Asian Research Journal of Mathematics

11(1): 1-5, 2018; Article no.ARJOM.43569

ISSN: 2456-477X

Sg-continuity in Topological Ordered Spaces

V. V. S. Ramachandram[1*] and B. Sankara Rao[2]

[1]Department of Science and Humanities, B.V.C. College of Engineering, Rajamahendravaram, India.
[2]Department of Mathematics, Adikavi Nannaya University, Rajamahendravaram, India.

Authors' contributions

This work was carried out in collaboration between both authors. Author VVSR designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript. Author BSR managed the analysis of the study. Both authors read and approved the final manuscript.

Article Information

DOI: 10.9734/ARJOM/2018/43569
Editor(s):
(1) Dr. Radoslaw Jedynak, Department of Computer Science and Mathematics, Kazimierz Pulaski University of Technology and Humanities, Poland.
(2) Dr. Xingting Wang, Department of Mathematics, Temple University, Philadelphia, USA.
Reviewers:
(1) Nihal Taş, Balikesir University, Turkey.
(2) Angelo Favini, Bologna University, Italy.
(3) G. Srinivasarao, Tirumala Engineering College, Jawaharlal Nehru Technological University, Kakinada, India.
(4) Vildan Çetkin, Kocaeli University, Turkey.
Complete Peer review History: http://www.sciencedomain.org/review-history/26595

Original Research Article

Received: 02 June 2018
Accepted: 29 August 2018
Published: 10 October 2018

Abstract

Semi generalised closed set in a Topological space was first introduced by P. Bhattacharya and B.K.Lahiri in 1987. A subset A of a topological space (X,τ) is a semi generalised closed (sg-closed) set if $scl(A)\subseteq U$ whenever $A\subseteq U$ and U is semi-open in (X,τ). Some authors introduced the notion of sg-continuity in topological spaces. The same notion can be extended to topological ordered spaces. A topological ordered space is a topological space together with a partial order. In this paper, we introduce and study the notion of semi generalised increasing continuous function (sgi-continuous function), semi generalised decreasing continuous function (sgd-continuous function) and semi generalised balanced continuous function (sgb-continuous function) and study the relationships between them.

Keywords: Topological ordered space; increasing set; decreasing set; balanced set and semi generalised closed set.

Mathematics subject classification: 54A05

**Corresponding author: E-mail: vvsrch@gmail.com;*

1 Introduction

The study of topological ordered spaces (TOS) was introduced by L.Nachbin [2]. It is a triple $(\mathrm{X},\tau,\leq)$ where τ is a topology and $\leq$ is a partial order on X. Let $(\mathrm{X},\tau,\leq)$ be a TOS. For any $x\in\mathrm{X}, [x,\rightarrow]=\{y\in\mathrm{X}/x\leq y\}$ and. $[\leftarrow,x]=\{y\in\mathrm{X}/y\leq x\}$ A subset A of a TOS $(\mathrm{X},\tau,\leq)$ is increasing if $\mathrm{A}=i[\mathrm{A}]$ and decreasing if $\mathrm{A}=d[\mathrm{A}]$ where $i[\mathrm{A}]=\bigcup_{a\in\mathrm{A}}[a,\rightarrow]$ and $d[\mathrm{A}]=\bigcup_{a\in\mathrm{A}}[\leftarrow,a]$. The complement of an increasing set is a decreasing set and vice versa. A subset of a TOS $(X,\tau,\leq)$ is a balanced set if it is both increasing and decreasing.

2 Preliminaries

In the present paper (X,τ) represents a non-empty topological space on which no separation axioms are assumed unless otherwise mentioned. For a subset A of (X,τ), the closure is the intersection of all closed sets containing A and semi-closure is the intersection of all semi-closed sets containing A .They are denoted by $cl(\mathrm{A})$ and $scl(\mathrm{A})$ .respectively.

Definition 2.1. A subset A of a topological space (X,τ) is a semi-open set [5] if $\mathrm{A}\subseteq cl(\mathrm{int}(\mathrm{A}))$ and a *semi-closed* set if $\mathrm{int}(cl(\mathrm{A}))\subseteq\mathrm{A}$.

Definition 2.2. A subset A of a topological space (X,τ) is a sg-closed set [1] if $scl(\mathrm{A})\subseteq U$ whenever $\mathrm{A}\subseteq$ U and U is semi-open in (X,τ).

Definition 2.3. A subset A of a topological ordered space $(X,\tau,\leq)$ is a sgi-closed (resp. sgd-closed, sgb-closed) set [4] if A is sg-closed and increasing (resp. decreasing, balanced).

3 Sg-continuity in Topological Ordered Spaces

We define new types of semi generalised continuous functions in topological ordered spaces. We recall that a function $f:(X,\tau_1)\rightarrow(Y,\tau_2)$ is sg-continuous [1] if $f^{-1}(\mathrm{V})$ is sg-closed whenever V is a closed set in Y.

The following notions are introduced in a topological ordered space.

Definition 3.1. A function $f:(X,\tau,\leq)\rightarrow(Y,\tau^1,\leq^1)$ is

(1) a semi generalised increasing continuous function (briefly sgi-continuous) if $f^{-1}(\mathrm{V})$ is a sgi-closed set in X whenever V is an i-closed set in Y.
(2) a semi generalised decreasing continuous function (briefly sgd-continuous) if $f^{-1}(\mathrm{V})$ is a sgd-closed set in X whenever V is a d-closed set in Y.
(3) a sg-balanced continuous function (briefly sgb-continuous) if $f^{-1}(\mathrm{V})$ is a sgb-closed set in X whenever V is a b-closed set in Y.

The following examples support the above definitions.

Example 3.2. Let $X = Y = \{a,b,c\}$, $\tau_8 = \{\phi, X, \{a,b\}\}$, $\leq_3 = \{(a,a),(b,b),(c,c),(a,b),(a,c)\}$ $\leq_4 = \{(a,a),(b,b),(c,c),(a,b),(c,a),(c,b)\}$. Then $(X, \tau_8, \leq_3)$ and $(Y, \tau_8, \leq_4)$ are topological ordered spaces. The i-closed sets in Y are ϕ, X and sgi-closed sets in X are $\phi, X, \{c\}, \{b,c\}$. Define $f : X \to Y$ as $f(a) = b$, $f(b) = c$ and $f(c) = a$ then, f is a sgi-continuous function.

Example 3.3. Let $X = Y = \{a,b,c\}$, $\tau_7 = \{\phi, X, \{a\}, \{b\}, \{b,c\}, \{a,b\}\}$ $\tau_8 = \{\phi, X, \{a,b\}\}$, $\leq_7 = \{(a,a),(b,b),(c,c),(b,a)\}$, $\leq_8 = \{(a,a),(b,b),(c,c)\}$, then $(X, \tau_7, \leq_7)$ and $(Y, \tau_8, \leq_8)$ are topological ordered spaces. The d-closed sets in Y are $\phi, X, \{c\}$ and sgd-closed sets in X are $\phi, X, \{c\}, \{b,c\}$ .Define $f : X \to Y$ as $f(a) = b$, $f(b) = a$ and $f(c) = c$ then, f is a sgd-continuous function.

Example 3.4. Let $X = Y = \{a,b,c\}$, $\tau_8 = \{\phi, X, \{a,b\}\}$, $\tau_9 = \{\phi, X, \{b\}, \{c\}, \{b,c\}\}$ $\leq_1 = \{(a,a),(b,b),(c,c),(a,b),(b,c),(a,c)\}$, $\leq_5 = \{(a,a),(b,b),(c,c),(a,c),(b,c)\}$ Then, $(X, \tau_8, \leq_5)$ and $(Y, \tau_9, \leq_1)$ are topological ordered spaces. The b-closed sets in Y are ϕ, X and the sgb-closed sets in X are ϕ, X. Define $f : X \to Y$ as $f(a) = b$, $f(b) = c$ and $f(c) = a$. Then, f is a sgb-continuous function.

4 Independency of the functions

Remark 4.1: The notions sgi-continuity and sgd-continuity are independent as seen in the following example.

Example 4.2: Let $X = Y = \{a,b,c\}$, $\tau_8 = \{\phi, X, \{a,b\}\}$, $\leq_3 = \{(a,a),(b,b),(c,c),(a,b),(a,c)\}$ $\leq_4 = \{(a,a),(b,b),(c,c),(a,b),(c,a),(c,b)\}$ .Then, $(X, \tau_8, \leq_3)$ and $(Y, \tau_8, \leq_4)$ are topological ordered spaces. The i-closed sets in Y are ϕ, X and sgi-closed sets in X are $\phi, X, \{c\}, \{b,c\}$. Define $f : X \to Y$ as $f(a) = a$, $f(b) = c$ and $f(c) = b$ then, f is a **sgi-continuous function**. The d-closed sets in Y are $\phi, X, \{c\}$ and the sgd-closed sets in X are $\phi, X, \{a,c\}$ Then, f is **not a sgd-continuous function**.

If we take $X = Y = \{a,b,c\}$, $\tau_8 = \{\phi, X, \{a,b\}\}$, $\leq_4 = \{(a,a),(b,b),(c,c),(a,b),(c,a),(c,b)\}$, $\leq_5 = \{(a,a),(b,b),(c,c),(a,c),(b,c)\}$ then, $(X, \tau_8, \leq_4)$ and $(Y, \tau_8, \leq_5)$ are topological ordered spaces. The i-closed sets in Y are $\phi, X, \{c\}$ and sgi-closed sets in X are ϕ, X. Define $f : X \to Y$ as $f(a) = a$, $f(b) = a$ and $f(c) = c$. Then, f is **not a sgi-continuous function**. The d-closed sets in Y are ϕ, X and sgd-closed sets in X are $\phi, X, \{a,c\}, \{c\}$. Then, f is a **sgd-continuous function**.

Remark 4.3: The notions sgi-continuity and sgb-continuity are independent as seen in the following example.

Example 4.4. In $(X,\tau_8,\leq_4)$ and $(Y,\tau_8,\leq_5)$ where $X=Y=\{a,b,c\}$, $\tau_8=\{\phi,X,\{a,b\}\}$, $\leq_4=\{(a,a),(b,b),(c,c),(a,b),(c,a),(c,b)\}$ and $\leq_5=\{(a,a),(b,b),(c,c),(a,c),(b,c)\}$, the i-closed sets in Y are $\phi,X,\{c\}$ and sgi-closed sets in X are ϕ,X. Define $f:X\to Y$ as $f(a)=a$, $f(b)=a$ and $f(c)=c$. Then, f is **not a sgi-continuous function**. The b-closed sets in Y are ϕ,X and sgb-closed sets in X are ϕ,X. Then, f is **a sgb-continuous function.**

On the other hand, in the spaces $(X,\tau_{11},\leq_7)$ and $(Y,\tau_{11},\leq_8)$ where $X=Y=\{a,b,c\}$, $\tau_{11}=\{\phi,X,\{c\},\{c,b\}\}$, $\leq_7=\{(a,a),(b,b),(c,c),(b,a)\}$ and $\leq_8=\{(a,a),(b,b),(c,c)\}$, the i-closed sets in Y are $\phi,X,\{a\},\{a,b\}$ and sgi-closed sets in X are $\phi,X,\{a\},\{a,b\}$. Define $f:X\to Y$ as $f(a)=a$, $f(b)=b$ and $f(c)=c$. Then, f is a **sgi-continuous function**. The b-closed sets in Y are $\phi,X,\{a\},\{a,b\}$ and sgb-closed sets in X are $\phi,X,\{a,b\}$.Then, f is **not a sgb-continuous function**.

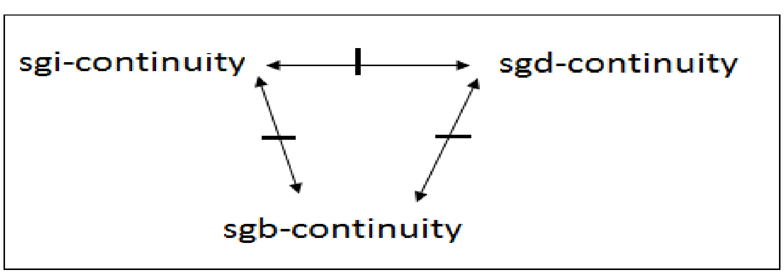
Remark 4.5: The notions sgd-continuity and sgb-continuity are independent as seen in the following example.

Example 4.6: In the spaces $(X,\tau_{11},\leq_8)$ and $(Y,\tau_{11},\leq_9)$ where $X=Y=\{a,b,c\}$, $\leq_8=\{(a,a),(b,b),(c,c)\}$ and $\leq_9=\{(a,a),(b,b),(c,c),(a,c)\}$, the b-closed sets in Y are ϕ,X and sgb-closed sets in X are $\phi,X,\{a\},\{b\},\{a,b\}$. Define $f:X\to Y$ as $f(a)=c$, $f(b)=b$ and $f(c)=a$. Then, f is a **sgb-continuous function**. The d-closed sets in Y are $\phi,X,\{a\},\{a,b\}$, and sgd-closed sets in X are $\phi,X,\{a\},\{b\},\{a,b\}$. Then, f is **not a sgd-continuous function.**

For the other part, consider the topological ordered spaces $(X,\tau_{11},\leq_6)$ and $(Y,\tau_{11},\leq_7)$ where $X=Y=\{a,b,c\}$, $\leq_6=\{(a,a),(b,b),(c,c),(b,a),(a,c),(b,c)\}$ and $\leq_7=\{(a,a),(b,b),(c,c),(b,a)\}$. The d-closed sets in Y are $\phi,X,\{a,b\}$ and sgd-closed sets in X are $\phi,X,\{b\},\{a,b\}$. Define $f:X\to Y$ as $f(a)=a$, $f(b)=b$ and $f(c)=c$. Then, f is a **sgd-continuous function.** The b-closed sets in Y are $\phi,X,\{a,b\}$ and sgb-closed sets in X are ϕ,X .Then, f is **not a sgb-continuous function.**

5 Conclusion

The following results were proved in this paper.



Here the symbol A $\nleftrightarrow$ B indicates A and B are independent notions.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Bhattacharyya P, Lahiri BK. Semi generalized closed sets in topology. Indian J. Pure. Appl. Math. 1987;29:375–382.

[2] Nachbin L. Topology and order, D.Van Nostrand Inc., Princeton, New Jersey; 1965.

[3] Veera Kumar MKRS. Homeomorphisms in topological ordered spaces. Acta Ciencia Indica. 2002;1:67-76.

[4] Ramachnadram VVS, Sankara Rao B, Veera Kumar MKRS. g-closed type, g*-closed type and sg-closed type sets in topological ordered spaces. Diophantus J. Math. 2015;4(1):1-9.

[5] Levine N. Semi-open sets and semi-continuity in topological spaces. Amer. Math. Monthly. 1963;70:36-41. a semi-open set [17] if A $\subseteq$ cl(int(A)) and a semi-closed set if int (cl(A)) $\subseteq$ A.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
http://www.sciencedomain.org/review-history/26595