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Full Length Research Paper

Thermal deflection of a thin circular plate with radiation

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This paper deals with the determination of thermal deflection of a thin circular plate defined as $0 \le r < a$; $-h \le z \le h$. A circular plate is considered having arbitrary initial temperature and subjected to radiation type boundary condition which is fixed at $(r = a)$. The non homogeneous type boundary conditions are maintained at plane surfaces of the plate. The governing heat conduction equation has been solved by using integral transform technique. The results are obtained in series form in terms of Bessel's functions. As a special case, aluminum metallic plate has been considered and the results for temperature distribution and thermal deflection have been computed numerically and are illustrated graphically.

Key words: Heat generation, non homogeneous heat conduction equation, thermal deflection, thermoelastic problem.

INTRODUCTION

Nowacki (1957) has determined steady-state thermal stresses in a circular plate subjected to an axisymmetric temperature distribution on the upper face with zero temperature on the lower face and the circular edge respectively. Roy (1973) discussed the normal deflection of a thin clamped circular plate due to ramp type heating of a concentric circular region of the upper face. This satisfies the time – dependent heat conduction equation. Recently Kedar and Deshmukh (2005) have determined the thermal deflection of a thin circular plate.

In this paper, the work of Deshmukh et al. (2005) has been extended for two-dimensional non-homogeneous boundary value problem of heat conduction and studied the thermal deflection of the plate defined as $0 \le r \le a$; $-h \le z \le h$. This inverse problem deals with the determination of temperature distribution, unknown temperature gradient and thermal deflection.

The plate is considered having arbitrary initial temperature and subjected to radiation type boundary conditions which are fixed at $(r = 0)$ and $(r = a)$. The non homogeneous type boundary conditions are maintained on plane surfaces of the disc. The governing heat conduction equation has been solved by using integral transform technique. The results are obtained in series form in terms of Bessel's functions. The results for thermal deflection have been computed numerically and are illustrated graphically. It is believed that this particular problem has not been considered by any one. This is new and novel contribution to the field. According to Lamba and Khobragade (2011), analytical approach is establish to construct solution in terms of stresses in a thin circular plate subjected to steady and unsteady state thermoelasticity due to diametrical compression. In this approach stress distribution are expressed by means of theoretical technique. The result is obtained as series of Bessel functions.

The results presented here will be useful in engineering problems particularly in aerospace engineering for stations of a missile body not influenced by nose tapering. The missile skill material is assumed to have physical properties independent of temperature, so the temperature $T(r, z, t)$ is a function of radius, thickness and time only.

STATEMENT OF PROBLEM

Consider a circular plate of thickness 2*h* occupying space

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D defined by $0 \le r \le a$; $-h \le z \le h$. The plate is considered having arbitrary initial temperature and subjected to radiation type boundary condition which is fixed at $(r = a)$. The non homogeneous type boundary conditions are maintained at plane surfaces of the plate.

The differential equation satisfying the deflection function $w(r,t)$ is given by:

$$\nabla^4 \omega = \frac{\nabla^2 M_T}{D(1-v)} \tag{1}$$

where M_T is the thermal moment of the plate which is defined as;

$$M_T = a_t E \int_{-h}^{h} T(r,z,t) z \quad dz \tag{2}$$

D is the flexural rigidity of the disc denoted as;

$$D = \frac{Eh^3}{12(1-v^2)} \tag{3}$$

a_t, E and v are the coefficients of the linear thermal expansion, Young's modules and Poisson's ratio of the disc material respectively and

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} \tag{4}$$

Since the edge of an annular disc is fixed and clamped,

$$\omega = \frac{\partial \omega}{\partial r} = 0, \text{ at } r = a \tag{5}$$

Initially $T = \omega = F(r,z)$, at $t = 0$

The temperature $T(r,z,t)$ of the plate at time t satisfies the differential equation,

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{6}$$

with the boundary conditions,

$$[T(r,z,t)]_{r=0} = g_1(z,t), \quad t > 0 \tag{7}$$

$$[T(r,z,t)]_{r=a} = g_2(z,t), \quad -h \le z \le h \tag{8}$$

$$\left[T + k_1 \frac{\partial T}{\partial z}\right]_{z=-h} = f_1(r,t), \quad t > 0 \tag{9}$$

$$\left[T + k_2 \frac{\partial T}{\partial z}\right]_{z=h} = f_2(r,t), \quad t > 0 \tag{10}$$

and initial condition is;

$$T(r,z,t) = F(r,z), \quad 0 \le r \le a, \ -h \le z \le h, \quad t = 0 \tag{11}$$

where k_1 and k_2 are radiation constants on the plane surfaces of the plate respectively and α is thermal diffusivity of the material of the plate. Equations 1 to 11 constitute the mathematical formulation of the problem under consideration.

SOLUTION OF THE PROBLEM

Applying Marchi-Fasulo transform (Marchi and Fasulo, 1964] (Appendix) to Equation 6, one obtains;

$$\frac{d^2\overline{T}}{dr^2} + \frac{1}{r}\frac{d\overline{T}}{dr} - a_n^2 \overline{T} = \frac{1}{k}\frac{d\overline{T}}{dt} + \Psi \tag{12}$$

Further applying finite Hankel transform to Equation 12, we get:

$$\frac{d\overline{T}^*}{dt} + kp^2 \overline{T}^* = \Psi_1^* \tag{13}$$

where $a_n^2 + \mu_m{}^2 = p^2$, $-k\Psi^* = \Psi_1^*$

Equation 13 is a first order differential equation, whose solution is given by:

$$\overline{T}^*(m,n,t) = e^{-kp^2 t}\left[\int_0^t \psi_1{}^* e^{kp^2 t^1} \ dt^1 + \overline{f}^*(m,n)\right] \tag{14}$$

Applying inversion of Hankel transform and Marchi-Fasulo transform to Equation 14, one obtains;

$$T(r,z,t) = \frac{2}{a^2}\sum_{m=1}^{\infty} \frac{J_0(\mu_m r)}{[J_1(\mu_m a)]^2}$$

$$\sum_{n=1}^{\infty} \frac{P_n(z)}{\lambda_n} e^{-kp^2 t}\left[\int_0^t \psi_1{}^* e^{kp^2 t^1} \ dt^1 + \overline{f}^*(m,n)\right] \tag{15}$$

Equation 15 is the desired solution of the given problem.

DETERMINATION OF THERMAL DEFLECTION

Using the value of temperature distribution from Equation 15 into Equation 2, one obtains;

$$M_T(r,t) = \frac{2a_t E}{a^2} \sum_{m=1}^{\infty} \frac{J_0(\mu_m r)}{[J_1(\mu_m a)]^2} \sum_{n=1}^{\infty} e^{-kp^2 t} \left[\int_0^t \psi_1^* e^{kp^2 t^1} \, dt^1 + \overline{f}^*(m,n) \right] \int_{-h}^{h} \frac{z P_n(z)}{\lambda_n} dz \tag{16}$$

We assume the solution of Equation 1, satisfying condition 5 as;

$$\omega(r,t) = \sum_{m=1}^{\infty} C_n(t) \, Sin\left(\frac{m\pi r}{a}\right) r(r-a) \tag{17}$$

where μ_m are the positive roots of the transcendental equation,

$$J_1(\mu_m a) = 0 . \tag{18}$$

It can be easily seen that

$$\omega = \frac{\partial \omega}{\partial r} = 0 \quad \text{at } r = a$$

Hence solution 17 satisfies condition 5.

Now

$$\nabla^4 \omega = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \right)^2 \sum_{m=1}^{\infty} c_m [J_0(\mu_m r)] \tag{19}$$

We use the well known result,

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \right) J_0(\mu_m r) = -\mu_m^2 \, J_0(\mu_m r) \tag{20}$$

In Equation 19, we get;

$$\nabla^4 \omega = \sum_{m=1}^{\infty} c_m \mu_m^4 \, J_0(\mu_m r) \tag{21}$$

Also

$$\nabla^2 M_T = -a_t E \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} [J_0(\mu_m r) \frac{1}{\lambda_n^2} (\overline{F}^* + \int_0^t e^{\alpha p^2 t^1} \overline{\Psi}^* dt^1)(z-1) \int_{-h}^{h} P_n(z) \, dz] \tag{22}$$

Using Equations 21 and 22 in Equation 1, one obtains;

$$\sum_{m=1}^{\infty} c_m \mu_m^4 J_0(\mu_m r) = -\frac{a_t E}{D(1-v)}$$

$$\times \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} [J_0(\mu_m r) \frac{1}{\lambda^2} (\overline{F}^* + \int_0^t \overline{\Psi}^* e^{\alpha p^2 t^1} dt^1)(z-1) \int_{-h}^{h} P_n(z) \, dz] \tag{23}$$

On solving Equation 23 we get;

$$c_m(t) = -\frac{a_t E}{D(1-v)} \sum_{n=1}^{\infty} \left[\frac{1}{\lambda_n^2 \mu_m^4} (\overline{F}^* + \int_0^t \overline{\Psi}^* e^{\alpha p^2 t^1} dt^1)(z-1) \int_{-h}^{h} P_n(z) \, dz \right] \tag{24}$$

Substituting Equation 24 into Equation 17, we get

$$\omega(r,t) = -\frac{a_t E}{D(1-v)} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left[\frac{1}{\lambda_n^2 \mu_m^4} (\overline{F}^* + \int_0^t \overline{\Psi}^* e^{\alpha p^2 t^1} dt^1)(z-1) \int_{-h}^{h} P_n(z) \, dz \right] \times [J_0(\mu_m r)] \tag{25}$$

The expression (25) is represented graphically, that is Figures 1 and 2.

SPECIAL CASE AND NUMERICAL RESULTS

Setting

$$F(r,z) = \delta(r - r_0) \times (z-h)^2 \times (z+h)^2 \tag{26}$$

where r is the radius of the disc and δ is the Dirac – delta function.

$$\Rightarrow \overline{F}^* = 3(k_3 + k_4) J_0(\mu_m r_0) \left[\frac{a_n h \cos^2(a_n h) - \cos(a_n h) \sin(a_n h)}{a_n^2} \right] \tag{27}$$

Using Equation 27 in Equation 16, one obtains;

$$T = \sum_{m=1}^{\infty} \frac{1}{\mu_m} J_0(\mu_m r) \left[\sum_{n=1}^{\infty} \frac{1}{\lambda_n} P_n(z) \left(\overline{F}^* + \int_0^t \overline{\Psi}^* \, e^{\alpha p^2 t^1} dt^1 \right) \right] \tag{28}$$

The expression (28) is represented graphically, that is Figures 3 and 4.

CONCLUSION

In this paper, the temperature distribution and thermal

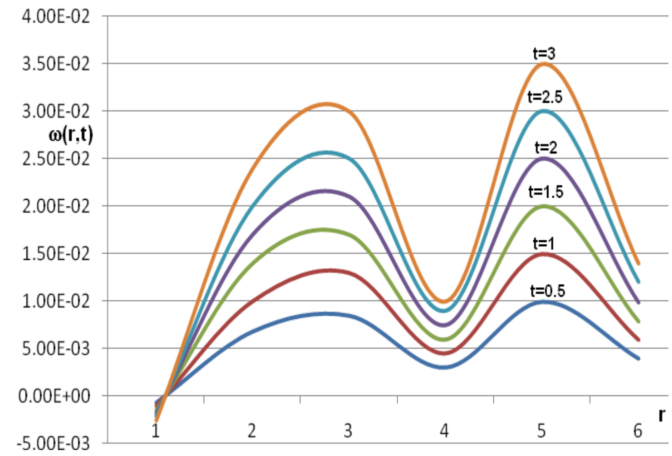

Figure 1. Thermal deflection ω(r,t) versus r for different values of t.

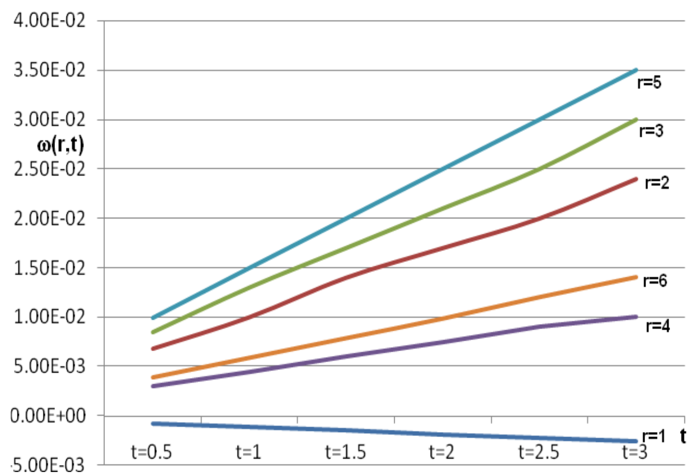

Figure 2. Thermal deflection ω(r,t) versus t for different values of r.

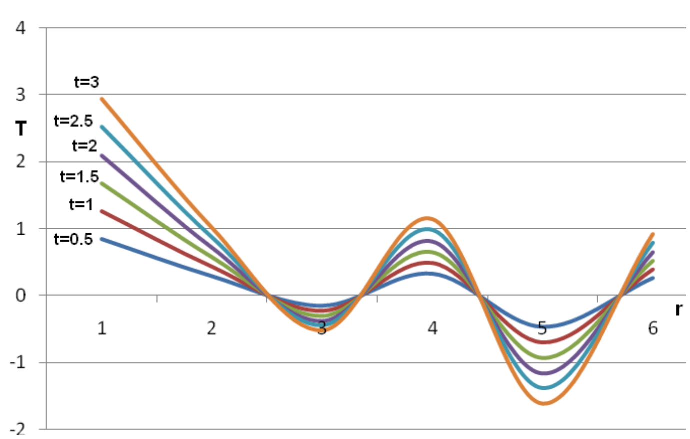

Figure 3. Temperature distribution T versus r for different values of t.

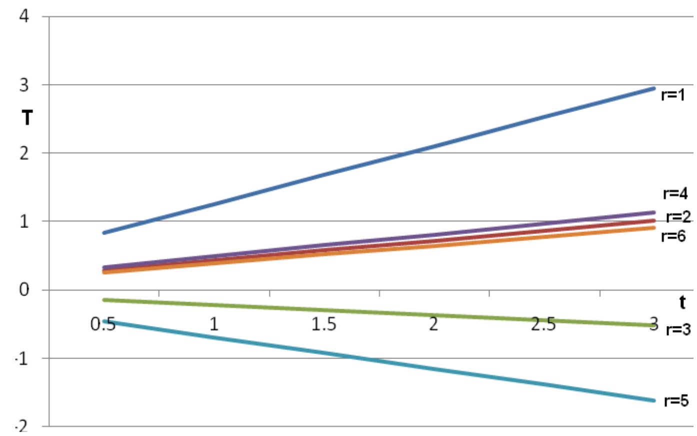

Figure 4. Temperature distribution T versus t for different values of r.

deflection have been investigated and the results are depicted graphically. Mathematical calculations have been made in MathCAD, and graphs have been plotted for temperature distribution and thermal deflection versus r, t and z for different values of time and radius.

The temperature distribution and thermal deflection of a thin circular plate made of aluminium have been determined by using the conditions given in the problem and applying integral transform techniques.

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APPENDIX

The finite Marchi-Fasulo integral transform of f (z), -h < z < h is defined to be

$$\overline{F}(n) = \int_{-h}^{h} f(z) P_n(z) dz \quad (29)$$

then at each point of (-h, h) at which f(z) is continuous,

$$f(z) = \sum_{n=1}^{\infty} \frac{\overline{F}(n)}{\lambda_n} P_n(z) \quad (30)$$

where

$$P_n(z) = Q_n \cos(a_n z) - W_n \sin(a_n z)$$

$$Q_n = a_n(\alpha_1 + \alpha_2)\cos(a_n h) + (\beta_1 - \beta_2)\sin(a_n h)$$

$$W_n = (\beta_1 + \beta_2)\cos(a_n h) + (\alpha_2 - \alpha_1) a_n \sin(a_n h)$$

$$\lambda_n = \int_{-h}^{h} P_n^{\,2}(z)\, dz = h\left[Q_n^{\,2} + W_n^{\,2}\right] + \frac{\sin(2a_n h)}{2a_n}\left[Q_n^{\,2} - W_n^{\,2}\right]$$

The Eigen values a_n are the solutions of the equation

$$\left[\alpha_1 a \cos(ah) + \beta_1 \sin(ah)\right] \times \left[\beta_2 \cos(ah) + \alpha_2 a \sin(ah)\right] = \left[\alpha_2 a \cos(ah) - \beta_2 \sin(ah)\right] \times \left[\beta_1 \cos(ah) - \alpha_1 a \sin(ah)\right]$$

α_1, α_2, β_1 and β_2 are constants.

Moreover the integral transform has the following property:

$$\int_{-h}^{h} \frac{\partial^2 f(z)}{\partial z^2} P_n(z) dz = \frac{P_n(h)}{\alpha_1}\left[\beta_1 f(z) + \alpha_1 \frac{\partial f(z)}{\partial z}\right]_{z=h} - \frac{P_n(-h)}{\alpha_2}\left[\beta_2 f(z) + \alpha_2 \frac{\partial f(z)}{\partial z}\right]_{z=-h} - a_n^{\,2}\,\overline{F}(n).$$