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Multi-Criteria Group Decision-Making Method Based on an Improved Single-Valued Neutrosophic Hamacher Weighted Average Operator and Grey Relational Analysis

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Abstract

This paper proposes a multi-criteria decision-making (MCGDM) method based on the improved single-valued neutrosophic Hamacher weighted averaging (ISNHWA) operator and grey relational analysis (GRA) to overcome the limitations of present methods based on aggregation operators. First, the limitations of several existing single-valued neutrosophic weighted averaging aggregation operators (*i.e.*, the single-valued neutrosophic weighted averaging, single-valued neutrosophic weighted algebraic averaging, single-valued neutrosophic weighted Einstein averaging, single-valued neutrosophic Frank weighted averaging, and single-valued neutrosophic Hamacher weighted averaging operators), which can produce some indeterminate terms in the aggregation process, are discussed. Second, an ISNHWA operator was developed to overcome the limitations of existing operators. Third, the properties of the proposed operator, including idempotency, boundedness, monotonicity, and commutativity, were analyzed. Application examples confirmed that the ISNHWA operator and the proposed MCGDM method are rational and effective. The proposed improved ISNHWA operator and MCGDM method can overcome the indeterminate results in some special cases in existing single-valued neutrosophic weighted averaging aggregation operators and MCGDM methods.

Keywords

Single-Valued Neutrosophic Numbers, Single-Valued Neutrosophic Hamacher Weighted Averaging Operator, Grey Relational Analysis, Multi-Criteria Decision-Making, Multi-Criteria Group Decision-Making

1. Introduction

Multi-criteria group decision-making (MCGDM) refers to a decision-making process in which several decision-makers (DMs) compare and select finite alternatives by using specific methods according to the evaluation information of multiple criteria [1] [2]. Due to the complex uncertainty of environmental conditions and the cognitive limitations of DMs, traditional quantitative methods usually cannot accurately describe decision-making problems. Many scholars have focused on fuzzy sets (FSs) [3] [4] [5] and their extensions (e.g., intuitionistic fuzzy sets (IFSs) [6] [7] [8], hesitant fuzzy sets (HFSs) [9] [10] [11], picture fuzzy sets (PFSs) [12] [13] [14]), and fuzzy rough sets (FRSs) and so on [15] [16]) to solve this issue. However, compared those extensions, neutrosophic sets (NSs) [17] [18] [19] characterized by truth-membership, indeterminacy-membership, and falsity-membership function can better describe the fuzzy uncertainty of decision-making provided by DMs. Wang *et al.* [20] and Ye [21] extended the scope of membership degrees from the non-standard unit interval]0-, 1+[to standard unit interval [0, 1] and defined single-valued neutrosophic sets (SNSs) to further promote the application of NSs.

The single-valued neutrosophic MCGDM and multi-criteria decision-making (MCDM) methods have been investigated mainly from three aspects: aggregation operators [22] [23] [24], information measures [25] [26] [27] [28], and outranking relations [29] [30] [31]. The aggregation operator, which is fundamental for the MCGDM and MCDM methods, can effectively present decision-making results in most cases. Ye [21] defined the single-valued neutrosophic weighted averaging (SNWA) operator and developed the corresponding MCDM method; meanwhile, Peng *et al.* [22] developed the single-valued neutrosophic weighted algebraic averaging (SNWAA) and single-valued neutrosophic weighted Einstein averaging (SNEWA) operators; Garg [24] defined the single-valued neutrosophic Frank weighted averaging (SNFWA) operator and applied it to handle MCDM problems; moreover, Liu *et al.* [32] developed an MCGDM method based on the single-valued neutrosophic Hamacher weighted averaging (SNHWA) operator. Finally, Kilic *et al.* [33] developed a single-valued neutrosophic leanness assessment methodology based on SNWAA operator and Mishra *et al.* [34] proposed a CRiteria Importance through Intercriteria Correlation (CRITIC) method using the SNWAA operator, while Pani *et al.* developed a multi-objective optimization based on ratio analysis with the full multiplicative form (MULTIMOORA) method based on the SNWAA operator.

However, existing single-valued neutrosophic weighed averaging operators and their corresponding MCGDM and MCDM methods have the following limitations. 1) The SNWA [21], SNWAA [22], SNEWA [22], SNFWA [23], and SNHWA operators [30] may produce unreasonable results in the aggregation process (*i.e.*, indeterminate aggregated terms in some cases). 2) The corresponding decision-making methods, including the MCDM method using the SNWA operator [21], the MCDM method using the SNWAA and SNEWA operators [22], the MCDM method using the SNFWA operator [24], and the MCGDM method using the SNHWA operator [32] can also produce unreasonable decision-making results, which do not allow to the real decision-making.

In this paper, we present an improved SNHWA (ISNHWA) operator that was developed to exceed the limitations of existing aggregation operators. An improved single-valued neutrosophic MCGDM method based on the ISNHWA operator and grey relational analysis (GRA) is also proposed to overcome the limitations of existing MCGDM and MCDM methods based on aggregation operators.

The structure of this paper is organized as follows. Section 2 reviews related concepts, including SNSs, single-valued neutrosophic numbers (SNNs), the comparison method of SNNs, some existing single-valued neutrosophic weighted averaging operators, and GRA. Section 3 analyzes the limitations of existing aggregation operators. Section 4 describes the proposed ISNHWA operator and discusses its properties. Section 5 introduces a single-valued neutrosophic MCGDM method based on the ISNHWA operator and GRA. Section 6 provides some application examples to verify the effectiveness and feasibility of the proposed method. Finally, conclusions are drawn in Section 7.

2. Preliminaries

2.1. Single-Valued Neutrosophic Averaging Operators

In this section, related concepts, including SNSs, SNNs, the comparison method of SNNs, and some existing single-valued neutrosophic averaging operators, are reviewed.

Definition 1 [19]. Let X be a space of points (objects) with a generic element in X, denoted by x. An SNS in X is characterized as $S=\left\{\left\langle x,t_S(x),f_S(x),k_S(x)\right\rangle \middle| x\in X\right\}$, where $t_S(x)$, $f_S(x)$, and $k_S(x)$ are the truth-membership (satisfying $0\le t_S(x)\le 1$), indeterminacy-membership (satisfying $0\le f_S(x)\le 1$), and falsity-membership (satisfy $0\le k_S(x)\le 1$) respectively. If X has only one element, then $S=\left\langle t_S(x),f_S(x),k_S(x)\right\rangle$ is called an SNN and is denoted by $S=\langle t,f,k\rangle$.

Definition 2 [22]. Assuming that S_1 and S_2 are two SNNs, then their comparison can be described as follows:

1) If $p(S_1)>p(S_2)$, then $S_1\succ S_2$;

2) If $p(S_1)=p(S_2)$ and $q(S_1)>q(S_2)$, then $S_1\succ S_2$;

3) If $p(S_1)=p(S_2)$ and $q(S_1)=q(S_2)$, then $S_1=S_2$.

Here $p(S_i)=\dfrac{t_i+1-f_i+1-k_i}{3}$ and $q(S_i)=t_i-k_i\ (i=1,2)$ denote the score and accuracy functions, respectively.

Definition 3. It is hypothesized that $S_j=\left\langle t_j,f_j,k_j\right\rangle (j=1,2,\cdots,n)$ is a group of SNNs and ω_j the corresponding weight of S_j, $\omega_j\in[0,1]$ and $\sum_{j=1}^{n}\omega_j=1$. In this case, the following single-valued neutrosophic weighted average operators can be defined.

1) SNWA operator [21]:

$$\begin{aligned}&\mathrm{SNWA}_{\omega}\left(S_1,S_2,\cdots,S_n\right)\\&=\left\langle 1-\prod_{j=1}^{n}\left(1-t_j\right)^{\omega_j},1-\prod_{j=1}^{n}\left(1-f_j\right)^{\omega_j},1-\prod_{j=1}^{n}\left(1-k_j\right)^{\omega_j}\right\rangle.\end{aligned}\tag{1}$$

2) SNWAA operator [22]:

$$\mathrm{SNWAA}_{\omega}\left(S_1,S_2,\cdots,S_n\right)=\left\langle 1-\prod_{j=1}^{n}\left(1-t_j\right)^{\omega_j},\prod_{j=1}^{n}f_j^{\omega_j},\prod_{j=1}^{n}k_j^{\omega_j}\right\rangle.\tag{2}$$

3) SNEWA operator [22]:

$$\begin{aligned}&\mathrm{SNEWA}_{\omega}\left(S_1,S_2,\cdots,S_n\right)\\&=\left\langle \frac{\prod_{j=1}^{n}\left(1+t_j\right)^{\omega_j}-\prod_{j=1}^{n}\left(1-t_j\right)^{\omega_j}}{\prod_{j=1}^{n}\left(1+t_j\right)^{\omega_j}+\prod_{j=1}^{n}\left(1-t_j\right)^{\omega_j}},\frac{2\prod_{j=1}^{n}f_j^{\omega_j}}{\prod_{j=1}^{n}\left(2-f_j\right)^{\omega_j}+\prod_{j=1}^{n}f_j^{\omega_j}},\frac{2\prod_{j=1}^{n}k_j^{\omega_j}}{\prod_{j=1}^{n}\left(2-k_j\right)^{\omega_j}+\prod_{j=1}^{n}k_j^{\omega_j}}\right\rangle.\end{aligned}\tag{3}$$

4) SNFWA operator [24]:

$$\begin{aligned}&\mathrm{SNFWA}_{\omega}\left(S_1,S_2,\cdots,S_n\right)\\&=\left\langle 1-\log_{\lambda}\left(1+\prod_{j=1}^{n}\left(\lambda^{1-t_j}-1\right)^{\omega_j}\right),\log_{\lambda}\left(1+\prod_{j=1}^{n}\left(\lambda^{f_j}-1\right)^{\omega_j}\right),\right.\\&\qquad\left.\log_{\lambda}\left(1+\prod_{j=1}^{n}\left(\lambda^{k_j}-1\right)^{\omega_j}\right)\right\rangle(\lambda>1).\end{aligned}\tag{4}$$

5) SNHWA operator [32]:

$$\begin{aligned}\mathrm{SNHWA}_{\omega}\left(S_1,S_2,\cdots,S_n\right)=&\left\langle \frac{\prod_{j=1}^{n}\left(1+(\gamma-1)t_j\right)^{\omega_j}-\prod_{j=1}^{n}\left(1-t_j\right)^{\omega_j}}{\prod_{j=1}^{n}\left(1+(\gamma-1)t_j\right)^{\omega_j}+(\gamma-1)\prod_{j=1}^{n}\left(1-t_j\right)^{\omega_j}},\right.\\&\frac{\gamma\prod_{j=1}^{n}f_j^{\omega_j}}{\prod_{j=1}^{n}\left(1+(\gamma-1)\left(1-f_j\right)\right)^{\omega_j}+(\gamma-1)\prod_{j=1}^{n}f_j^{\omega_j}},\\&\left.\frac{\gamma\prod_{j=1}^{n}k_j^{\omega_j}}{\prod_{j=1}^{n}\left(1+(\gamma-1)\left(1-k_j\right)\right)^{\omega_j}+(\gamma-1)\prod_{j=1}^{n}k_j^{\omega_j}}\right\rangle.\end{aligned}\tag{5}$$

In Equation (5), $\gamma>0$. If $\gamma=1$, then the SNHWA operator would be reduced to the SNWAA operator in Equation (2); if $\gamma=2$, then the SNHWA operator would be reduced to the SNEWA operator in Equation (3).

2.2. Grey Relational Analysis

GRA is a method to quantify the grey and fuzzy information association relationship between sequences, which is suitable to solve the problem with uncertainty information or insufficient information. The comparison sequence and the reference sequence are determined firstly, and then the correlation degree between the sequences can be obtained based on the fitting degree of the geometry between different sequence curves. The more similar the geometry, the greater the degree of correlation between them. The mathematical expression for the grey correlation coefficients is presented as follows [35]:

$$\tau_i(k)=\frac{\min_i\min_k\left|\varepsilon_0(k)-\varepsilon_i(k)\right|+\varsigma\max_i\max_k\left|\varepsilon_0(k)-\varepsilon_i(k)\right|}{\left|\varepsilon_0(k)-\varepsilon_i(k)\right|+\varsigma\max_i\max_k\left|\varepsilon_0(k)-\varepsilon_i(k)\right|}. \quad (6)$$

where $\varsigma\in(0,1)$ denotes the distinguishable coefficient, and $\left|\varepsilon_0(k)-\varepsilon_i(k)\right|$ denotes the absolute distance between the comparison values and the reference values. For the study of the specific value of distinguishable coefficient, most scholars have verified the fluctuation hours of the sequence data through examples, and ς should take a larger value in the range (0, 1); otherwise, when the sequence data fluctuates greatly, it should take a smaller value. $\varsigma=0.5$ is determined in this paper.

3. Limitations of Existing Single-Valued Neutrosophic Weighted Average Operators

In this section, the limitations of existing single-valued neutrosophic weighted average operators (*i.e.*, the SNWA [21], SNWAA [22], SNEWA [22], SNFWA [24], and SNHWA [32] operators) are discussed.

1) It is assumed that $S_1=\langle 1,0,0\rangle$, $S_2=\langle 0,1,0\rangle$, and $S_3=\langle 0,0,1\rangle$ are three SNNs, and that their weight vector is $\omega=(0,0,1)^{\mathrm{T}}$. Based on the SNWA operator in Equation (1), $\mathrm{SNWA}_\omega(S_1,S_2,S_3)=\langle 1-0^0\times 1^0\times 1^1,1-1^0\times 0^0\times 1^1,1-1^0\times 1^0\times 0^1\rangle$ can be obtained. Since indeterminate value (0^0) can be found in the aggregated results, the SNWA operator [21] is unreasonable in some cases.

2) It is assumed that $S_1=\langle 1,0,0.2\rangle$, $S_2=\langle 0.3,0.1,0\rangle$, and $S_3=\langle 0.2,0.3,0.5\rangle$ are three SNNs, and that their weight vector is $\omega=(0,0,1)^{\mathrm{T}}$. Based on the SNWAA operator in Equation (2),
$\mathrm{SNWA}_\omega(S_1,S_2,S_3)=\langle 1-0^0\times 0.3^0\times 0.2^1,0^0\times 0.1^0\times 0.3^1,0.2^0\times 0^0\times 0.5^1\rangle$ can be obtained. The three terms in the aggregated results all include 0^0, which is an indeterminate value: the SNWAA operator [22] has limitations in some cases.

3) It is assumed that $S_1=\langle 0.5,0.3,0.4\rangle$, $S_2=\langle 0.3,0,0.5\rangle$, and $S_3=\langle 0.4,0.4,0\rangle$ are three SNNs, and that their weight vector is $\omega=(1,0,0)^{\mathrm{T}}$. Based on the SNEWA operator in Equation (3), two terms $0.3^1\times 0^0\times 0.4^0$ and $0.4^1\times 0.5^0\times 0^0$ can be found in the aggregated results. Since they are indeterminate values, the SNEWA operator [22] is unreasonable in some cases.

4) It is assumed that $S_1=\langle 1,0,0.5\rangle$ and $S_2=\langle 0.5,0.3,0.4\rangle$ are two SNNs, and that their weight vector is $\omega=(0,1)^{\mathrm{T}}$. Based on the SNFWA operator in Equation (4), two indeterminate terms, *i.e.*, $0^0\times\left(\lambda^{0.5}-1\right)$ and $0^0\times 0.3^1$, can be found in the aggregated results: the SNFWA operator [24] is unreasonable in

some cases.

5) It is assumed that $S_1 = \langle 1, 0.2, 0.3 \rangle$, $S_2 = \langle 0.5, 0, 0.5 \rangle$, and $S_3 = \langle 0.5, 0.6, 0 \rangle$, and that weight vector is $\omega = (1,0,0)^{\mathrm{T}}$. Based on the SNHWA operator in Equation (5), two indeterminate terms ($0.2^1 \times 0^0 \times 0.6^0$ and $0.3^1 \times 0.5^0 \times 0^0$) can be obtained in the aggregated results: the SNHWA operator [32] is unreasonable in some cases.

4. The Improved Single-Valued Neutrosophic Weighted Average Operator

In this section, we defined an ISNHWA operator that can overcome the limitations of the SNWA, SNWAA, SNEWA, SNFWA, and SNHWA operators.

Definition 4. Assuming that $S_j = (t_j, f_j, k_j)(j = 1, 2, \cdots, n)$ is a group of SNNs and ω_j the weight of S_j, $\omega_j \in [0,1]$ and $\sum_{j=1}^{n} \omega_j = 1$, then the ISNHWA operator can be defined as a function $S^n \to S$:

$$\mathrm{ISNHWA}_\omega(S_1, S_2, \cdots, S_n) = \left\langle \frac{\prod_{j=1}^{n}\left(1+(\gamma-1)t_j\right)^{\omega_j} - \left(1-\frac{1}{\rho}\left(1-\prod_{j=1}^{n}\left(1-\rho t_j\right)^{\omega_j}\right)\right)}{\prod_{j=1}^{n}\left(1+(\gamma-1)t_j\right)^{\omega_j} + (\gamma-1)\left(1-\frac{1}{\rho}\left(1-\prod_{j=1}^{n}\left(1-\rho t_j\right)^{\omega_j}\right)\right)}, \right.$$
$$\frac{\gamma\left(1-\frac{1}{\rho}\left(1-\prod_{j=1}^{n}\left(1-\rho\left(1-f_j\right)\right)^{\omega_j}\right)\right)}{\prod_{j=1}^{n}\left(1+(\gamma-1)f_j\right)^{\omega_j} + (\gamma-1)\left(1-\frac{1}{\rho}\left(1-\prod_{j=1}^{n}\left(1-\rho\left(1-f_j\right)\right)^{\omega_j}\right)\right)}, \tag{7}$$
$$\left. \frac{\gamma\left(1-\frac{1}{\rho}\left(1-\prod_{j=1}^{n}\left(1-\rho\left(1-k_j\right)\right)^{\omega_j}\right)\right)}{\prod_{j=1}^{n}\left(1+(\gamma-1)\left(1-k_j\right)\right)^{\omega_j} + (\gamma-1)\left(1-\frac{1}{\rho}\left(1-\prod_{j=1}^{n}\left(1-\rho\left(1-k_j\right)\right)^{\omega_j}\right)\right)} \right\rangle.$$

In the above equation, $\gamma > 0$ and $0 < \rho < 1$.

1) If $\gamma = 1$, then the ISNHWA operator can be reduced to the improved single-valued neutrosophic weighted average (ISNWA) operator:

$$\mathrm{ISNWA}_\omega(S_1, S_2, \cdots, S_n) = \left\langle \frac{1}{\rho}\left(1-\prod_{j=1}^{n}\left(1-\rho t_j\right)^{\omega_j}\right), 1-\frac{1}{\rho}\left(1-\prod_{j=1}^{n}\left(1-\rho\left(1-f_j\right)\right)^{\omega_j}\right), \right. \tag{8}$$
$$\left. 1-\frac{1}{\rho}\left(1-\prod_{j=1}^{n}\left(1-\rho\left(1-k_j\right)\right)^{\omega_j}\right) \right\rangle.$$

2) If $\gamma = 2$, then the ISNHWA operator can be reduced to the improved single-valued neutrosophic Einstein weighted average (ISNEWA) operator:

$$\text{ISNEWA}_{\omega}\left(S_1,S_2,\cdots,S_n\right) = \left\langle \frac{\prod_{j=1}^{n}\left(1+t_j\right)^{\omega_j}-1+\frac{1}{\rho}\left(1-\prod_{j=1}^{n}\left(1-\rho t_j\right)^{\omega_j}\right)}{\prod_{j=1}^{n}\left(1+t_j\right)^{\omega_j}+1-\frac{1}{\rho}\left(1-\prod_{j=1}^{n}\left(1-\rho t_j\right)^{\omega_j}\right)}, \frac{2\left(1-\frac{1}{\rho}\left(1-\prod_{j=1}^{n}\left(1-\rho\left(1-f_j\right)\right)^{\omega_j}\right)\right)}{\prod_{j=1}^{n}\left(2-f_j\right)^{\omega_j}+1-\frac{1}{\rho}\left(1-\prod_{j=1}^{n}\left(1-\rho\left(1-f_j\right)\right)^{\omega_j}\right)}, \frac{2\left(1-\frac{1}{\rho}\left(1-\prod_{j=1}^{n}\left(1-\rho\left(1-k_j\right)\right)^{\omega_j}\right)\right)}{\prod_{j=1}^{n}\left(2-k_j\right)^{\omega_j}+1-\frac{1}{\rho}\left(1-\prod_{j=1}^{n}\left(1-\rho\left(1-k_j\right)\right)^{\omega_j}\right)} \right\rangle. \quad (9)$$

If $\rho=1$, then the ISNHWA operator in Equation (9) can be reduced to the SNHWA operator in Equation (5). For convenience, in this study we considered $\rho=0.98$ and $\gamma=3$. Apparently, the larger the value of ρ, the higher will be the similarity degree between the aggregated results using the ISNHWA operator and those using the SNHWA operator.

Here, we will utilize the proposed ISNHWA operator to handle the problems presented in Section 3, overcoming the limitations of the SNWA [21], SNWAA [22], SNEWA [22], SNFWA [24], and SNHWA [32] operators. The corresponding results are summarized below.

1) We assumed that $S_1=\langle 1,0,0\rangle$, $S_2=\langle 0,1,0\rangle$, and $S_3=\langle 0,0,1\rangle$ were three SNNs, and that their weight vector was $\omega=(0,0,1)^{\mathrm{T}}$. In this case, using the ISNHWA operator, we obtained $\text{ISNHWA}_{\omega}(S_1,S_2,S_3)=\langle 0,0,1\rangle$, which is in accord with the actual decision-making process: the ISNHWA operator can overcome the limitations of the SNWA operator [21].

2) We assumed that $S_1=\langle 1,0,0.2\rangle$, $S_2=\langle 0.3,0.1,0\rangle$, and $S_3=\langle 0.2,0.3,0.5\rangle$ were three SNNs, and that their weight vector was $\omega=(0,0,1)^{\mathrm{T}}$. In this case, using the ISNHWA operator, we obtained $\text{ISNHWA}_{\omega}(S_1,S_2,S_3)=\langle 0.2,0.3,0.5\rangle$, which is in accord with the actual decision-making process: the ISNHWA operator overcomes the shortcomings of the SNWAA operator [22].

3) We assumed that $S_1=\langle 0.5,0.3,0.4\rangle$, $S_2=\langle 0.3,0,0.5\rangle$, and $S_3=\langle 0.4,0.4,0\rangle$ were three SNNs, and that their weight vector was $\omega=(1,0,0)^{\mathrm{T}}$. In this case, using the ISNHWA operator, we obtained $\text{ISNHWA}_{\omega}(S_1,S_2,S_3)=\langle 0.5,0.3,0.4\rangle$, which is also in accord with the actual decision-making process: the ISNHWA operator overcomes the limitations of the SNEWA operator [22].

4) We assumed that $S_1=\langle 1,0,0.5\rangle$ and $S_2=\langle 0.5,0.3,0.4\rangle$ were two SNNs, and that their weight vector was $\omega=(0,1)^{\mathrm{T}}$. In this case, using the ISNHWA operator, we obtained $\text{ISNHWA}_{\omega}(S_1,S_2,S_3)=\langle 0.5,0.3,0.4\rangle$, which is in accord with the actual decision-making process: the ISNHWA operator overcomes the

limitations of the SNFWA operator [24].

5) We assumed that $S_1=\langle 1,0.2,0.3\rangle$, $S_2=\langle 0.5,0,0.5\rangle$, and $S_3=\langle 0.5,0.6,0\rangle$ are three SNNs, and that their weight vector was $\omega=(1,0,0)^{\mathrm{T}}$. In this case, using the ISNHWA operator, we obtained $\mathrm{ISNHWA}_\omega(S_1,S_2,S_3)=\langle 1,0.2,0.3\rangle$, which is in accord with the actual decision-making process: the ISNHWA operator overcomes the shortcomings of the SNHWA operator [32].

To sum up, the ISNHWA operator overcomes the shortcomings of the SNWA [21], SNWAA [22], SNEWA [22], SNFWA [24], and SNHWA [32] operators.

Based on Definition 4, some properties of the ISNHWA operator will be discussed below.

Theorem 1 (Idempotency). Let $S_j\ (j=1,2,\cdots,n)$ be a group of SNNs and ω_j be the weight of S_j ($\omega_j\in[0,1]$ and $\sum_{j=1}^n\omega_j=1$). If $S_1=S_2=\cdots=S_n=S=\langle t,f,k\rangle$, then $\mathrm{ISNHWA}_\omega(S_1,S_2,\cdots,S_n)=S$.

Proof. Since $S_1=S_2=\cdots=S_n=S=\langle t,f,k\rangle$:

$$\mathrm{ISNHWA}_\omega(S_1,S_2,\cdots,S_n)$$

$$=\left\langle \frac{(1+(\gamma-1)t)^{\sum_{i=1}^n\omega_i}-\left(1-\frac{1}{\rho}\left(1-(1-\rho t)^{\sum_{i=1}^n\omega_i}\right)\right)}{(1+(\gamma-1)t)^{\sum_{i=1}^n\omega_i}+(\gamma-1)\left(1-\frac{1}{\rho}\left(1-(1-\rho t)^{\sum_{i=1}^n\omega_i}\right)\right)},\right.$$

$$\frac{\gamma\left(1-\frac{1}{\rho}\left(1-(1-\rho(1-f))^{\sum_{i=1}^n\omega_i}\right)\right)}{(1+(\gamma-1)(1-f))^{\sum_{i=1}^n\omega_i}+(\gamma-1)\left(1-\frac{1}{\rho}\left(1-(1-\rho(1-f))^{\sum_{i=1}^n\omega_i}\right)\right)},$$

$$\left.\frac{\gamma\left(1-\frac{1}{\rho}\left(1-(1-\rho(1-k))^{\sum_{i=1}^n\omega_i}\right)\right)}{(1+(\gamma-1)(1-k))^{\sum_{i=1}^n\omega_i}+(\gamma-1)\left(1-\frac{1}{\rho}\left(1-(1-\rho(1-k))^{\sum_{i=1}^n\omega_i}\right)\right)}\right\rangle$$

$$=\left\langle \frac{1+(\gamma-1)t-(1-t)}{1+(\gamma-1)t+(\gamma-1)(1-t)},\frac{\gamma(1-(1-f))}{1+(\gamma-1)(1-f)+(\gamma-1)f},\frac{\gamma(1-(1-k))}{1+(\gamma-1)(1-k)+(\gamma-1)k}\right\rangle$$

$$=\langle t,f,k\rangle.$$

Theorem 2 (Boundedness). Let $S_j\ (j=1,2,\cdots,n)$ be a group of SNNs. If $S^-=\langle \min t_j,\max f_j,\max k_j\rangle (j=1,2,\cdots,n)$ and $S^+=\langle \max t_j,\min f_j,\min k_j\rangle (j=1,2,\cdots,n)$, then $S^-\le \mathrm{ISNHWA}_\omega(S_1,S_2,\cdots,S_n)\le S^+$.

Proof. Assuming that $x=\prod_{j=1}^n(1+(\gamma-1)t_j)^{\omega_j}$, $y=1-\frac{1}{\rho}\left(1-\prod_{j=1}^n(1-\rho t_j)^{\omega_j}\right)$, and $\mathrm{ISNHWA}_\omega(S_1,S_2,\cdots,S_n)=\langle p(t_j),p(f_j),p(k_j)\rangle$, then

$$p\left(t_j\right)=\frac{x-y}{x+(\gamma-1)y}=1-\frac{\gamma}{x/y+(\gamma-1)}.$$

Since $\frac{x}{y}=\frac{\prod_{j=1}^{n}\left(1+(\gamma-1)t_j\right)^{\omega_j}}{1-\frac{1}{\rho}\left(1-\prod_{j=1}^{n}\left(1-\rho t_j\right)^{\omega_j}\right)}$ and $\frac{x}{y}$ is a monotonous increasing function of t_j, also $p\left(t_j\right)$ is a monotonous increasing function of t_j. Thus,

$$p\left(t_j\right)\geq\frac{\prod_{j=1}^{n}\left(1+(\gamma-1)\min t_j\right)^{\omega_j}-\left(1-\frac{1}{\rho}\left(1-\prod_{j=1}^{n}\left(1-\rho\min t_j\right)^{\omega_j}\right)\right)}{\prod_{j=1}^{n}\left(1+(\gamma-1)\min t_j\right)^{\omega_j}-(\gamma-1)\left(1-\frac{1}{\rho}\left(1-\prod_{j=1}^{n}\left(1-\rho\min t_j\right)^{\omega_j}\right)\right)}$$
$$=\min t_j$$

and
$$p\left(t_j\right)\leq\frac{\prod_{j=1}^{n}\left(1+(\gamma-1)\max t_j\right)^{\omega_j}-\left(1-\frac{1}{\rho}\left(1-\prod_{j=1}^{n}\left(1-\rho\max t_j\right)^{\omega_j}\right)\right)}{\prod_{j=1}^{n}\left(1+(\gamma-1)\max t_j\right)^{\omega_j}-(\gamma-1)\left(1-\frac{1}{\rho}\left(1-\prod_{j=1}^{n}\left(1-\rho\max t_j\right)^{\omega_j}\right)\right)}$$
$$=\max t_j.$$

Similarly, we can prove that $\min f_j\leq p\left(f_j\right)\leq\max f_j$ and $\min k_j\leq p\left(k_j\right)\leq\max k_j$. Hence, $\min t_j+1-\max f_j+1-\max k_j\leq \text{ISNHWA}_{\omega}\left(S_1,S_2,\cdots,S_n\right)\leq\max t_j+1-\min f_j+1-\min k_j$, *i.e.*, $p\left(S^-\right)\leq p\left(\text{ISNHWA}_{\omega}\left(S_1,S_2,\cdots,S_n\right)\right)\leq p\left(S^+\right)$. So $S^-\leq\text{ISNHWA}_{\omega}\left(S_1,S_2,\cdots,S_n\right)\leq S^+$ is true.

Theorem 3 (Monotonicity). Let $S_j\left(j=1,2,\cdots,n\right)$ and $\bar{S}_j\left(j=1,2,\cdots,n\right)$ be two groups of SNNs. If $S_j\leq\bar{S}_j$, $t_j\leq\bar{t}_j$, $f_j\geq\bar{f}_j$, and $k_j\geq\bar{k}_j\left(j=1,2,\cdots,n\right)$, then $\text{ISNHWA}_{\omega}\left(S_1,S_2,\cdots,S_n\right)\leq\text{ISNHWA}_{\omega}\left(\bar{S}_1,\bar{S}_2,\cdots,\bar{S}_n\right)$.

Proof. Since $t_j\leq\bar{t}_j$, $f_j\geq\bar{f}_j$ and $k_j\geq\bar{k}_j\left(j=1,2,\cdots,n\right)$, based on Theorem 2, we can obtain $p\left(\text{ISNHWA}_{\omega}\left(S_1,S_2,\cdots,S_n\right)\right)\leq p\left(\text{ISNHWA}_{\omega}\left(\bar{S}_1,\bar{S}_2,\cdots,\bar{S}_n\right)\right)$. Therefore, $\text{ISNHWA}_{\omega}\left(S_1,S_2,\cdots,S_n\right)\leq\text{ISNHWA}_{\omega}\left(\bar{S}_1,\bar{S}_2,\cdots,\bar{S}_n\right)$.

Theorem 4 (Commutativity). Let $S_j\left(j=1,2,\cdots,n\right)$ be a group of SNNs. If $\tilde{S}_j\left(j=1,2,\cdots,n\right)$ is an arbitrary permutation of S_j, then $\text{ISNHWA}_{\omega}\left(S_1,S_2,\cdots,S_n\right)=\text{ISNHWA}_{\omega}\left(\tilde{S}_1,\tilde{S}_2,\cdots,\tilde{S}_n\right)$.

The process of proof is omitted here.

5. Improved Single-Valued Neutrosophic MCGDM Method Based on the ISNHWA Operator and GRA

Let $S=\left\{S_1,S_2,\cdots,S_n\right\}$ be a group of alternatives, $C=\left\{C_1,C_2,\cdots,C_m\right\}$ a group of criteria, and $D=\left\{D_1,D_2,\cdots,D_z\right\}$ a group of DMs. The weight vectors of the criteria and DMs will be $w=\left(w_1,w_2,\cdots,w_m\right)^{\mathrm{T}}$ and $\lambda=\left(\lambda_1,\lambda_2,\cdots,\lambda_z\right)^{\mathrm{T}}$, respectively, and satisfy the following conditions: $w_j,\lambda_l\in[0,1]$, $\sum_{j=1}^{m}\omega_j=1$, and

$\sum_{l=1}^{z}\lambda_l = 1$. It is assumed that $R_{ij}^l = \langle t_{ij}^l, f_{ij}^l, k_{ij}^l \rangle$ $(i = 1,2,\cdots,n; j = 1,2,\cdots,m; l = 1,2,\cdots,z)$ is the criteria value provided by the DM D_l for the alternative S_i under criterion C_j. The steps involved in the application of the new MCGDM method, which is based on such promises, are described in the following subsections. Notably, if there is only one DM, then Step 2 can be omitted: the single-valued neutrosophic MCDM method is a special case of the MCGDM method. Moreover, if the weight information of criteria is completely known, then Step 3 can be omitted here; otherwise, if the weight information of criteria is completely unknown, the weight of criteria will be determined using the grey correlation analysis method. The steps for selecting the alternative(s) are provided in the following and the flowchart of the proposed method is shown in Figure 1 and Algorithm 1 respectively.

Step 1. Normalization of the decision matrix.

Since the criteria may be of the cost or benefit type, the criteria normalization method can be determined as:

$$\bar{R}_{ij}^l = \begin{cases} R_{ij}^l, & \text{for the benefit criteria } C_j \\ \left(R_{ij}^l\right)^c, & \text{for the cost criteria } C_j \end{cases}, \quad (i = 1,2,\cdots,n; j = 1,2,\cdots,m), \qquad (10)$$

where $\left(R_{ij}^l\right)^c = \left(k_{ij}^l, f_{ij}^l, t_{ij}^l\right)$ denotes the complement of S_{ij}^l.

Step 2: Calculation of the comprehensive evaluation values of all DMs.

From the ISNHWA operator, let $\rho = 0.98$ and $\gamma = 3$. In this case, the aggregated comprehensive evaluation values $\delta_{ij}\left(i = 1,2,\cdots,n; j = 1,2,\cdots,m\right)$ of all

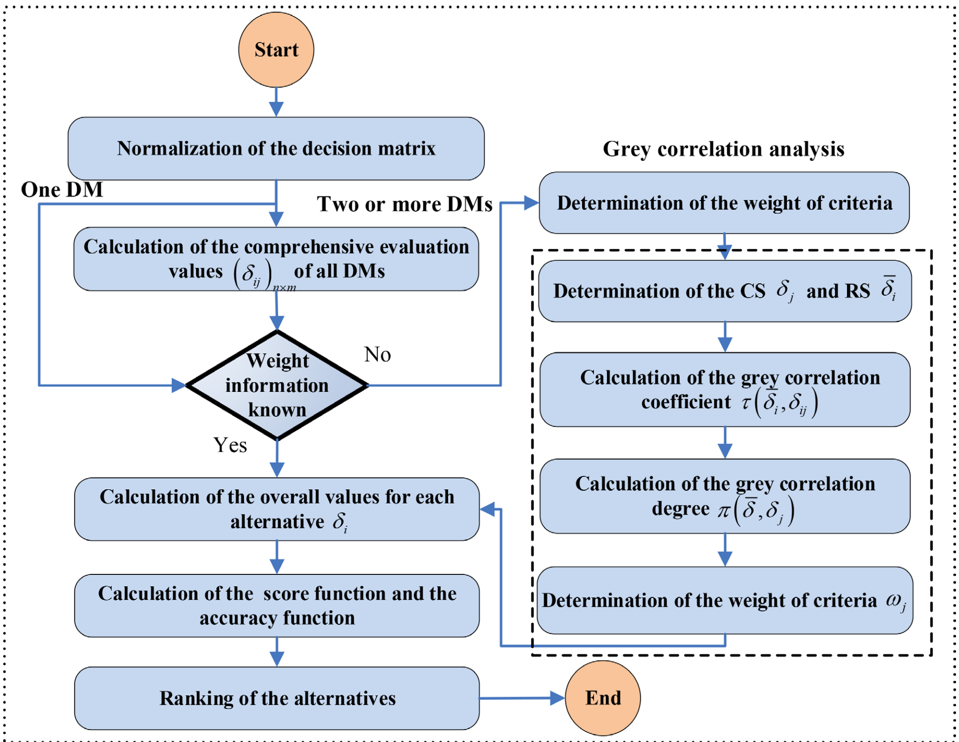

Figure 1. Flowchart of the proposed method.

Algorithm 1. The score function and the accuracy function values of each alternative.

Input: The single-valued neutrosophic decision matrix $\left(R_{ij}^{l}\right)_{n\times m}\left(l=1,2,\cdots,z\right)$, the alternatives is $S=\left\{S_1,S_2,\cdots,S_n\right\}$, and the distinguish coefficient is $\varsigma=0.5$.

Output: The score function and the accuracy function values of each alternative.

function the score function and the accuracy function values of each alternative $\left(\left(R_{ij}^{l}\right)_{n\times m},\varsigma,\rho\right)$

for each $\text{DM}=1\rightarrow z$ **do**

function the general decision matrix $\left(\delta_{ij}\right)_{n\times m}$

calculate the aggregated decision matrix $\left(\delta_{ij}\right)_{n\times m}$ in Equation (11)

end for

return $\left(\delta_{ij}\right)_{n\times m}$

end function

function the weight of criteria w_j

for each criterion $j=1\rightarrow m$ **do**

calculate the arithmetic mean value of the alternative under criteria $\overline{\delta}_i$

end for

calculate the $\overline{\delta}_i$ in Equation (12)

return $\overline{\delta}_i$

for each alternative under criterion $i,j=1\left(i=1,2,\cdots,n;j=1,2,\cdots,m\right)$, determine the grey relation coefficient $\tau\left(\overline{\delta}_i,\delta_{ij}\right)$ **do**

Calculate the $\tau\left(\overline{\delta}_i,\delta_{ij}\right)$ in Equation (13), calculate the grey correlation degree $\pi\left(\overline{\delta}_i,\delta_{ij}\right)$ in Equation (14), calculate the weight of criteria w_j in Equation (15)

end for

return w_j

end function

for each alternative $i\leftarrow zeros\left(1,n\right)$ **do**

calculated the aggregated overall values δ_i of each alternative in Equation (16), calculated the score function and accuracy function values of each alternative $p\left(\delta_i\right)$ and $q\left(\delta_i\right)$ in Equation (17)

end for

return $p\left(\delta_i\right)$ and $q\left(\delta_i\right)$

end function

DMs are the following:

$$\delta_{ij}=\langle t_{ij},f_{ij},k_{ij}\rangle=\mathrm{ISNHWA}_{\lambda}\left(R_{ij}^{1},R_{ij}^{2},\cdots,R_{ij}^{z}\right)$$

$$=\left\langle\frac{\prod_{l=1}^{z}\left(1+2t_{ij}^{l}\right)^{\lambda_l}-\left(1-\frac{1}{0.98}\left(1-\prod_{l=1}^{z}\left(1-0.98t_{ij}^{l}\right)^{\lambda_l}\right)\right)}{\prod_{l=1}^{z}\left(1+2t_{ij}^{l}\right)^{\lambda_l}+2\left(1-\frac{1}{0.98}\left(1-\prod_{l=1}^{z}\left(1-0.98t_{ij}^{l}\right)^{\lambda_l}\right)\right)},\right.$$

$$\frac{3\left(1-\frac{1}{0.98}\left(1-\prod_{l=1}^{z}\left(1-0.98\left(1-f_{ij}^{l}\right)\right)^{\lambda_l}\right)\right)}{\prod_{l=1}^{z}\left(1+(\gamma-1)f_{ij}^{l}\right)^{\lambda_l}+2\left(1-\frac{1}{0.98}\left(1-\prod_{l=1}^{z}\left(1-0.98\left(1-f_{ij}^{l}\right)\right)^{\lambda_l}\right)\right)}, \tag{11}$$

$$\left.\frac{3\left(1-\frac{1}{0.98}\left(1-\prod_{l=1}^{z}\left(1-0.98\left(1-k_{ij}^{l}\right)\right)^{\lambda_l}\right)\right)}{\prod_{l=1}^{z}\left(1+2\left(1-k_{ij}^{l}\right)\right)^{\lambda_l}+2\left(1-\frac{1}{0.98}\left(1-\prod_{l=1}^{z}\left(1-0.98\left(1-k_{ij}^{l}\right)\right)^{\lambda_l}\right)\right)}\right\rangle.$$

Step 3: Determination of the weight of criteria

Grey correlation analysis describes the proximity between comparison sequence (CS) and reference sequence (RS), which can be measured by the grey correlation coefficient [36] [37]. First, we will use the corresponding column of the comprehensive decision-making matrix provided by all DMs as the CS, *i.e.*, $\delta_j=\left(\delta_{1j},\delta_{2j},\cdots,\delta_{nj}\right)(j=1,2,\cdots,m)$. Second, the RS is the key factor to determine the weight of criteria. According to the maximum entropy principle, the weight of all criteria is equal without any prior information. Then the RS can be considered as the arithmetic mean value of the alternative under all criteria and denoted as: $\bar{\delta}=\left(\bar{\delta}_1,\bar{\delta}_2,\cdots,\bar{\delta}_n\right)$, where

$$\bar{\delta}_i=\mathrm{ISNHWA}_{1/n}\left(\delta_{i1},\delta_{i2},\cdots,\delta_{im}\right)$$

$$=\left\langle\frac{\prod_{j=1}^{m}\left(1+2t_{ij}\right)^{1/n}-\left(1-\frac{1}{0.98}\left(1-\prod_{j=1}^{m}\left(1-0.98t_{ij}\right)^{1/n}\right)\right)}{\prod_{j=1}^{m}\left(1+2t_{ij}\right)^{1/n}+2\left(1-\frac{1}{0.98}\left(1-\prod_{j=1}^{m}\left(1-0.98t_{ij}\right)^{1/n}\right)\right)},\right.$$

$$\frac{3\left(1-\frac{1}{0.98}\left(1-\prod_{j=1}^{m}\left(1-0.98\left(1-f_{ij}\right)\right)^{1/n}\right)\right)}{\prod_{j=1}^{m}\left(1+(\gamma-1)f_{ij}\right)^{1/n}+2\left(1-\frac{1}{0.98}\left(1-\prod_{j=1}^{m}\left(1-0.98\left(1-f_{ij}\right)\right)^{1/n}\right)\right)}, \tag{12}$$

$$\left.\frac{3\left(1-\frac{1}{0.98}\left(1-\prod_{j=1}^{m}\left(1-0.98\left(1-k_{ij}\right)\right)^{1/n}\right)\right)}{\prod_{j=1}^{m}\left(1+2\left(1-k_{ij}\right)\right)^{1/n}+2\left(1-\frac{1}{0.98}\left(1-\prod_{j=1}^{m}\left(1-0.98\left(1-k_{ij}\right)\right)^{1/n}\right)\right)}\right\rangle.$$

Then the corresponding grey correlation coefficient between $\bar{\delta}_i$ and δ_{ij} can be obtained as:

$$\tau\left(\overline{\delta}_i,\delta_{ij}\right)=\frac{\min_j\min_i d\left(\overline{\delta}_i,\delta_{ij}\right)+\varsigma\max_j\max_i d\left(\overline{\delta}_i,\delta_{ij}\right)}{d\left(\overline{\delta}_i,\delta_{ij}\right)+\varsigma\max_j\max_i d\left(\overline{\delta}_i,\delta_{ij}\right)}. \quad (13)$$

where $d\left(\overline{\delta}_i,\delta_{ij}\right)=\frac{1}{3}\left(\left|t_{\overline{\delta}_i}-t_{\delta_{ij}}\right|+\left|f_{\overline{\delta}_i}-f_{\delta_{ij}}\right|+\left|k_{\overline{\delta}_i}-k_{\delta_{ij}}\right|\right)$. $\varsigma\left(0<\varsigma<1\right)$ represents the distinguished degree. The smaller the parameter value is, the higher the distinguished degree between the correlation coefficients is. In general, if the sequence data fluctuation is small, then the parameter should take a larger value in the interval of (0, 1); otherwise, when the sequence data fluctuation is large, the smaller value should be determined. Based on the empirical value, $\rho=0.5$ will be used in this paper.

Then the grey correlation degree between the RS and the CS can be determined:

$$\pi\left(\overline{\delta},\delta_j\right)=\frac{1}{n}\sum_{i=1}^{n}\tau\left(\overline{\delta}_i,\delta_{ij}\right). \quad (14)$$

The RS is the aggregation information of each alternative that the weight of all criteria is equal, which can be regarded as a virtual criterion. Similar to the consensus problem in group decision-making, the higher the correlation between the DM and the consensus opinion of the group, the higher the weight should be given to the corresponding DM. Therefore, the weight of criteria can be determined as:

$$w_j=\frac{\pi\left(\overline{\delta},\delta_j\right)}{\sum_{j=1}^{m}\pi\left(\overline{\delta},\delta_j\right)}. \quad (15)$$

Step 4: **Calculation of the overall values for each alternative.**

From Step 2, the aggregated overall values $\delta_i\left(i=1,2,\cdots,n\right)$ of each alternative can be determined using the ISNHWA operator as follows:

$$\delta_i=\left\langle t_i,f_i,k_i\right\rangle=\text{ISNHWA}_w\left(R_{i1},R_{i2},\cdots,R_{im}\right). \quad (16)$$

Step 5: **Calculation of the score function and of the accuracy function.**

The score function $p\left(\delta_i\right)$ and accuracy function $q\left(\delta_i\right)\left(i=1,2,\cdots,n\right)$ can be determined from Definition 2.

Step 6. Ranking of the alternatives.

The larger the value of $p\left(\delta_i\right)$, the better is the alternative S_i. Based on Step 4, the final ranking can be obtained.

6. Application Examples

With the rapid development of urbanization and industrialization in China, the environmental problems are becoming more and more serious. Under the background of rapid global climate change, the surface wind speed in most regions has decreased, and the air quality has touched the red line many times, which has seriously damaged the health of the people. Especially in autumn and winter, the degree of air pollution and the number of air pollution days increase more, so it is very important to predict and estimate the air quality. The 19th Asian Games 2022 will be held in Hangzhou during September 10-25. In order to forecast the air quality during the 19th Asian Games, the local environmental department intends to present a comprehensive evaluation of the air quality in Hangzhou during September 2018, 2019, 2020, and 2021. To this aim, three air-quality monitoring stations (*i.e.*, three DMs) can be considered: $D=\{D_1,D_2,D_3\}$, and the corresponding weight of three stations is $\lambda=(0.31,0.4,0.29)^{\mathrm{T}}$. The monthly air quality during September 2018, 2019, 2020, and 2021 can be denoted as alternatives: $S=\{S_1,S_2,S_3,S_4\}=$ {September 2017, September 2018, September 2019, September 2020}. From the Implementing Details for Urban Environmental Comprehensive Treatment and Quantitative Examination (General Office of State Environmental Protection Administration (GOSEPA, 2006), the important criteria for evaluation can be denoted instead as $C=\{C_1,C_2,C_3,C_4,C_5\}=\{\mathrm{NO_2},\mathrm{PM_{2.5}},\mathrm{PM_{10}},\mathrm{SO_2},\mathrm{CO},\mathrm{O_3}\}$.

For the data in experiment, all members of the monitoring station department should be provided the evaluation information in linguistic terms based on the actual data for attributes of air quality and DMs' knowledges respectively. The transformation rules between the linguistic terms and SNNs are presented in Table 1. For example, one DM in D_1 provides the language evaluation information "higher" for the CO_2 emissions in 2018, then the corresponding SNN is $\langle 0.8,0.1,0.2\rangle$. Second, according to the evaluation information for all DMs in D_1, the average values for three parameters can be obtained as $\langle 0.5,0.3,0.2\rangle$. Finally, since the criterion for CO_2 emissions is cost-type, then the normalized value of evaluation information is $\langle 0.2,0.3,0.5\rangle$ based on Equation (10).

In order to rank the four alternatives, the measured values can be converted to SNNs based on the monitoring stations: $R^l=\left(R_{ij}^l\right)_{4\times 3}(l=1,2,3)$.

$$R^1=\begin{pmatrix}\langle 0.2,0.3,0.5\rangle & \langle 0.5,0.3,0.5\rangle & \langle 0.4,0.4,0.3\rangle & \langle 0.2,0.1,0.3\rangle & \langle 0.5,0.1,0.2\rangle & \langle 0.3,0.1,0.3\rangle \\ \langle 0.2,0.4,0.4\rangle & \langle 0.2,0.4,0.4\rangle & \langle 0.4,0.3,0.3\rangle & \langle 0.3,0.1,0.2\rangle & \langle 0.4,0.2,0.1\rangle & \langle 0.5,0.2,0.2\rangle \\ \langle 0.5,0.4,0.3\rangle & \langle 0.4,0.3,0.3\rangle & \langle 0.5,0.3,0.2\rangle & \langle 0.4,0.2,0.3\rangle & \langle 0.4,0.2,0.3\rangle & \langle 0.4,0.2,0.3\rangle \\ \langle 0.3,0.5,0.2\rangle & \langle 0.4,0.3,0.3\rangle & \langle 0.5,0.2,0.3\rangle & \langle 0.1,0.4,0.3\rangle & \langle 0.3,0.3,0.3\rangle & \langle 0.3,0,0.3\rangle \end{pmatrix};$$

Table 1. The transformation rules used for the evaluation of air quality.

Linguistic term	SNN
Very high	(0.9, 0, 0.1)
High	(0.8, 0.1, 0.2)
Medium high	(0.7, 0, 0.1)
Medium	(0.5, 0.1, 0.1)
Medium low	(0.4, 0, 0.1)
Low	(0.3, 0.1, 0.2)
Very low	(0.2, 0, 0.1)

$$R^2 = \begin{pmatrix} \langle 0.3,0.5,0.1\rangle & \langle 0.1,0.5,0.4\rangle & \langle 0.2,0.4,0.4\rangle & \langle 0.2,0.4,0.2\rangle & \langle 0.5,0.1,0.1\rangle & \langle 0.3,0.1,0.2\rangle \\ \langle 0.2,0.6,0.2\rangle & \langle 0.4,0.3,0.3\rangle & \langle 0.3,0.4,0.3\rangle & \langle 0.4,0.1,0.1\rangle & \langle 0.6,0,0.2\rangle & \langle 0.4,0.1,0\rangle \\ \langle 0.5,0.5,0.2\rangle & \langle 0.5,0.4,0.3\rangle & \langle 0.5,0.6,0.1\rangle & \langle 0.3,0.2,0.2\rangle & \langle 0.4,0,0.1\rangle & \langle 0.4,0.2,0.2\rangle \\ \langle 0.4,0.5,0.1\rangle & \langle 0.4,0.4,0.3\rangle & \langle 0.4,0.3,0.3\rangle & \langle 0.2,0.3,0.2\rangle & \langle 0.5,0.1,0.2\rangle & \langle 0.5,0.1,0.2\rangle \end{pmatrix};$$

$$R^3 = \begin{pmatrix} \langle 0.3,0.5,0.2\rangle & \langle 0.5,0.4,0.3\rangle & \langle 0.4,0.2,0.3\rangle & \langle 0.2,0.1,0.3\rangle & \langle 0.4,0.1,0\rangle & \langle 0.5,0.5,0\rangle \\ \langle 0.1,0.6,0.3\rangle & \langle 0.5,0.4,0.4\rangle & \langle 0.6,0.2,0.2\rangle & \langle 0.3,0.2,0.4\rangle & \langle 0.5,0.1,0.1\rangle & \langle 0.4,0.2,0.1\rangle \\ \langle 0.5,0.3,0.1\rangle & \langle 0.5,0.2,0.3\rangle & \langle 0.5,0.6,0.2\rangle & \langle 0.4,0.1,0.3\rangle & \langle 0.5,0.2,0.2\rangle & \langle 0.4,0,0.1\rangle \\ \langle 0.6,0.5,0.1\rangle & \langle 0.5,0.3,0.3\rangle & \langle 0.4,0.5,0.2\rangle & \langle 0.5,0,0.2\rangle & \langle 0.6,0.1,0.1\rangle & \langle 0.4,0.1,0\rangle \end{pmatrix}.$$

Step 1. Normalization of the decision matrix.

Since these criteria are of the cost type, the normalized decision matrix can be obtained from Equation (9): $\overline{R}_l = R_l$.

Step 2. Calculation of the comprehensive evaluation values of all DMs.

Based on the MCGDM method using the ISNHWA operator, and considering $\rho = 0.98$ and $\gamma = 3$ (see Equation (10)), the comprehensive values of all DMs can be determined as:

$$R = \begin{pmatrix} \langle 0.269,0.4712,0.4452\rangle & \langle 0.3442,0.4601,0.4032\rangle & \langle 0.3209,0.4226,0.3291\rangle \\ \langle 0.1706,0.5081,0.5346\rangle & \langle 0.3695,0.4403,0.3577\rangle & \langle 0.4236,0.4067,0.301\rangle \\ \langle 0.5,0.4612,0.4074\rangle & \langle 0.4698,0.4067,0.3\rangle & \langle 0.5,0.4911,0.4878\rangle \\ \langle 0.4323,0.5,0.5023\rangle & \langle 0.4297,0.4298,0.3368\rangle & \langle 0.4318,0.41,0.309\rangle \end{pmatrix.$$

$$\begin{matrix} \langle 0.2,0.3023,0.1788\rangle & \langle 0.4718,0.2143,0.1026\rangle & \langle 0.36,0.2856,0.1722\rangle \\ \langle 0.3404,0.246,0.1261\rangle & \langle 0.5125,0.1203,0.0513\rangle & \langle 0.4318,0.2824,0.1571\rangle \\ \langle 0.3604,0.296,0.1652\rangle & \langle 0.4297,0.1434,0.0655\rangle & \langle 0.4,0.1865,0.0912\rangle \\ \langle 0.2579,0.2378,0.1369\rangle & \langle 0.4723,0.268,0.1443\rangle & \langle 0.4109,0.1194,0.0516\rangle \end{matrix}\Bigg).$$

Step 3. Determination of the weight of criteria

Form Equation (11)-Equation (12), the reference sequence is determined as: $\overline{f}_1 = \langle 0.3286,0.4327,0.2479\rangle$, $\overline{f}_2 = \langle 0.3766,0.4052,0.2122\rangle$, $\overline{f}_3 = \langle 0.4442,0.404,0.2114\rangle$ and $\overline{f}_4 = \langle 0.4068,0.4014,0.2078\rangle$. The grey relational coefficient can be obtained:

$$\tau\left(\overline{f}_i, R_{ij}\right) = \begin{pmatrix} 0.6788 & 0.8071 & 1 & 0.6442 & 0.5042 & 0.7279 \\ 0.4380 & 0.8239 & 0.9158 & 0.6944 & 0.4621 & 0.7559 \\ 0.6640 & 0.9589 & 0.5644 & 0.7490 & 0.5630 & 0.5945 \\ 0.5648 & 0.8363 & 0.9209 & 0.5933 & 0.7175 & 0.5472 \end{pmatrix}.$$

Based on Equation (13)-Equation (14), the grey correlation degree between the RS and the CS can be obtained as: $\pi\left(\overline{\delta},\delta_1\right) = 0.5864$, $\pi\left(\overline{\delta},\delta_2\right) = 0.8565$, $\pi\left(\overline{\delta},\delta_3\right) = 0.8503$, $\pi\left(\overline{\delta},\delta_4\right) = 0.6702$, $\pi\left(\overline{\delta},\delta_5\right) = 0.5617$, and $\pi\left(\overline{\delta},\delta_6\right) = 0.6563$. Then the weight of criteria can be determined as: $w_1 = 0.1402$, $w_2 = 0.2048$, $w_3 = 0.2033$, $w_4 = 0.1603$, $w_5 = 0.1343$, and $w_6 = 0.1571$.

Step 4. Calculation of the overall values for each alternative.

Based on the ISNHWA operator and Equation (15), the overall aggregated values for each alternative can be obtained: $\delta_1 = \langle 0.3262,0.4381,0.2593\rangle$,

$\delta_2 = \langle 0.3785, 0.4137, 0.2235 \rangle$, $\delta_3 = \langle 0.4472, 0.4130, 0.2271 \rangle$, and $\delta_4 = \langle 0.4066, 0.4063, 0.2150 \rangle$.

Step 5. Determination of the score and accuracy functions.

From the comparison method in Definition 2, the score value of each alternative can be obtained: $p(\delta_1) = 0.5430$, $p(\delta_2) = 0.5804$, $p(\delta_3) = 0.6024$, and $p(\delta_4) = 0.5951$.

Step 6. Ranking of the alternatives.

Since $p(\delta_3) > p(\delta_4) > p(\delta_2) > p(\delta_1)$, $S_3 \succ S_4 \succ S_2 \succ S_1$. The best and worst alternatives would be S_3 and S_1.

In order to further explore the effectiveness of the proposed method, the sensitivity analysis of different parameters will be carried out below to explore the influence of different parameter value on the final decision-making results.

If $\rho = 0.98$, $\varsigma = 0.5$ and $0 < \gamma < 10$, then the corresponding weight and the final rankings are shown in Figure 2 & Figure 3. Obviously, the change of parameter γ have less impact on w_4 and more effects on other five weights. However, according to the results in Figure 3, as the parameter value γ become larger, the final ranking of four alternatives gradually tends to stabilize, *i.e.*, $S_3 \succ S_4 \succ S_2 \succ S_1$. The best and worst alternatives would be S_3 and S_1. If $\rho = 0.98$, $0 < \varsigma < 1$, and $\gamma = 3$, then the corresponding weight and the final rankings are shown in Figure 4 & Figure 5. From Figure 4, the change of parameter ς have more impacts on w_2 and w_3, and less effects on other four weights. Moreover, the different values of the parameter ς do not affect the ordering of four alternatives, and the final ranking is always $S_3 \succ S_4 \succ S_2 \succ S_1$. If $0 < \rho < 1$, $\varsigma = 0.5$, and $\gamma = 3$, then the corresponding weight and the final rankings are shown in Figure 6 & Figure 7. According to Figure 5, different values of the parameter ρ have less effect on the weights, and the final rankings of four alternatives are not affected by the parameter ρ.

Therefore, from the analysis presented above, although the parameter changes

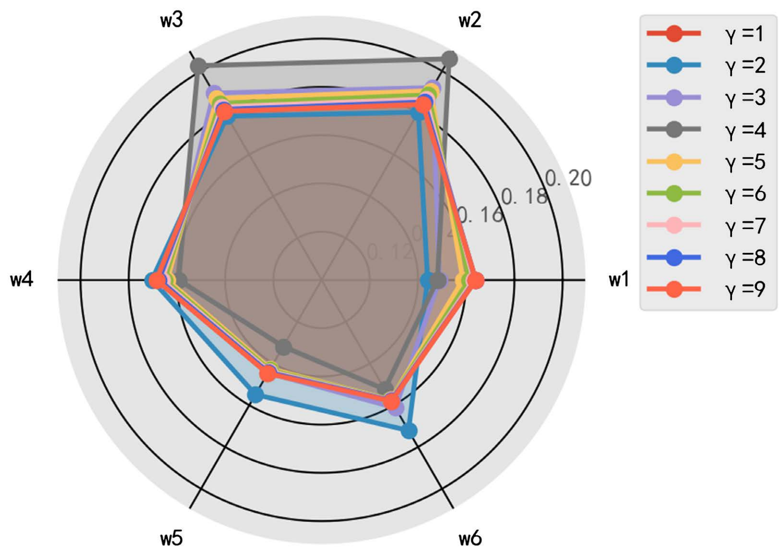

Figure 2. Weight with $\rho = 0.98$, $\varsigma = 0.5$ and $0 < \gamma < 10$.

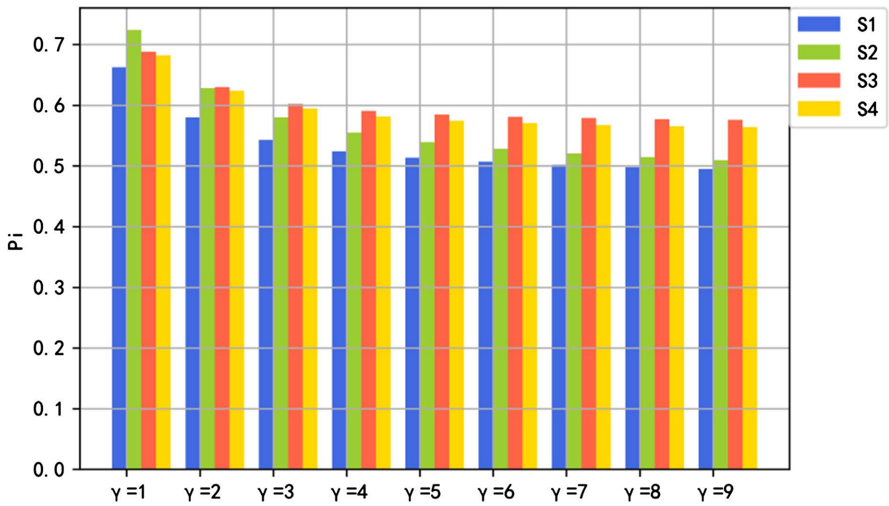

Figure 3. Final rankings with $\rho = 0.98$, $\varsigma = 0.5$ and $0 < \gamma < 10$.

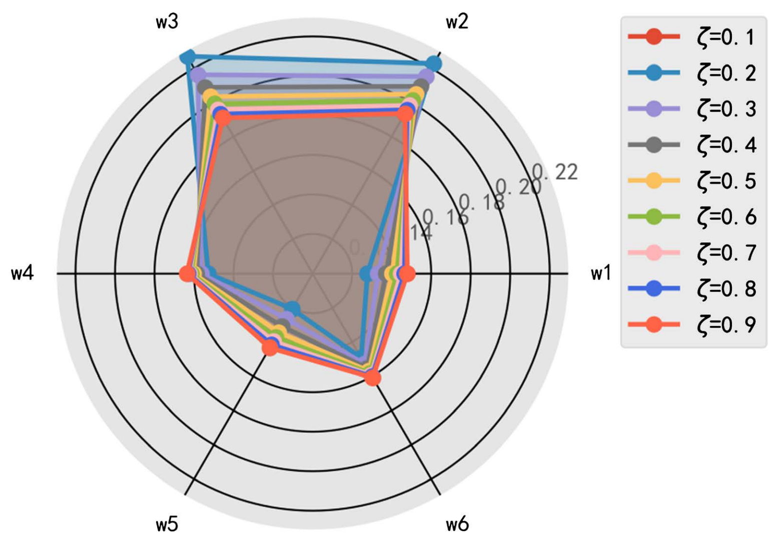

Figure 4. Weights with $\rho = 0.98$, $0 < \varsigma < 1$, and $\gamma = 3$.

Figure 5. Final rankings with $\rho = 0.98$, $0 < \varsigma < 1$, and $\gamma = 3$.

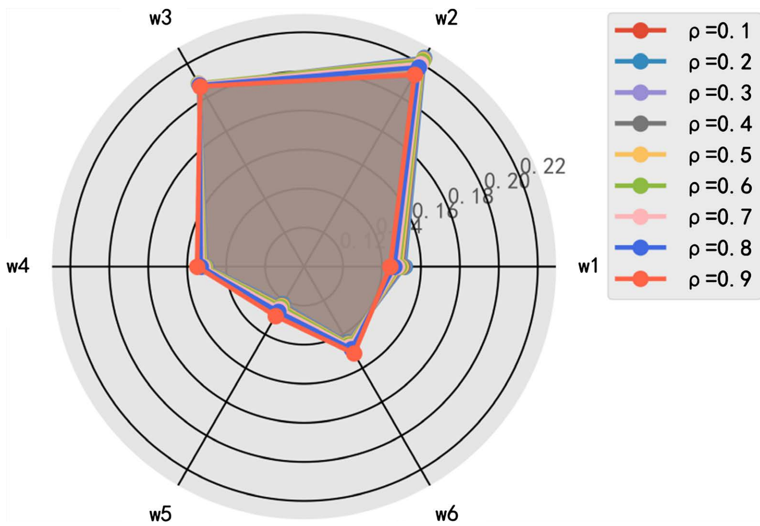

Figure 6. Weights with $0<\rho<1$, $\varsigma=0.5$, and $\gamma=3$.

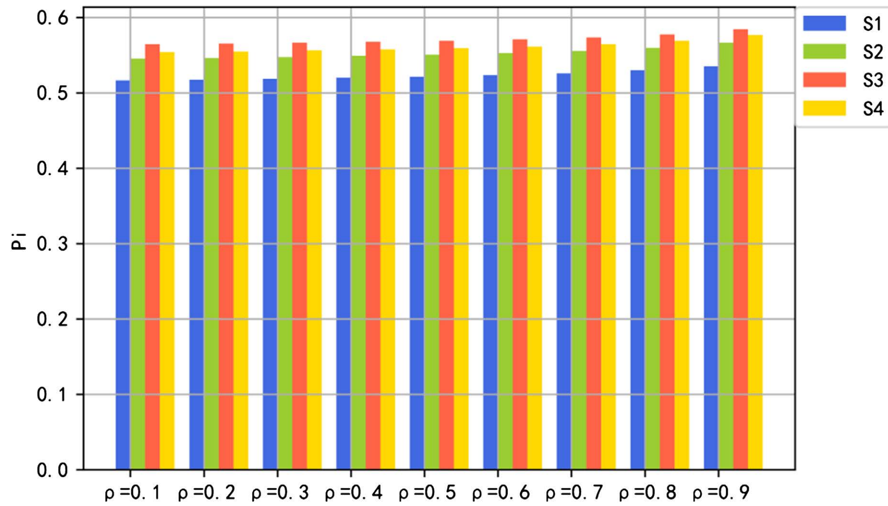

Figure 7. Final rankings with $0<\rho<1$, $\varsigma=0.5$, and $\gamma=3$.

affect the value of the corresponding weights, it did not influence the final rankings of four alternatives, ensuring the stability of the proposed decision method.

At the same time, DMs can choose different parameter values according to their own preferences, providing more choices for different decision-making problems.

Moreover, since the existing MCDM methods based on those operators discussed earlier cannot handle decision-making problems where the weight information is completely unknown. Then if Example 6.1 is solved by applying the SNWA-operator based MCDM method [21], the SNWAA- and SNEWA-operator based MCDM methods [22], the SNFWA-operator based MCDM method [24], and the SNHWA-operator based MCGDM method [32], the weight of criteria in Step 2 is used: $\omega=(0.2942,0.3367,0.3691)^{\mathrm{T}}$. The corresponding compared results shown in Table 2 can be obtained. Notably, the aggregation operators in the compared MCDM methods can be used twice to handle MCGDM problems.

Table 2. Compared results.

MCDM methods	Ranks
SNWA-operator based MCDM method [21]	$S_4 \succ S_3 \succ S_2 \succ S_1$
SNWAA-operator based MCDM method [22]	$S_3 \succ S_4 \succ S_2 \succ S_1$
SNEWA-operator based MCDM method [22]	$S_4 \succ S_3 \succ S_1 \succ S_2$
SNFWA-operator based MCDM method ($\lambda = 2$) [24]	$S_4 \succ S_3 \succ S_2 \succ S_1$
SNHWA-operator based MCGDM method ($\gamma = 3$) [32]	$S_3 \succ S_4 \succ S_2 \succ S_1$
ISNHWA-operator based MCGDM method ($\gamma = 3$)	$S_3 \succ S_4 \succ S_2 \succ S_1$

From **Table 1**, it can be seen that the ranking obtained by using the proposed single-valued neutrosophic MCGDM method is the same as that obtained by using the SNWAA-operator based MCDM method [22] and SNHWA-operator based MCGDM method ($\gamma = 3$) [31]: $S_3 \succ S_4 \succ S_1 \succ S_2$ and the best alternative is S_3. However, the ranking obtained by applying the proposed MCGDM method based on the ISNHWA operator is different from that obtained by applying the SNWA-operator based MCDM method [19], the SNEWA-operator based MCDM method [22], and the SNFWA-operator based MCDM method ($\lambda = 2$) [25], and the best alternative would be S_4. Moreover, the SNWA-operator based MCDM method [21], the SNEWA-operator based MCDM method [22], and the SNFWA-operator based MCDM method [24] have limitations, as discussed in Examples 5.1 and 5.2. Thus, the ISNHWA-operator based MCGDM method, the SNWAA-operator based MCDM method [22], and the SNHWA-operator can produce more reasonable results in this case.

In the following, another example with known weight information is provided to make a comparative analysis to further verify the effectiveness and superiority of the proposed method.

7. Conclusion

A novel single-valued neutrosophic MCGDM method based on the ISNHWA operator is proposed in this paper. The proposed ISNHWA operator avoids the limitations of existing aggregation operators (*i.e.*, the SNWA [21], SNWAA [22], SNEWA [22], SNFWA [24], and SNHWA [31] operators). Additionally, the novel single-valued neutrosophic MCGDM method overcomes the limitations of the SNWA-operator based MCDM method [21], the SNWAA- and SNEWA-operator based MCDM methods [22], the SNFWA-operator based MCDM method [24], and the SNHWA-operator based MCGDM method [31]. Several application examples have been provided to demonstrate that the ISNHWA operator and the single-valued neutrosophic MCGDM method are effective and feasible for solving MCDM and MCGDM problems. Future research should focus on the improvement of other single-valued neutrosophic aggregation operators (*i.e.*, the Bonferroni mean operator and Heronian mean operators), and of the corresponding MCDM and MCGDM methods.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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