_3) = \{1,2,3,4\} \triangle \{1,3\} = \{2,4\}$.

Hence,

$$\begin{aligned} & e((F_1,B_1),(F_2,B_2)) \\ = \; & ||B_1 \triangle B_2|| + \sqrt{\sum_{\varepsilon \in B_1 \cap B_2} ||\, F_1(\varepsilon) \triangle F_2(\varepsilon)||^2} \\ = \; & ||\{e_1\}|| + \sqrt{||F_1(e_2) \triangle F_2(e_2)||^2 + ||F_1(e_3) \triangle F_2(e_3)||^2} \\ = \; & 1 + \sqrt{2} \\ & e((F_1,B_1),(F_3,B_3)) \\ = \; & ||B_1 \triangle B_3|| + \sqrt{\sum_{\varepsilon \in B_1 \cap B_3} ||\, F_1(\varepsilon) \triangle F_3(\varepsilon)||^2} \\ = \; & 1 + \sqrt{14} \\ & e((F_2,B_2),(F_3,B_3)) \\ = \; & ||B_2 \triangle B_3|| + \sqrt{\sum_{\varepsilon \in B_2 \cap B_3} ||\, F_2(\varepsilon) \triangle F_3(\varepsilon)||^2} \\ = \; & ||\{e_2.e_3\}|| + \sqrt{||F_2(e_2) \triangle F_3(e_2)||^2 + ||F_2(e_3) \triangle F_3(e_3)||^2} \\ = \; & 2 + \sqrt{5} \end{aligned}$$

$$
\begin{aligned}
& q((F_1,B_1),(F_2,B_2)) \\
= \; & \frac{||B_1 \triangle B_2||}{\sqrt{||B_1 \cup B_2||}} + \sqrt{\sum_{\varepsilon \in B_1 \cap B_2} \chi(\varepsilon)} \\
= \; & \frac{1}{\sqrt{3}} + \sqrt{\frac{||F_1(e_2) \triangle F_2(e_2)||^2}{||F_1(e_2) \cup F_2(e_2)||} + \frac{||F_1(e_3) \triangle F_2(e_3)||^2}{||F_1(e_3) \cup F_2(e_3)||}} \\
= \; & \frac{\sqrt{3}}{3} + \frac{\sqrt{3}}{2} \\
& q((F_1,B_1),(F_3,B_3)) \\
= \; & \frac{||B_1 \triangle B_3||}{\sqrt{||B_1 \cup B_3||}} + \sqrt{\sum_{\varepsilon \in B_1 \cap B_3} \chi(\varepsilon)} \\
= \; & \frac{1}{\sqrt{4}} + \sqrt{\frac{||F_1(e_1) \triangle F_3(e_1)||^2}{||F_1(e_1) \cup F_3(e_1)||} + \frac{||F_1(e_2) \triangle F_3(e_2)||^2}{||F_1(e_2) \cup F_3(e_2)||} + \frac{||F_1(e_3) \triangle F_3(e_3)||^2}{||F_1(e_3) \cup F_3(e_3)||}} \\
= \; & \frac{1}{2} + \frac{\sqrt{141}}{6} \\
& q((F_2,B_2),(F_3,B_3)) \\
= \; & \frac{||B_2 \triangle B_3||}{\sqrt{||B_2 \cup B_3||}} + \sqrt{\sum_{\varepsilon \in B_2 \cap B_3} \chi(\varepsilon)} \\
= \; & \frac{2}{\sqrt{4}} + \sqrt{\frac{||F_2(e_2) \triangle F_3(e_2)||^2}{||F_2(e_2) \cup F_3(e_2)||} + \frac{||F_2(e_3) \triangle F_3(e_3)||^2}{||F_2(e_3) \cup F_3(e_3)||}} \\
= \; & 1 + \frac{2\sqrt{3}}{3}
\end{aligned}
$$

Definition 2.8. Let (G, τ, M) be a soft topological space, in which τ is a finite soft topology. For all $(F_i, B_i) \in \tau, (i = 1, 2, 3)$, $m : \tau \times \tau \to G$ is a mapping, which satisfies the following conditions:

(1) $m((F_1, B_1), (F_2, B_2)) \geq 0$;
(2) $m((F_1, B_1), (F_2, B_2)) = m((F_2, B_2), (F, B_1))$;
(3) $m((F_1, B_1), (F_2, B_2)) + m((F_2, B_2), (F_3, B_3)) \geq m((F_1, B_1), (F_3, B_3))$.

Then m is called the **metric** over (G, τ, M), and the pair $(\mathcal{T}, m)$ is a **metric soft topological space**; simply speaking, $(\mathcal{T}, m)$ is a **metric soft space**.

Example 2.3. *Clearly, the mappings e and q, which are metrics over (G, τ, M), are defined as the above example from this definition.*

Lemma 2.2. *Let A, B, C be any set, then*
(1) $||A \triangle B|| = ||B \triangle A||$;
(2) $||A \triangle C|| \leq ||A \triangle B|| + ||B \triangle C||$.

Proof. (1) is clear because

$$A \triangle B = A \cup B - A \cap B = B \cup A - B \cap A = B \triangle A.$$

We prove (2) by Venn Diagram.

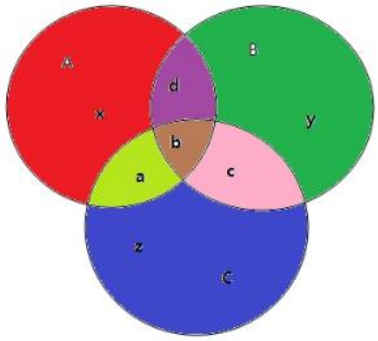

Fig. 1.Veen diagram of sets

From Fig.1, we can see

$$\begin{aligned}||A\triangle B|| &= x+a+y+c,\\ ||B\triangle C|| &= y+d+z+a,\\ ||A\triangle C|| &= x+d+z+c,\end{aligned}$$

Obviously,

$$\begin{aligned}||A\triangle C|| &= x+d+z+c\\ &= (x+z)+(d+c)\\ &\leq (x+2y+z)+(2a+d+c)\\ &= ||A\triangle B||+||B\triangle C||\end{aligned}$$

□

Proposition 2.1. *Let (G,τ,M) be a countable soft topological space. (F_1,B_1) and (F_2,B_2) are arbitrary soft sets in τ, in which B_1 and B_2 are not identically void. The Euclidean distance $e((F_1,B_1),(F_2,B_2))$ between soft set (F_1,B_1) and (F_2,B_2) is defined as above definition, and the mapping $e:\tau\times\tau\to\mathbb{R}^+$ is a metric.*

Proof. Using the definition of Euclidean distance, Remark 2.2 and Lemma 2.2, it is obvious that the mapping e is a metric. □

Definition 2.9. Let $\mathcal{T}=(G,\tau,M)$ be a countable soft topological space , the mapping $e:\tau\times\tau\to\mathbb{R}^+$ be a metric determined by Euclidcan distance over $\mathcal{T}$, then τ is **induced by the metric** e, simply speaking , τ is a **measurable countable soft topology**, and the pair $(\mathcal{T},e)$ is called as a **metric countable soft topological space**.

For any soft topological (G,τ,M), we also hope there is a metric such that (G,τ,M) can be a measurable soft topological space, for example:

Example 2.4. *Let (G,τ,M) be the soft topological space, soft sets $(F_1,B_1),(F_2,B_2)\in\tau$, define*

$$d((F_1,B_1),(F_2,B_2))=\begin{cases}1, & if(F_1,B_1)\neq(F_2,B_2)\\ 0, & otherwise\end{cases}$$

then $\tau=\{(F,B)|(F,B)$ is a soft open set over $G\}$ is a soft discrete topology induced by d.

The example above shows that a definite metric induces a soft topology, but it is very special. So far we have not found a proper metric which can induce a metric soft topology in a random soft topological space, and we will discuss it later.

3 Conclusion

Our aim is to discuss the metric on the countable soft topological space. We hope to offer a new idea for data classification.

This paper discusses metric on the countable soft topological space. Three points are mainly covered: firstly, we point out the Euclidean distance defined on the soft space by ATHAR KAHRAL which has a defect, and we modify it. Secondly, we define the finite and countable soft topology and define the metric countable soft topology. Lastly, we point out the limitation of Euclidean distance which can not be applied to the infinite universe and propose the further research.

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Competing Interests

Authors have declared that no competing interests exist.

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