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A Comparability Investigation of Numerical Techniques and Scientific Computation for Financial Engineering

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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ABSTRACT

This research explores the comparability of various numerical techniques and scientific computing methods applied to financial engineering. Financial engineering relies heavily on advanced mathematical models and computational analysis to value complex financial instruments, manage risk, and optimize investment strategies. This study critically examines the efficiency, accuracy, and computational feasibility of prominent finite difference methods and Monte Carlo simulations. Additionally, it assesses the integration of these methods with modern scientific computing frameworks, to enhance performance and scalability. The investigation includes a series of benchmark tests on common financial problems such as option pricing, portfolio optimization, and risk management. Our findings reveal that while traditional numerical methods like finite differences offer robustness and precision, they often lack scalability compared to Monte Carlo simulations which, despite their computational intensity, benefit significantly from parallelization enhancement. As such, this study gives best practices in selecting and combining numerical techniques and computing frameworks, aiming to equip financial engineers with effective tools for tackling modern financial challenges.

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1 INTRODUCTION

Financial services are one of the fastest-growing sectors in the business world. The rapid transformation resulted in the creation of modern financial instruments that are very complex and require new mathematical models for their implementation and pricing. The field of corporate finance was initially controlled by management students, and currently it is increasingly dominated by mathematicians and computing scholars. In the 1970s, Merton Robert, Scholes Murray, and Black Fisher developed the Black Scholes model, a major breakthrough in pricing of complex financial instruments. In 1997, the model formulator was given the Nobel Prize for Economics by Scholes Myron and Moore Robert, and recognized for the importance of their work worldwide. The Black-Scholes formulation emphasizes the crucial role of mathematics in financial services, opening the way for the development and development of mathematics, also known as financial engineering.

Owners of call options have the responsibility to sell (buy) basic assets pricing exercise, but do not have the obligation. The European options can be executed at the expiration only, while American pricing option may be executed at any time until expiry. The solution for the European option in closed-form is derived from the papers [1,2]. In the case of the United States, early exercise possibility creates complications in the analysis calculation. The authors [3,4] have shown that the assessment of American options involves a problem of free boundaries, and the boundaries change with maturity and are called optimal training boundaries. For this reason, financial researchers have studied methods to determine this limit quickly and accurately [5,6]. These methods are generally classified into two categories: analytic approximations developed by [7-9] and numerical methods proposed by [10,11]. Wu and Kwok [12] have found an accurate and explicit solution to the Black-Scholes formulation for evaluating American placement option via the infinite Taylor series. Their work is an important step in the evaluation of the options offered by the United States, but the implementation of their numerical solutions is difficult due to potential computational errors.

Michael et al. [13] expanded Wu and Kwok's work [12] to pricing American options in general dissemination processes. Most of the computing schemes employed in the calculating of American option are based on the Finite Difference method of Brennan and Schwartz [14] and the binomial method of Cox et al. Grant and Glassman's Monte Carlo simulation method [15], Tilley's smallest square method [16,17], Brandimarte's integral equation method [18], and Boyle's Laplace transformation method. [19], are time-recursive. These methods discretize the life of an option and calculate the optimal exercise limit backwards over time. These methods require fast calculations and minimum price errors due to repeated calculations at each step of the time. Furthermore, the front-fixing methods developed by Wu and Kwok [12] and Han and Wu [20] use nonlinear transformations to fix boundaries and solve resulting nonlinear problems. Wilmott et al. Secant method. The nonlinear problem is treated [21], and Geske and Johnson's moving boundary approach [22] converts the linear differential equation of the partial differential equation of the free boundary (PDE) to a sequence of the linear fixed boundary PDE problem. Until recently, Han and Wu [23] introduced a new predictor correction system that will price the options placed by Americans under the Black Castle model. Wilmott et al. [21] proposes an extension of Han and Wu's [23] theory to value American option positions based on a stochastic volatility model [24-26].

2 NUMERICAL TECHNIQUES FOR FINANCIAL ENGINEERING

In this study, survey of numerical method based on finite difference method and Monte Carlo Simulation (MCS) to overcome the difficulty in the valuation American option. Especially, the technique minimizes the necessary applicable approaches of locating the optimum boundary exercise before introducing discretization of finite difference. This technique is flexible, efficient, and accurate for all pay-off cases, and implementation is easy when other techniques are comparatively considered. The early outcomes depict that the scheme have high intrinsic accuracy to discretization of finite difference; as such, the techniques are stronger tool for determining American option. The techniques are finite volume schemes, spectral methods, Monte Carlo simulation, finite difference schemes, and many others.

In finance, a partial differential equation or partial integro-differential equation (PIDE) can be applied for pricing option. To approximate the outputs, different classes of computing techniques are applicable: finite volume schemes, spectral schemes, finite element schemes, finite difference schemes.

2.1 Finite Difference Approximation

$$\Phi_x|_{i,j} \simeq \frac{\phi(i+1,j)-\phi(i-1,j)}{2\Delta x},$$
$$\Phi_t|_{i,j} \simeq \frac{\phi(i,j+1)-\phi(i,j-1)}{2\Delta t},$$
$$\Phi_{xx}|_{i,j} \simeq \frac{\Phi(i-1,j)+\phi(i+1,j)-2\phi(i,j)}{(\Delta x)^2},$$
$$\Phi_{tt}|_{i,j} \simeq \frac{\Phi(i,j-1)+\phi(i,j+1)-2\phi(i,j)}{(\Delta x)^2}.$$

There are different types of finite difference methods; such as, finite Crank-Nicolson difference technique, finite implicit difference technique, and finite explicit difference technique.

The Crank-Nicolson approach is categorized a θ-scheme particular case, which is taken as an implicit ($\theta = 0$) and explicit($\theta = 1$) of average θ-weighted for finite difference techniques. When $\theta = \frac{1}{2}$, the Crank-Nicolson θ-method is gotten.

When considering boundary conditions and payoff function for the evaluation of a price option price with finite difference approach, the Black-Scholes equation is transformed into system of equations that is solved by adopting matrix approach. Therefore, the explicit scheme is changed to:

$$V_{1+n} = C_n + AV_n,$$

and the implicit scheme is changed to:

$$AV_{1+n} = C_n + V_n,$$

while the Crank-Nicolson technique is changed to:

$$AV_{1+n} = C_n + BV_n.$$

The explicit scheme is directly solved using a matrix A, meanwhile the Crank-Nicolson and implicit schemes are indirectly solved with matrix inversion A.

2.2 Implicit Method

Next, considering implicit scheme for solving PDE. employing implicit technique, a backward difference model is applied to approximate V_t, the equivalent model gives:

$$0 = rS\frac{V(S+\delta S,t)-V(S-\delta S,t)}{2\delta S} + \frac{V(S,t)-V(S,t-\delta t)}{\delta t} + \frac{1}{2}\sigma^2 S^2 \frac{V(S-\delta S,t)+V(S+\delta S,t)-2V(S,t)}{(\delta S)^2} - rV(S,t) \quad (1)$$

$P(S,t)$ can be approximated by $V(S_n, T_m) \equiv V_n^m$. As such we have have:

$$V_n^{m-1} = a_n V_{n-1}^m + b_n V_n^m + c_n V_{n+1}^m, \quad \text{for m = 1,...,M, and n = 1,...,N-1} \quad (2)$$

where

$$a_n = \frac{1}{2}rn\delta t - \frac{1}{2}\sigma^2 n^2 \delta t$$
$$b_n = 1 + \sigma^2 n^2 \delta t + r\delta t$$
$$c_n = -\frac{1}{2}rn\delta t - \frac{1}{2}\sigma^2 n^2 \delta t$$

An initial condition is obtained (since backward difference model is used, the final condition gives initial condition), and the boundary conditions are equivalent to explicit scheme.

2.3 Monte Carlo Computation

Computation is an approach for generating random numbers inline with the assumed probabilities associating with an uncertainty source, such as interest rates, purpose approximations, price stocks, product sales, commodity prices or exchange rates. Outputs related to random drawings are determined to evaluate the possible outcomes and the related risk. The major steps for Monte Carlo computation techniques are as follows.

- The needed time horizon of the neutral risk constraint for asset underlying path is computed.
- The payoff discount is corresponding to the interest free-risk rate path.
- Sample simulation path for the high number procedures are repeated.
- The cash flow discount average over the options value sample is calculated.

A Monte Carlo computation algorithm is adopted for the stock price random sampling results is given according [6] to

$$dS = \mu S dt + \sigma S dW(t) \tag{3}$$

where stock price is S and Wiener process is dW_t. Assume δS is the stock price increase for small next time interval δt therefore

$$\frac{\delta S}{S} = \mu \delta t + \sigma Z \sqrt{\delta t} \tag{4}$$

where neutral-risk expected return is μ, stock price volatility is σ, and $Z \sim N(0, 1)$ are confirmed to

$$S(t + \delta t) - S(t) = \mu S(t) \delta t + Z \sigma S(t) \sqrt{\delta t} \tag{5}$$

then with the initial value S and in time $t + \delta t$, the value S is calculated, and from $t + \delta t$ value, in time $t + 2\delta t$, the value of S is computed, and so on. With random sample of N in normal distribution, the trial complete path is simulated for S. It is preferable to compute $\ln S$ than S, by transforming the price asset process through Ito's lemma

$$d(\ln S) = \left(\mu - \frac{\sigma^2}{2}\right) dt + \sigma dW(t)$$

such that

$$\ln S(t + \delta t) - \ln S(t) = \left(\mu - \frac{\sigma^2}{2}\right) \delta t + \sigma Z \sqrt{\delta t}$$

or

$$S(t + \delta t) = S(t) \exp\left[\left(\mu - \frac{\sigma^2}{2}\right) \delta t + \sigma Z \sqrt{\delta t}\right] \tag{6}$$

The Monte Carlo computation is relevant for a depending financial payoff derivative on the path and life asset underlying option for a dependent path option. Considering a maturity stock price for an Asian option process at time T taken as

$$S_T^j = S \exp\left[\left(\mu - \frac{\sigma^2}{2}\right) T + \sigma Z \sqrt{T}\right] \tag{7}$$

where M and $j = 1, 2, ..., M$ and M describes word difference states or trial numbers. The computation of M gives the possible stock price path at T maturity date. The estimated call value Asian option is

$$C = \frac{1}{M} \sum_{j=1}^{M} \max[S_T^j - S_t, 0] e^{-rT} \tag{8}$$

This is a price derivative unbiased estimate. For high trial numbers M, the limit central theorem gives an estimated interval confidence based on the payoff discount sample variance. Given payoff discount mean as $\bar{\mu}$ and standard deviation as ω, therefore, the estimated standard error is $\frac{\omega}{\sqrt{M}}$. A 0.95% interval price derivative confidence f is taken as

$$\bar{\mu} - \frac{1.96\omega}{\sqrt{M}} < f < \bar{\mu} + \frac{1.96\omega}{\sqrt{M}} \tag{9}$$

with normally distributed f assumption

3 SCIENTIFIC COMPUTING FOR ASIAN OPTIONS

Asian or Average options are options whose payoff depends on the average price of the asset underlying for life part option. If trading day option is given as N, option maturing date is T, and $S(t_j)$ the end price security's day is j, where $t_N = T$ and $j = 1, 2, ..., N,$. The asset price average underlying is computed via two schemes, namely the average geometric and arithmetic.

- **Average Arithmetic:** Let the average arithmetic value be $S_A(t)$ for asset underlying evaluation over the life option. The average arithmetic is determined by

$$\begin{aligned} S_A(t) &= \frac{S(t_1) + S(t_2) + ... + S(t_N)}{N} \\ &= \frac{1}{N} \sum_{i=1}^{N} S(t_i) \end{aligned} \tag{10}$$

- **Average Geometric:** Let the average geometric value be $SG(t)$ for asset underlying calculation over the life option. Therefore, the average geometric is defined as [7]

$$\begin{aligned} S_G(t) &= \left[\prod_{i=1}^{N} S(t_i) \right]^{1/N} \\ &= [S(t_1) S(t_2) \cdots S(t_N)]^{1/N} \end{aligned} \tag{11}$$

The Asian standard option types gotten from the average geometric or arithmetic of asset underlying are:

(i) Price Average Option

- A payoff call price average is $\max(\bar{S}(t) - K, 0)$.
- A payoff put price average is $\max(K - \bar{S}(t), 0)$.

(ii) Price Strike Average Option

- A payoff call strike average is $\max(S_T - \bar{S}(t), 0)$.
- An payoff put strike average is $\max(\bar{S}(t) - S_T, 0)$.

where average arithmetic or average geometric in (10) or (11) is $\bar{S}(t)$.

The investor needs determine the Asian option types that will the adopted. The price call strike standard average for Asian payoff option is

$$f_c(S,T) = \max\left[S(T) - \frac{1}{T}\int_0^T S(\tau)d\tau, 0\right], \tag{12}$$

where its price asset value relies on the history, not on its final value. The put Asian is described as

$$f_p(S,T) = \max\left[\frac{1}{T}\int_0^T S(\tau)d\tau - S(T), 0\right] \tag{13}$$

The frequency is major basic concerns for observing the price over the average period. For Monte Carlo price of equation (12), a N positive integer is time subdivide in the range [0, T] into N equal Δt = T/N and subranges, this compute the asset price

$$S[(k+1)\Delta t] = \exp\left[\Delta t(r - \frac{\sigma^2}{2}) + \sigma\sqrt{Z_k \Delta t}\right] S(k\Delta t) \tag{14}$$

where $k = 0, 1, ..., N-1$ and $Z_k \sim N(0, 1)$. Set $S(k\Delta t) = S_k$: As such, (16) indicates

$$In\left[\frac{S_{1+k}}{S_k}\right] = X_k = \left[(r - \frac{\sigma^2}{2})\Delta t + \sigma Z_k\sqrt{\Delta t}\right]$$

$$\mu\Delta t + \sigma Z_k\sqrt{\Delta t} \tag{15}$$

where $X_k \sim N(\Delta t\mu, \sigma^2\Delta t)$ and GBM neutral-risk drift term is μ = $(r - \sigma^2/2)$. Then

$$\ln\left[\frac{S_{1+k}}{S_k}\right] = X_k$$

therefore it gives

$$\begin{aligned} S_{1+k} &= S_k e^{X_k} \\ &= S_{k-1}e^{X_{k-1}}e^{X_k} \\ &= S_0 e^{X_{0+\ldots+X_k}} \end{aligned} \tag{16}$$

Equation (16) denotes the explicit equation, while S_k defines recurrence relation as given (14). The integral average time approximated using trapezium scheme

$$\int_0^T S(\tau)d\tau \approx \frac{1}{N}\left[\frac{1}{2}S(0) + \frac{1}{2}S(T) + \sum_{k=1}^{N-1} S(k\Delta t)\right] \tag{17}$$

and $\bar{S}_t$ is the approximated discrete. The call Asian option discretely guided as the path value estimate ith gives

$$c^i = \max[S_T - \bar{S}_t, 0]e^{-Tr}. \tag{18}$$

A repetition is done for for j = $1, 2, ..., M$ and the estimated option final value is

$$C = \frac{1}{M}\sum_{j=1}^{M} c^j \tag{19}$$

4 SCIENTIFIC COMPUTING FOR AMERICAN OPTION

A call American option do not give obligation but right to its holder the right, to buy any time a particular asset for a given price from the beginning date to a given future expiration date. At any time, the strength for the option exercise to be extended is of to the owner rights; thus, the American option has larger values potential.

Given S lies between $\max(E-S,0) > P(S,t)$, the the demand of the arbitragers pushes the value option in a short time. Hence, early exercise is allowed under the imposed constraint

$$\max(S-E,0) \leq V(S,t)$$

The European and American options have diverse values.

4.1 The Option Price of the American Put

Once $\bar{S}_f(p)$ is found, D_1, D_2 and D_4 can be determined easily and expressed in the form of Laplace term p as

$$\begin{aligned} D_1 &= \tfrac{\gamma}{p(p+\gamma)} \cdot \tfrac{q_2}{(q_2-q_1)} \cdot \tfrac{1}{(p\bar{S}_f)^{q1}} \\ &= -\tfrac{\gamma}{p(p+\gamma)} \cdot \tfrac{b-\sqrt{p+a^2}}{2\sqrt{p+a^2}} \cdot \left[1-\tfrac{p+\gamma}{\gamma(b-\sqrt{p+a^2})}\right]. \end{aligned} \tag{20}$$

$$\begin{aligned} D_2 &= \tfrac{\gamma}{p(p+\gamma)} \cdot \tfrac{q_1}{(q_1-q_2)} \cdot \tfrac{1}{(p\bar{S}_f)^{q2}} \\ &= \tfrac{\gamma}{p(p+\gamma)} \cdot \tfrac{b+\sqrt{p+a^2}}{2\sqrt{p+a^2}} \cdot \left[1-\tfrac{p+\gamma}{\gamma(b-\sqrt{p+a^2})}\right]^{\frac{q_2}{q_1}}. \end{aligned} \tag{21}$$

$$\begin{aligned} D_4 &= D_1 + D_2 - \tfrac{\gamma}{(\gamma+p)p}, \\ &= \tfrac{\gamma}{(\gamma+p)p} \cdot \left\{-\tfrac{b-\sqrt{a^2+p}}{2\sqrt{a^2+p}} \cdot \left[1-\tfrac{p+\gamma}{\gamma(b-\sqrt{p+a^2})}\right]\right. \\ &\left. + \tfrac{\sqrt{a^2+p}+b}{2\sqrt{p+a^2}} \cdot \left[1-\tfrac{p+\gamma}{\gamma(b-\sqrt{p+a^2})}\right]^{\frac{q_2}{q_1}} - 1\right\}. \end{aligned} \tag{22}$$

As such, $U(S,\tau)$ is expressed as

$$U(S,\tau) = \frac{1}{2\pi i}\int_{\mu-\infty i}^{\mu+\infty i} \frac{\gamma e^{p\tau}}{p(p+\gamma)} F_1(p)dp, \tag{23}$$

for $S_f(\tau) \leq S \leq 1$ and,

$$U(S,\tau) = \frac{1}{2\pi i}\int_{\mu-\infty i}^{\mu+\infty i} \frac{\gamma e^{p\tau}}{p(p+\gamma)} F_2(p)dp, \tag{24}$$

for $S > 1$

In Eq. (23) and Eq. (24), $F_1(p)$ and $F_2(p)$ are obtained and can be written as

$$\begin{aligned} F_1(p) &= \tfrac{1}{2}\left(1-\tfrac{b}{\sqrt{a^2+p}}\right) \cdot S^{q_1} \cdot \left[1-\tfrac{p+\gamma}{\gamma(b-\sqrt{a^2+p})}\right] \\ &-\tfrac{1}{2}\left(\tfrac{b}{\sqrt{p+a^2}}-1\right) \cdot \left[1-\tfrac{p+\gamma}{\gamma(\sqrt{p+a^2})+b}\right]^{\frac{q_2}{q_1}} \cdot S^{q_2} - 1. \end{aligned} \tag{25}$$

$$\begin{aligned} F_2(p) = &\left\{\tfrac{1}{2}\left(1-\tfrac{b}{\sqrt{a^2+p}}\right) \cdot \left[1-\tfrac{p+\gamma}{\gamma(b-\sqrt{a^2+p})}\right]\right. \\ &\left. -\tfrac{1}{2}\left(\tfrac{b}{\sqrt{a^2+p}}-1\right) \cdot \left[1-\tfrac{p+\gamma}{\gamma(b+\sqrt{a^2+p})}\right]^{\frac{q_2}{q_1}} - 1\right\} \cdot S^{q_2}, \end{aligned} \tag{26}$$

4.2 Numerical Finite Difference Based Front Tracking Method for American Option

If $v = v(x,t)$ is defined in a reference fixed frame for time t and co-ordinate $\mathbf{x}$. The space derivatives is only involved in the differential operator L^1.

Instead of Eulerian (fixed) frame, a Lagrangian point of view of $\mathbf{x}$ is considered for a motion axis $\mathbf{x}(t)$. If a mapping invertible is defined for the fixed axes $\mathbf{a}$ and the motion axes $\mathbf{x}$ at time t.

$$\mathbf{x} = \hat{\mathbf{x}}(\mathbf{a}, t) \tag{27}$$

We have

$$v(\mathbf{x}, t) = \hat{v}(\mathbf{a}, t) = v(\hat{\mathbf{x}}(\mathbf{a}, t), t) \tag{28}$$

where Eulerian are $\hat{\mathbf{x}}$ and $\hat{v}$. Applying the function of function rule to (28) gives

$$\frac{\partial \hat{v}}{\partial t} = \frac{\partial v}{\partial \hat{\mathbf{x}}} \cdot \frac{\partial \hat{\mathbf{x}}}{\partial t} + \frac{\partial v}{\partial t} \tag{29}$$

Meanwhile, $v_\tau = v_{xx} + g$. Thus, gives

$$\frac{\partial \hat{v}}{\partial t} = g(x, \tau) + \frac{\partial v}{\partial x} \cdot \frac{\partial \hat{\mathbf{x}}}{\partial t} + \frac{\partial^2 v}{\partial x^2} \tag{30}$$

It is the time-dependent model, the call price American option solution is obtained.

Discretizing the model using finite difference schemes. Let N number divides the S space subintervals.

$$S_i = i\delta S, \qquad i = 0, ..., N \tag{31}$$

$$\delta S = \frac{B(\tau) - x^-}{N} \tag{32}$$

L denotes the number dividing the time interval such that

$$\tau_j = j\delta\tau, \qquad j = 0, ..., L$$

$$\delta\tau = \frac{1}{2}\sigma^2 T/L$$

$$\frac{\partial V}{\partial \tau} \approx \frac{V_i^{j+1} - V_i^j}{\delta\tau} \tag{33}$$

The second spatial derivative, $\frac{\partial^2 V}{\partial S^2}$ is approximated by

$$\frac{\partial^2 V}{\partial S^2} \approx \frac{V_{i-1}^j - 2V_i^j + V_{i+1}^j}{(\delta S)^2} \tag{34}$$

Approximating the 'velocity nodal', $\dot{S}$ as

$$\frac{\partial S}{\partial \tau} \approx \frac{S_i^{j+1} - S_i^j}{\delta\tau} \tag{35}$$

We first discretised the PDE (30)

$$\frac{V_i^{j+1} - V_i^j}{\delta\tau} = \theta_1 \left(\frac{V_{i+1}^{j+1} - 2V_i^j + V_{i-1}^{j+1}}{\delta S^2} \right) + \theta_2 \left(\frac{V_{i+1}^j - 2V_i^j + V_{i-1}^j}{\delta S^2} \right)$$

$$+\left[\left(\frac{V_i^j - V_{i-1}^j}{\delta S}\frac{S_i^{j+1} - S_i^j}{\delta \tau}\right)\right]$$
$$+\theta_3 G_i^{j+1} + \theta_4 G_i^j \tag{36}$$

For $1 \leq n \leq N-1$ and $1 \leq j \leq J-1$. The term θ_i governs the implicit procedure.
For consistency,

$$\theta_1 + \theta_2 = \theta_3 + \theta_4 \tag{37}$$

Increasing time expanded the domain $B(\tau)$. The appropriating the grid to determine the free boundary position then equally divide domain into linear equal grid space nodes. Given the free boundary position as x_N^{j+1}, $x_f(t)$, the grid nodes at time-step $j+1$ are denoted by $x_i^{j+1} = x^- + \frac{i}{N}(x_N^{j+1} - x^-)$ where $i = 1, 2, ..., N$.
Differentiating gives

$$\dot{x}_i = \frac{i}{N}(\dot{x}_N) \tag{38}$$

The nodal point velocity is determined, θ-weighted discretized finite difference.

When θ= 0 gives explicit discretization, for $\theta = \frac{1}{2}$ denotes Crank-Nicolson technique, and implicit method wehn θ = 1. Thus, $\theta = \frac{1}{2}$ dissertation is done.

$$V_i^{j+1} - V_i^j = \alpha_i[\theta_1(V_{i+1}^{j+1} - 2V_i^{j+1} + V_{i-1}^{j+1}) + \theta_2(V_{i+1}^j - 2V_i^j + V_{i-1}^j)]$$
$$+\beta_i[\theta_3 G_i^{j+1} + \theta_4 G_i^j] + \gamma_i[(V_i^j - V_{i-1}^j)(X_{N+1}^j - X_N^j)]$$

Where

$$\alpha_i = \frac{\delta\tau}{(\delta S)^2} > 0, \quad \gamma_i = \frac{i}{N\delta S} \quad \beta_i = 2k > 0$$

Rearranging (39) resulted into

$$c_i V_{i-1}^{j+1} + a_i V_i^{j+1} + b_i V_{i+1}^{j+1} + f_i(V_i^j - V_{i-1}^j)X_{N+1}^j$$
$$= \acute{c}_i V_{i-1}^j + \acute{a}_i V_i^j + \acute{b}_i V_{i+1}^j + \acute{f}_i(V_i^j - V_{i-1}^j)X_N^j + e_i G_i^{j+1} + \acute{e}_i G_i^j$$

where

$$\begin{aligned} c_i &= -\alpha_i\theta_1 & \acute{c}_i &= \theta_2\alpha_i \\ a_i &= 1 + 2\alpha_i\theta_1 & \acute{a}_i &= 1 - 2\alpha_i\theta_2 \\ b_i &= -\alpha_i\theta_1 & \acute{b}_i &= \alpha_i\theta_2 \\ e_i &= 2\theta_1\beta_i & \acute{e}_i &= 2\theta_2\beta_i \\ f_i &= \gamma_i\theta_1 & \acute{f}_i &= \gamma_i\theta_2 \end{aligned} \tag{39}$$

The problem reduces to system of equations

$$TV^{j+1} + \vec{\beta}X_N^{1+j} = \vec{d} + BV^j \tag{40}$$

To evaluate free boundary location at time-step, more information is required. The condition $\frac{\partial C(B(\tau),\tau)}{\partial x}$ = 0 result in $v_{N-1} = v_N$, to have

$$\vec{\beta}x_N + T\vec{v}^{j+1} = B\vec{v}^j + \vec{d} \tag{41}$$
$$\vec{v}h^T = 0$$

Where T, B, d and β components are considered

$$T = \begin{bmatrix} 2+2r & -r & 0 & \dots & 0 \\ -r & 2r+2 & -r & & \\ 0 & -r & \ddots & \ddots & 0 \\ \vdots & & \ddots & \ddots & -r \\ 0 & \dots & 0 & -r & 2r+2 \end{bmatrix} \quad (42)$$

$$T = \begin{bmatrix} 2-2r & r & 0 & \cdots & 0 \\ r & 2-2r & r & & \\ 0 & r & \ddots & \ddots & 0 \\ \vdots & & \ddots & \ddots & r \\ 0 & \cdots & 0 & r & 2-2r \end{bmatrix} \quad (43)$$

$$d_i = \frac{1}{2}\left(g(ih, j\Delta\tau + x^-) + (g(ih, \Delta\tau(1+j) + x^-)\right) - \left(\frac{v_i^j - v_{i-1}^j}{x_i - x_{i-1}}\right)\left(\frac{i}{N}\right)x_N^j$$

$$\beta_i = -\frac{i}{N}\left(\frac{v_i^j - v_{i-1}^j}{x_i - x_{i-1}}\right) \quad (44)$$

$$h^T = 0\,0\,\cdots\,\cdots -1\,1 \quad (45)$$

The notation is simplified the quantity Bv^j absorbed into the vector d. In explicitly matrix gives

$$\begin{bmatrix} 2-2r & -r & 0 & \cdots & 0 & -\beta_1 \\ -r & 2-2r & -r & & \cdots & -\beta_2 \\ 0 & -r & \ddots & \ddots & 0 & \vdots \\ \vdots & & \ddots & \ddots & -r & \vdots \\ 0 & \cdots & 0 & -r & 2-2r & -\beta_{N-1} \\ 0 & 0 & \cdots & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} v_1^j \\ v_2^j \\ \vdots \\ \vdots \\ v_{N-1}^j \\ v_N^{j+1} \\ 0 \end{bmatrix} = \begin{bmatrix} \acute{d}_1 \\ \acute{d}_2 \\ \vdots \\ \vdots \\ \acute{d}_{N-1} \\ 0 \\ 0 \end{bmatrix} \quad (46)$$

Symbolically given as

$$\begin{pmatrix} \mathbf{T} & \vec{\beta} \\ \vec{h}^T & 0 \end{pmatrix} \begin{pmatrix} \vec{v} \\ x_N^{j+1} \end{pmatrix} = \begin{pmatrix} \vec{d} \\ 0 \end{pmatrix} \quad (47)$$

Re-arranging equation (43) to solve determine x_N

$$\vec{\beta}x_N + T\vec{v} = \vec{d}$$
$$\vec{h}^T\vec{v} = 0$$
$$\Rightarrow \vec{v} = T^{-1}\left(\vec{d} - N - \vec{\beta}x\right)$$
$$\Rightarrow \vec{h}^T\left(T^{-1}\vec{d} - T^{-1}\vec{\beta}x_N\right) = 0$$
$$\Rightarrow x_N = \frac{T^{-1}\,\acute{}\,\tilde{h}\vec{d}^T}{T^{-1}\tilde{\beta}h^T}$$

A free boundary location method $x_f(\tau)$ is developed for time step successive procedure. On calculating x_N^{j+1}, the moving nodes velocity is determined for the equation

$$\dot{x_i} = \frac{i}{N}\dot{x_N} \quad (48)$$

On substitution gives

$$T\vec{v} = \vec{d} - \vec{\beta}x_N^{j+1} \tag{49}$$

The equation solution is achieved by a tridiagonal form, for

$$\vec{v} = T^{-1}(\vec{d} - \vec{\beta}x_N^{j+1}) \tag{50}$$

5 COMPARISON OF SCIENTIFIC COMPUTATIONS

Scientific simulating is the numerical analysis heart, this remain the major contention of this work. The study emphasis is on the modern and classical elements of computer scientists and computational mathematics, and based comparability analysis, the study is chosen for the scientific computing and numerical schemes for engineering finance. For algorithms comparison, an approximated standard numerical study us done with related call option examples.

5.1 Example 1:

The Table 1 illustrate the different results obtained from using finite difference and the Monte Carlo methods to call vanilla price computing with $r = 0.1, T = 1, N = 10$ and $\sigma = 0.2$ in relation to the obtained analytical outputsof Black-Scholes equations. The finite difference methods is built on time-steps of 50, and the Monte Carlo computing employes $20,000$ cases.

Table 1. Numerical of MCM and FDM comparison with the black-scholes analytical

	S_0	Analytic	Explicit	Implicit	Monte
Value	8	0.279	0.279	0.286	0.280
	10	1.327	1.324	1.327	1.344
	12	3.026	3.025	3.031	3.042
Error	8	0.279	0	0.007	0.001
	10	1.327	0.001	00	0.007
	12	3.026	0.001	0.005	0.016
Time (secs.)			0.0431	0.0573	1.4886

These results in Table 1 shows that the binomial technique is accurate and effective. However, the explicit finite difference scheme is highly accurate with fast computing time; thus, make it a strong scheme. Here, despite its method of explicit merits over the method of implicit, the explicit scheme step sizes must be chosen carefully to prevent instability.

Example 2:

Examine the two schemes performance on the true put European Black-Scholes price with

$$r = 0.05, K = 50, \sigma = 0.25, T = 3.$$

The Table 2 illustrates option price variation with the asset price underlying, S. The outcomes depict that the schemes are mutually consistent, effective, and satisfies Black-Scholes outputs. Meanwhile, such numerical schemes may not be needed in the existence of explicit formula.

Table 2. A put European black-scholes price comparison

S	Black-Scholes	Monte-Carlo	Implicit Euler
10	33.0363	33.0345	33.0369
20	23.2276	23.2291	23.2300
30	14.7739	14.7748	14.7749
40	8.7338	8.7374	8.7348
50	4.9564	4.9559	4.9563
60	2.7621	2.7602	2.7612
70	1.5328	1.5324	1.5325
80	0.8538	0.8543	0.8537
90	0.4797	0.4790	0.4794

Example 3:

Considering different numerical techniques effectiveness on the true put European Black-Scholes price with

$$r = 0.05, K = 50, \sigma = 0.25, T = 3$$

The outcomes obtained are given in the Table 3.

Table 3. A put European black-scholes price comparison

S	Black-Scholes	Finte Diff. Method	Monte Carlo Method
45	6.6021	6.6019	6.6014
50	4.9564	4.9563	4.9559
55	3.7046	3.7042	3.7076
60	2.7621	2.7613	2.7602
65	2.0574	2.0572	2.0581
70	1.5328	1.5326	1.5324
75	1.1430	1.1427	1.1407
80	0.8538	0.8537	0.8543
85	0.6392	0.6391	0.6405
90	0.4797	0.4795	0.4790

Table 3 narrates the option price variation with the asset price underlying, S. The outputs display the mutually consistent, efficient of the three computational techniques, and aligns with the Black-Scholes outcomes. Thus, finite difference method converges faster, accurate than the Monte Carlo technique.

6 CONCLUSION

Diverse computation schemes are used for derivatives valuing in the absence of close-form solution is available. These involve using finite difference technique and Monte Carlo computing. The Monte Carlo computation performs on derivative life forward from start to end. It uses derivative European-style with a complexity deal with payoff. The major findings includes:

- Finite difference method proved to be particularly effective in solving partial differential equations that are fundamental in pricing derivative securities.
- Monte Carlo methods excel in handling high-dimensional integrals and are highly flexible, making them suitable for a wide range of financial applications, including risk management, option pricing, and portfolio optimization.
- Finite difference methods are particularly accurate for PDE-based problems, while Monte

Carlo simulations provide robust solutions for complex, multi-dimensional scenarios.

- Monte Carlo simulations show superior scalability for high-dimensional problems, whereas finite difference methods struggle with increased dimensions.

No single numerical method stands out as universally superior. Instead, the optimal choice depends on the specific financial application and computational constraints. Future research should focus on developing hybrid methods that combine the strengths of different techniques, as well as exploring advancements in computational power and algorithms to further enhance the efficiency and applicability of these numerical methods in financial engineering.

DISCLAIMER (ARTIFICIAL INTELLIGENCE)

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc) and text-to-image generators have been used during writing or editing of manuscripts.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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