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MHD and Slip Boundary Condition Effects on Forced Convection Flow through a Microchannel Including Viscous Dissipation

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Authors' contributions

This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.

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Abstract

This study analyzes laminar forced convection flow in parallel plate micro-channel with slip velocity and temperature jump in the presence of transverse magnetic field. Closed-form solutions are obtained for velocity and thermal profiles when both walls of the channel are kept at unequal temperatures. The effects of various controlling parameters such as Hartman number, rarefaction parameter, fluid-wall interaction parameter and Brinkman number on the thermal behavior are discussed with the aid of line graphs. Interesting result from the present work is that increase in rarefaction parameter leads to enhancement in fluid temperature while increase in fluid-wall interaction parameter leads to increase in temperature jump on the walls of the channel.

Keywords: Knudsen number (Kn); forced convection; velocity slip; temperature jump; viscous dissipation; Magnetohydrodynamics (MHD).

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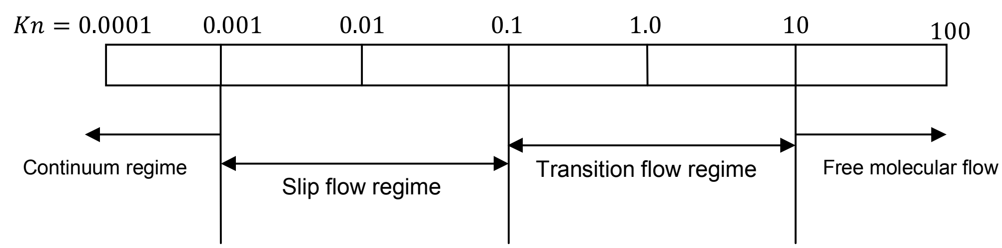
1 Introduction

Research in micro-coolers, micro-fuel cells, micro-biochips and micro-reactors has been growing at a tremendous pace due to rapid developments in micro-electronics and biotechnologies. In micro-fluidic systems, micro-channel has been noticed to be one of the important elements in fluid flow within a miniature area. In addition to been a reactant delivery, the micro-channels are also used to connect different chemical chambers. The design and process controls of micro-fluidic and microelectromechanical systems (MEMS) involve the effect of geometrical configurations on the pressure, velocity and thermal distributions of the fluid on the micrometer $(10^{-6}m)$ scale. Knudsen number (Kn) is an important factor in the flow of fluids through micro-channels and it is defined as the ratio of molecular free path (λ) to characteristic length (L). In the slip flow regime, the gas at the surface has a tangential velocity which causes it to slip along the surface and, the temperature of the gas at the surface is finitely different from the surface temperature resulting to a jump in temperature between the surface and the adjacent gas. However, Beskok and Karniadakis [1] proposed four flow regimes in gases as shown in Fig. 1.



Classification of different flow regimes

Recently, Biswal et al. [2] considered the hydrodynamic and heat transfer behavior in the developing region of a micro-channel. It was deduced that slip velocity and jump temperatures enhances the rate of heat transfer. Extensive research has been carried out recently in the field of micro geometric flows. Weng and Cheng [3] studied analytically the fully developed natural convection in open ended vertical parallel plate micro-channel, taking into account the effects of rarefaction and fluid wall interaction parameters on the volume flow rate and heat transfer. Fully developed thermocreep-driven gas micro-flow has been investigated by Weng et al. [4]. Laminar convection in a vertical channel with viscous dissipation and buoyancy effects have been examined by Barletta [5].

Similarly, the effect of viscous dissipation has received considerable attention owing to its importance in the heat transfer point of view. Citing few works, Mondal and Sanchayan [6] investigated viscous dissipation effect on the limiting value of Nusselt numbers for shear driven flow through asymmetrically heated parallel plates. Recently, Ramjee and Satyamurty [7] studied the Nusselt number for viscous dissipation flow between parallel plates kept at unequal temperatures. In another related work, Hatton and Turton [8] investigated Heat transfer in the thermal entry length with laminar flow between parallel walls at unequal temperatures. The Nusselt number for laminar forced convection in asymmetrically heated annuli with viscous dissipation was presented by Kumar and Satyamurty [9]. Tso et al. [10] and Aydin and Avci [11] studied viscous dissipation effects of power law fluid flow within parallel plates with constant heat fluxes and its effects on the rate of heat transfer in a Poiseuille flow respectively. Barletta [12] investigated laminar mixed convection with viscous dissipation in a vertical channel. Jha et al. [13] studied transient magnetohydrodynamic free convective flow in vertical micro-concentric annuli. Muhammad et al. [14] investigated squeezing nanofluid flow between two parallel plates under the influence of MHD and thermal radiation.

In another related work, the rotating flow of magneto hydrodynamic carbon nanotubes over a stretching sheet with the impact of non-linear thermal radiation and heat Generation/Absorption was studied by Muhammad et al. [15]. Hussain et al. [16] analyzed a bioconvection model for squeezing flow between parallel plates containing gyrotactic microorganisms with impact of thermal radiation and heat generation/absorption. The Combined Magneto Hydrodynamic and Electric Field Effect on an Unsteady Maxwell Nanofluid Flow over a Stretching Surface under the Influence of Variable Heat and Thermal Radiation was investigated by Khan et al. [17].

Therefore, the objective of the present work is to analyze the influence of transverse magnetic field, fluid-wall interaction parameter (ln), rarefaction parameter ($\beta_v Kn$) and viscous dissipation on the steady forced convective flow through a micro-channel. The present study extends the work of Ramjee and Satymurty [7] by taking into account the effects of Hartmann number, velocity slip and temperature jump boundary conditions on the fluid velocity and temperature.

2 Mathematical Formulation

Consider a fully developed flow of viscous incompressible fluid in a micro-channel formed by two parallel plates. The x coordinates coincides with the centerline of the channel and the y coordinate is normal to it and the distance between the two plates is L as shown in Fig. 1. The plates are positioned at $y = \pm 1/2$ and the average and inlet fluid temperatures are given as $\bar{T}$ and T_i respectively. The plates are kept at unequal temperatures of T_1 at $y = -1/2$ and T_2 at $y = +1/2$.

The mathematical model extended here is a generalization of Ramjee and Satymurty [7] in the presence of slip boundary conditions and transverse magnetic field as shown in Fig. 1 below.

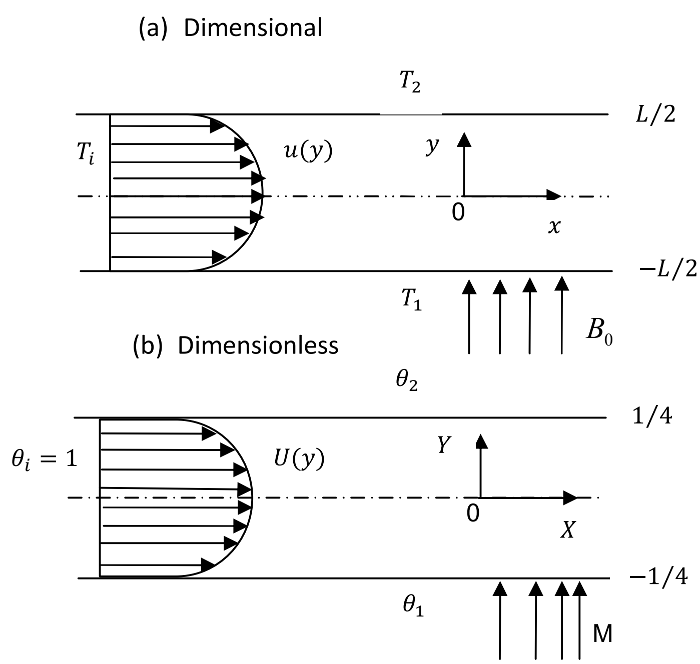


Fig. 1. Flow configuration and coordinate system

Under the usual Boussinesq approximation and neglecting convective terms, the governing equations in dimensional form are

$$\frac{d^2u}{dy^2} - \frac{1}{\rho\nu}\frac{dp}{dx} - \frac{\sigma_e B_0^2}{\rho} u = 0 \tag{1}$$

$$\frac{d^2T}{dy^2} + \frac{\mu}{k}\left(\frac{du}{dy}\right)^2 = 0 \tag{2}$$

The boundary conditions which describe velocity slip and temperature jump conditions at the fluid wall interface are

$$\begin{aligned} &u = \beta_v \lambda \frac{du}{dy} \ , T = T_1 + \beta_t \lambda \frac{dT}{dy} \ at \ y = -\frac{L}{2} \\ &u = -\beta_v \lambda \frac{du}{dy} \ , T = T_2 - \beta_t \lambda \frac{dT}{dy} \ at \ y = +\frac{L}{2} \end{aligned} \tag{3}$$

Using the following dimensionless quantities

$$\begin{aligned} &U = \frac{u}{u_{avg}}, Y = \frac{y}{2L}, P = \frac{1}{\rho v}\frac{dp}{dx}\frac{4L^2}{u_{avg}}, \theta = \frac{T-\bar{T}}{T_i-\bar{T}}, \bar{T} = \frac{T_1+T_2}{2} \\ &Br = \frac{\mu u_{avg}^2}{k(T_i-\bar{T})}, Kn = \frac{\lambda}{2L}, \beta_t = ln = \frac{\beta_t}{\beta_v} \end{aligned} \tag{4}$$

Equations $(1) - (3)$ in dimensionless form are given as

$$\frac{d^2U}{dY^2} = P + M^2 \tag{5}$$

$$\frac{d^2\theta}{dY^2} + Br\left(\frac{dU}{dY}\right)^2 = 0 \tag{6}$$

Subject to the relevant boundary conditions

$$\begin{aligned} &U = \beta_v Kn \frac{dU}{dY} \ , \theta = -\frac{(1-A)}{(1+A)} + \beta_v Knln \frac{d\theta}{dY} \ at \ Y = -\frac{1}{4} \\ &U = -\beta_v Kn \frac{dU}{dY} \ , \theta = \frac{(1-A)}{(1+A)} - \beta_v Knln \frac{d\theta}{dY} \ at \ Y = +\frac{1}{4} \end{aligned} \tag{7}$$

Where $A = \frac{(T_2-T_i)}{(T_1-T_i)}$

Solving (5) and (6) using (7) gives dimensionless velocity and temperature solutions respectively

$$\begin{aligned} &U = \frac{(P+M^2)Y^2}{2} + a_0 \qquad where \ a_0 = -\left(\frac{\beta_v Kn(P+M^2)}{4} + \frac{(P+M^2)}{32}\right) \\ &\theta = -Br\left(\frac{PY^4}{12} + \frac{2PM^2Y^4}{12} + \frac{M^4Y^4}{12}\right) + \frac{4EY}{(1+\beta_v Kn \ln)} + a_2 + a_3 \end{aligned} \tag{8}$$

$$Where \ \ a_2 = \frac{\beta_v Kn \ln Br}{96}\left(\frac{P^2}{2} + \frac{M^4}{2} + PM^2\right) \ \ and \ \ a_3 = \frac{Br}{1534}\left(\frac{P^2}{2} + \frac{M^4}{2} + PM^2\right)$$

3 Results and Discussion

The basic parameters that govern this flow are Brinkman number (Br), degree of asymmetry heating (A), rarefaction parameter $(R = \beta_v Kn)$ and fluid-wall interaction parameter (ln). To examine the effects of these basic parameters, the variations in velocity and temperature profiles at both walls of the channel are graphically presented and discussed. The present parametric study has been performed in continuum and slip flow regimes ($Kn \leq 0.1$). Also, for air and various surfaces, the values of β_t and β_v range from near 1.64 to more than 10 and from near 1 to 1.667 respectively. Therefore this study has been performed over the reasonable ranges of $0 \leq ln \leq 10$ and $0 \leq \beta_v Kn \leq 0.1$. The chosen reference values of $\beta_v Kn$ and ln for this analysis are 0.05 and 1.64 respectively as given in Weng and Chen [4]. Furthermore, three different degrees of asymmetry parameter $A = -0.5, 0.0$ and 0.5 have been considered.

Hence the results can be discussed in the interval $-1 \leq A \leq 1$, where $A = 1$ corresponds to symmetric heating/cooling, $T_1 = T_2 = T$ and $A \neq 1$ shall be taken as asymmetric heating/cooling. Furthermore, positive values of Brinkman number ($Br > 0$) are compatible with the fluid cooling case while negative values of Brinkman number ($Br < 0$) are compatible with the fluid heating case with reference to $\bar{T}$.

Fig. 3 shows variation in velocity for different values of rarefaction parameter($R = \beta_v Kn$). It is observed from Fig. 2 that as rarefaction parameter ($\beta_v Kn$) increases, the velocity slip at both walls of the channel increases. The rarefaction parameter ($\beta_v Kn$) reduces the retarding effects of the surface of the channel. This yields a significant increase in the fluid velocity near both surfaces of the channel.

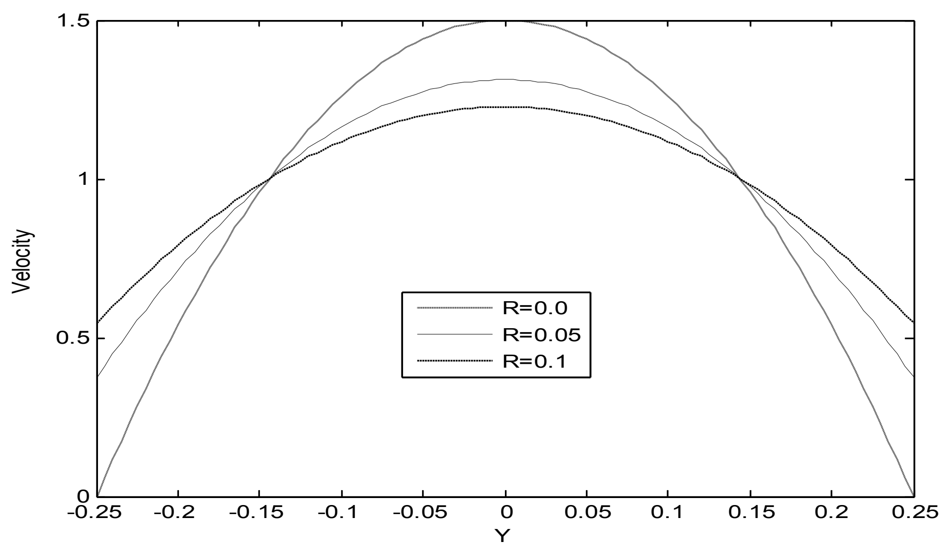


Fig. 2. Velocity distribution for different values of rarefaction parameter ($R = \beta_v Kn$)

Fig. 4 and 5 reveals the variations in temperature distribution for different values of rarefaction parameter ($R = \beta_v Kn$) at $A = -0.5$ and 0.5 respectively. It is observed from both figures that increase in rarefaction parameter ($R = \beta_v Kn$) leads to increase in temperature jump on the walls of the channel. This is attributed to the weak interaction between the fluid molecules and the heated wall of the channel. Also, an increase in Brinkman number Br leads to enhancement in fluid temperature. Viscous dissipation converts the kinetic energy of the moving fluid into internal energy thereby increasing the fluid temperature. It also accounts for the distortion on the temperature profile. In addition, when positive values of dissipation are considered, increase in rarefaction parameter ($R = \beta_v Kn$) leads to increase in temperature while at negative dissipation, increase in rarefaction parameter ($R = \beta_v Kn$) leads to decrease in temperature.

Fig. 6 and 7 depicts the variations in temperature distribution for different values of fluid-wall interaction parameter (ln) at two degrees of asymmetric heating $A = -0.5$ and 0.5 respectively. It is interesting to note that increase in fluid-wall interaction parameter (ln) brings about increase in temperature jump on both walls of the channel. In addition, when positive values of dissipation are considered, increase in fluid-wall interaction parameter (ln) leads to increase in temperature while at negative dissipation, increase in fluid-wall interaction parameter (ln) leads to decrease in temperature.

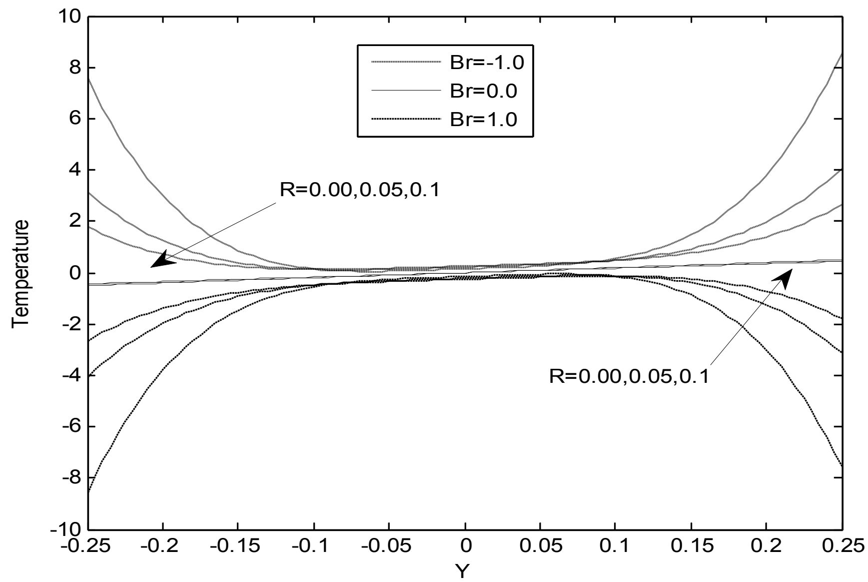


Fig. 3. Temperature distribution for different values of $(\beta_v K_n)$ for $A = 0.5$, $ln = 1.64$, $M = 2$

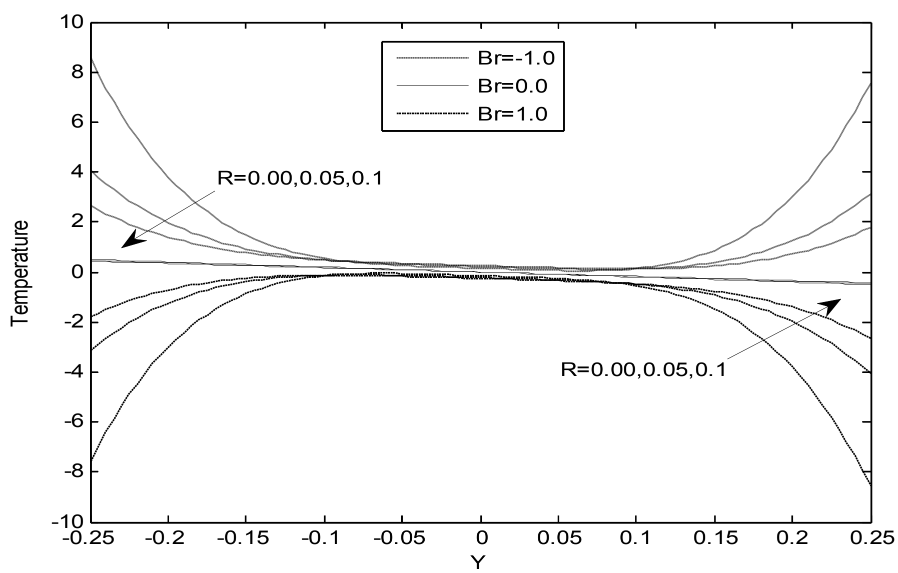


Fig. 4. Temperature distribution for different values of $\beta_v Kn$ for $A = -0.5$, $ln = 1.64$, $M = 2$

Fig. 7 and 8 depicts the variations in temperature distribution for different values of Hartman number (M) at two degrees of asymmetric heating $A = -0.5$ and 0.5 respectively. It is evident that increase in Hartman number (M) brings about increase in temperature on both walls of the channel for positive dissipation while the reverse is observed for negative dissipation. Furthermore, in absence of viscous dissipation, Hartman number is observed to have negligible effect on the temperature profile.

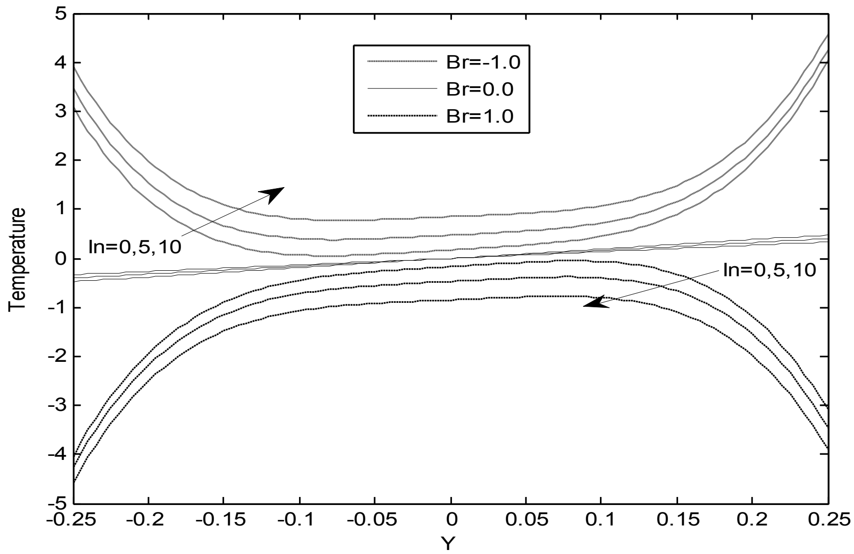


Fig. 5. Temperature distribution for different values of ln for $\beta_v Kn = 0.05, A = 0.5, M = 2$

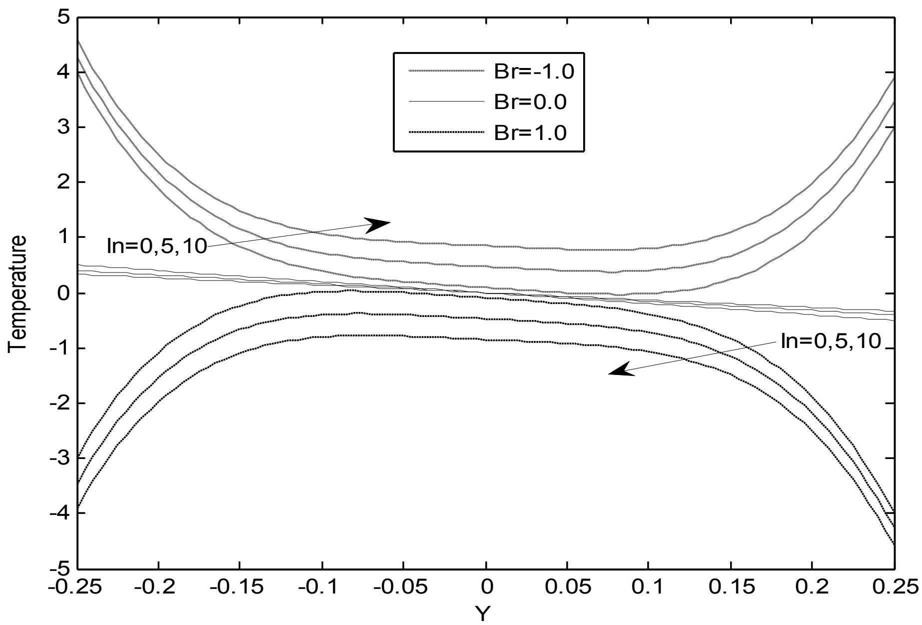


Fig. 6. Temperature distribution for different values of ln for $\beta_v Kn = 0.05, A = -0.5, M = 2$

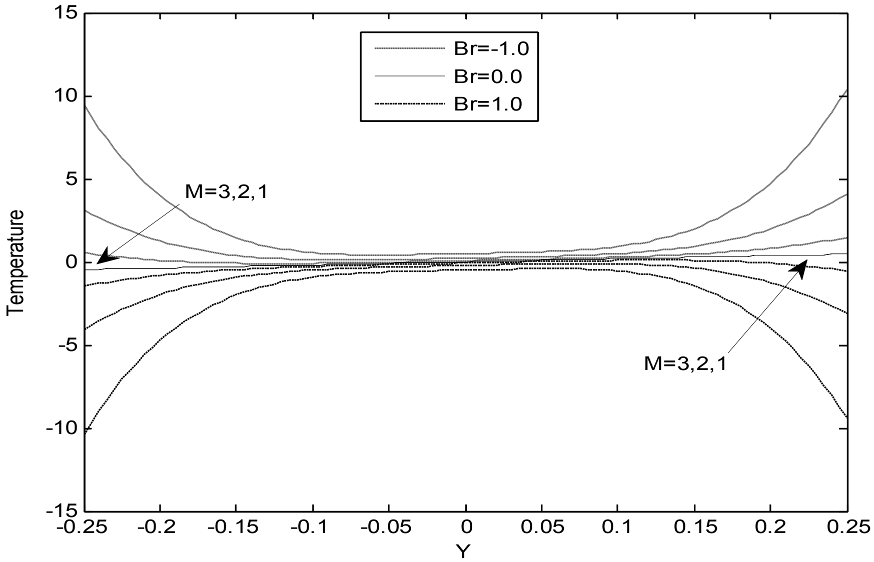


Fig. 7. Temperature profile for different values of M for $\beta_v Kn = 0.05, ln = 1.64, A = 0.5$.

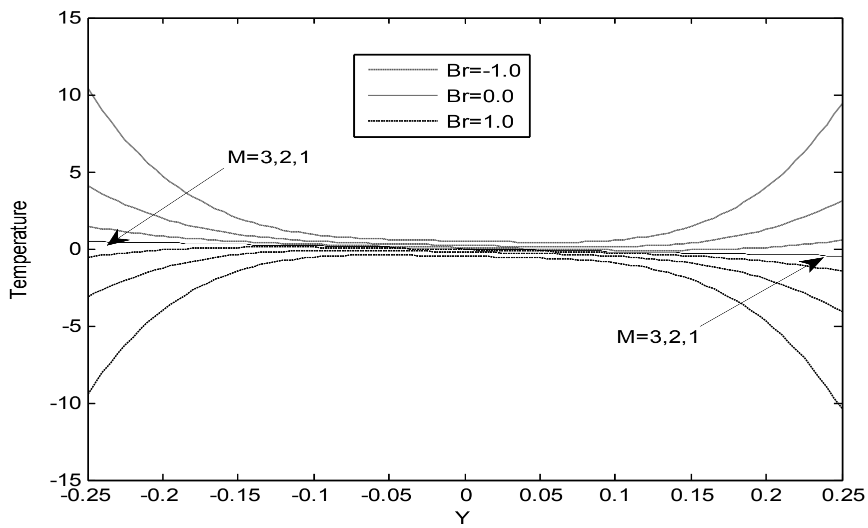


Fig. 8. Temperature profile for different values of M for $\beta_v Kn = 0.05, ln = 1.64,$ $A = -0.5$.

4 Conclusion

This study considered fully developed laminar forced convection flow through parallel plate micro-channel under slip and jump boundary conditions in the presence of transverse magnetic field. Exact solutions for the mathematical model governing the present physical situation and the expressions for the fluid velocity and temperature at both walls of the channel have been obtained. This study conforms to the findings of Ramjee and Satyamurty [7] when $M, ln, \beta_v Kn \rightarrow 0$. The following deductions were made from the present work:

I. The increase in rarefaction parameter $(\beta_v Kn)$ and fluid-wall interaction parameter (ln) leads to increase in temperature jump.
II. At positive dissipation, increase in fluid-wall interaction parameter (ln) leads to increase in temperature while at negative dissipation, fluid temperature decreases with increase in fluid-wall interaction parameter (ln).
III. Increase in Hartman number (M) brings about increase in temperature jump on both walls of the channel for positive dissipation while the reverse is observed for negative dissipation.

Competing Interests

Authors have declared that no competing interests exist.

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Appendix A

Nomenclatures used in the present work are:

A	*Degree of asymmetry*
Br	*Brinkman number*
D_h	*Hydraulic Diameter*
h_1	*Heat transfer coefficient at lower wall*
h_2	*Heat transfer coefficient at upper wall*
k	*Thermal conductivity of the fluid*
Kn	*Knudsen number*
L	*Length of wall*
ln	*Fluid-wall interaction parameter*
Nu_1	*Nusselt number at the lower wall of the channel*
Nu_2	*Nusselt number at the upper wall of the channel*
P	*Pressure gradient*
T	*Dimensional fluid temperature*
T_b	*Dimensional bulk temperature*
T	*Dimensional temperature*
T_1	*Dimensional temperature at the lower wall*
T_2	*Dimensional temperature at the upper wall*
U	*Dimensionless velocity*
u	*Dimensional velocity*
u_{avg}	*Reference velovcity*
X	*Dimensionless coordinate in axial direction*
Y	*Dimensionless coordinate normal to the axis of the channel*
y	*Dimensional coordinate normal to the axis of the channel*

Greek Symbols

θ	*Dimensionless temperature*
β_v, β_t	*Dimensionless variables*
λ	*Molecular mean free path*
μ	*Dynamic viscosity*
υ	*Kinematic viscosity*
ρ	*Fluid density*

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