



Comparison of Goodness of Fit Tests for Normal Distribution

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Authors' contributions

This work was carried out in collaboration between both authors. Authors IAF and EFR conceived and designed the study and wrote the protocol. Author IAF performed the statistical analysis. Author EFR managed the literature searches and the abstract. Both Authors contributed to the introduction, results, conclusion and recommendations. Both authors read, edit and approved the final manuscript.

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ABSTRACT

Goodness of fit test is a test that has attracted researchers' interest over the decades. This study is on goodness of fit test for normal distribution only. The Kolmogorov-Smirnov (K-St) and Pearson's Chi-square (χ^2 test) goodness of fit test were used to determine the normality of a given data. The result revealed that the data is normal under the two tests and that the Kolmogorov-Smirnov (K-S test) were preferred to Pearson's Chi-square (χ^2 test). The Kolmogorov-Smirnov (K-S) test of goodness of fit is the most suitable in terms of the p-value.

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1. INTRODUCTION

The common objectives in goodness of fit test are to investigate (or fit) the closeness of the observed data to that of the expected data which is sometimes based on theoretical models. This addresses the question of whether the data follows any pattern or fit a specified or assumed probability distribution. The measure of goodness of fit typically summarizes the discrepancy between the observed values and the values expected under the model in question.

Such measures can be used in hypothesis testing, also to test for normality of residuals, to test whether outcomes of frequencies follow a specified distribution. Quite often the observed data represent the fit of the saturated, the most complex model possible with the given data. Thus, quite often the alternative hypothesis (H1) will represent the saturated model which fits perfectly because each observation has a separate parameter.

When talking about goodness of fit, more references are made towards Pearson's chi-square only. However, Pearson's Chi-square approximation which is commonly used is not adequate for small samples. Therefore, the study looks into other forms of goodness of fit tests like the Kolmogorov-Smirnov (K-S) goodness of fit test.

2. NORMAL DISTRIBUTION

In probability theory, the normal (or Gaussian) distribution is a very common continuous probability distribution. Normal distributions are important in statistics and are often in the natural and social sciences to represent real valued random variables whose distributions are not known. The normal distribution is the only absolutely continuous distribution whose cumulants beyond the first two (i.e. other than the mean and variance) are zero. The normal curve was developed mathematically in 1733 by DeMoivre as an approximation to the binomial distribution primarily from the fact that distributions of many natural occurrences are at least approximately normally distributed [1]. The paper was discovered in 1924 by Karl Pearson [2]. Laplace used the normal curve (Gaussian distribution) in 1783 to describe the distribution of errors. The normal distribution was first applied

on the analysis of errors of measurement made in astronomical observations by Gauss [2].

3. KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST

The Kolmogorov-Smirnov of goodness of fit is one of the nonparametric tests. The test can be used to verify that a sample comes from a population with some known distribution and also that two populations have the same distribution. It is commonly used as a test for normality to verify if a particular data or datasets is normally distributed. The K-S test can also be used to check the assumption of normality in ANOVA. More specifically, the K-S test compares a known hypothetical probability distribution (e.g. normal distribution) to the distribution generated by your data the empirical distribution function in order to assess fit [3]. The goodness of fit test for a sample was introduced by [4] defined the empirical distribution function and find out how close this would be to the true distribution of $F(x)$ when this is continuous. The Kolmogorov-Smirnov test for two samples was introduced by Vladimir Ivanovich Smirnov [5].

4. CHI-SQUARE GOODNESS OF FIT TEST

The chi-square goodness of fit test is applied to binned data (i.e. data put into classes). This actually not a restriction since for non-binned data you can simply calculate a histogram or frequency table before generating the chi-square. Also, the chi-square goodness of fit test can be applied to discrete distributions such as the Binomial and the Poisson distribution while the Kolmogorov-Smirnov is restricted to continuous distributions. Pearson's paper of 1900 introduced what subsequently became known as chi-square goodness of fit to examine whether the observed data support a given specification [6]. His work motivated research in the testing of hypothesis and estimation unknown parameters and led to the development of statistics as a separated discipline. Goodness of fit test is highly used in business decision making. During the period 1899, Pearson statistical interest increased rapidly towards his system of frequency curve, which in return laid the foundation of multiple correlation and regression and obtain probable errors and correlation of estimated co-efficient in large samples.

Also in calculating goodness of fit, it is necessary to state the hypothesis (null, alternative), choose a significance level and to determine the critical value. Also in the analysis of variance, one of the components into which the variance is portioned may be a lack of fit sum of squares.

[1] study revealed that the Kambhampati test dominates and is significantly preferred to Chemoff-Lehmann test of goodness of fit test when compared.

[7] conducted a Monte Carlo study to determine the power of Pearson's overall goodness of fit test as well as the Pearson analog tests to detect rejections due to shifts in dispersion parameters, skewness and kurtosis as the location parameter vary. Simulations were conducted for large and small sample sizes. They recommended that to improve the power of the goodness of fit test the partition points are equiprobable and they observed that power can be improved by the use of non-equiprobable partitions.

[8] derives goodness of fit test based on chi-square statistic using simulation and examines the values of the chi square test statistic behaviour with the level of skewness for two different distributions and they observed that the value of chi-square test statistic decreases as the value of skewness increases and vice versa and concluded the result is relevant to the theories in which shape and skewness measure can be used to determine the validity of the assumed right skewed distribution to fit the data well.

[9] proposed a technique for testing multi-normality between an ordered set of transformed datasets and set of the population p th quantiles of the chi-square distribution. The critical values of the test were evaluated for different samples. The powers of the proposed test were compared some known tests and the result showed that the proposed test gives excellent results at large sample sizes.

[10] proposed a framework for constructing goodness of fit test in both low and high dimensional linear models. They show that simulation can be used to obtain the critical values for the test in the low dimensional settings and demonstrate using both theoretical results and extensive numerical studies that some form of the parametric bootstrap can do the same when the high dimensional linear model is under consideration.

5. STATEMENT PROBLEM

We tend to ask at times if a particular model follows or fit any assumed probability distribution. Hence, this study analysed Kolmogorov-Smirnov goodness of fit test and chi-square goodness of fit test for normal distribution. The result was compared and the most suitable goodness of fit test is selected to test normality based on p-value.

6. SIGNIFICANCE OF THE STUDY

The study would add more knowledge to the existing knowledge on goodness of fit tests and will also change the mind-set of researchers on the popularity of chi-square goodness of fit test over others.

7. RESULTS

7.1 Kolmogorov-Smirnov Goodness of Fit Test

The Kolmogorov-Smirnov (K-S) test is based on the empirical distribution function (ECDF). Given N ordered data points Y_1, Y_2, \dots, Y_N , the ECDF is defined as

$$EN = n(i)/N \quad (1)$$

Where $n(i)$ = the number of points is less than Y_i and the Y_i are ordered from smallest to largest value. This is a step function that increases by $1/N$ at the value of each ordered data point.

The Kolmogorov-Smirnov statistic for a given cumulative distribution function $F(x)$ is

$$D_n = \text{Sup}_x |F_n(x) - F(x)| \quad (2)$$

Where;

Sup_x = the supremum of the set of distances.
 $F_n(x)$ = the cdf of the hypothesized distribution.
 $F(x)$ = the empirical distribution function of the observed

By the Glivenko-Cantelli theorem, if the sample comes from distribution $F(x)$, then D_n converges to 0 almost surely in the limit when goes to infinity. For one-tailed test, omit the absolute values from the formula. If D_n is greater than the critical value, the null hypothesis is rejected. Critical values for D_n are found in the K-S Test P-

Value Table. The Kolmogorov–Smirnov test can be adapted for discrete variables. The form of the test statistic remains the same as in the continuous case, but the calculation of its value is more subtle.

While the Kolmogorov–Smirnov test is usually used to test whether a given $F(x)$, is the underlying probability distribution of $F_n(x)$, the procedure may be inverted to give confidence limits on $F(x)$, itself. If one chooses a critical value of the test statistic D_α such that $P(D_n > D_\alpha) = \alpha$, then a band of width $\pm D_\alpha$ around $F_n(x)$ will entirely contain $F(x)$ with probability $1 - \alpha$.

When the p-value is greater than ($\alpha = 0.05$), we accept the null hypothesis and when it is less than we reject the null hypothesis and draw conclusion.

7.2 Chi-Square Goodness of Fit Test

$$\chi^2 = \sum_{i=1}^n \left[\frac{(O_i - E_i)^2}{E_i} \right] \quad (3)$$

Where,

- O_i = observed frequency
- E_i = expected frequency
- N = the sample size.

7.3 Normal Distribution

The probability density function of the normal distribution is given by:

$$f(x, \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x - \mu)/2\sigma^2} \quad (4)$$

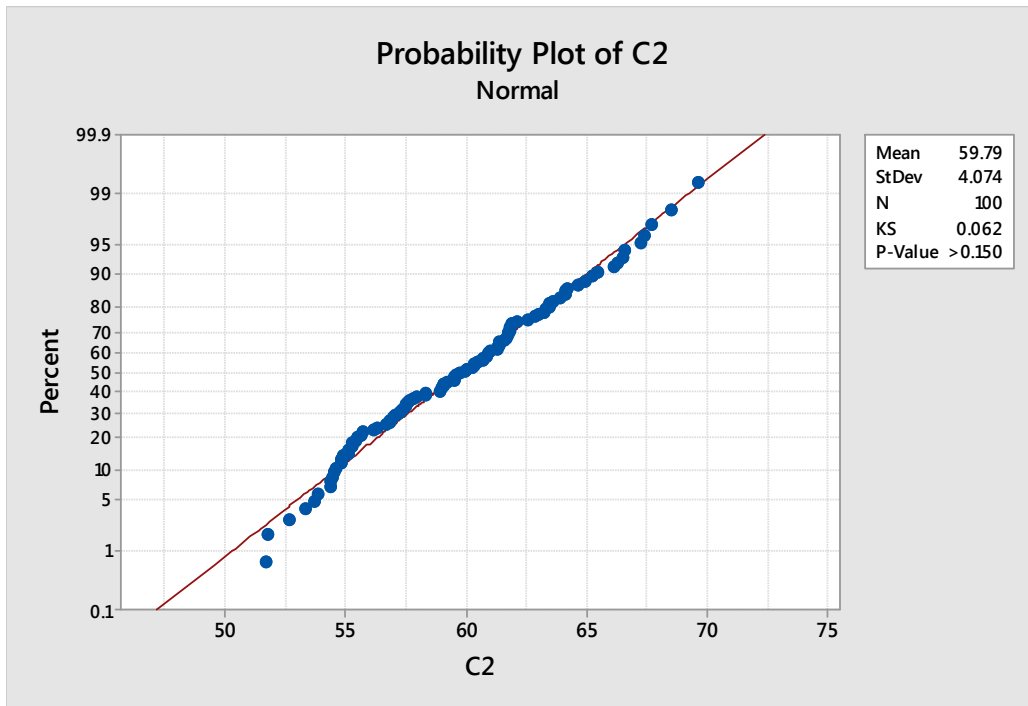
Where, μ = the mean or expectation of the distribution

- σ = is the standard deviation
- σ^2 = is the variance.

This applies single variables to continuous data, e.g. heights of plants, length of time etc. to calculate the value of the "standard score" or "z-score"

$$Z = \frac{x - \mu}{\sigma}$$

7.4 The Kolmogorov-Smirnov Test (100 Sample Sizes)



Decision Rule: Since the p-value is greater than 0.05, which is above our threshold of 5% (i.e. $0.150 \geq 0.05$) we accept the null hypothesis (H_0). Hence, we say that the distribution is normal (according to Kolmogorov-Smirnov test).

C1, C2,...., C15 are simulated data set used in the study to demonstrate the comparison of the two goodness of fit tests.

7.5 CHI-SQUARE GOODNESS OF FIT TEST FOR (100 sample sizes)

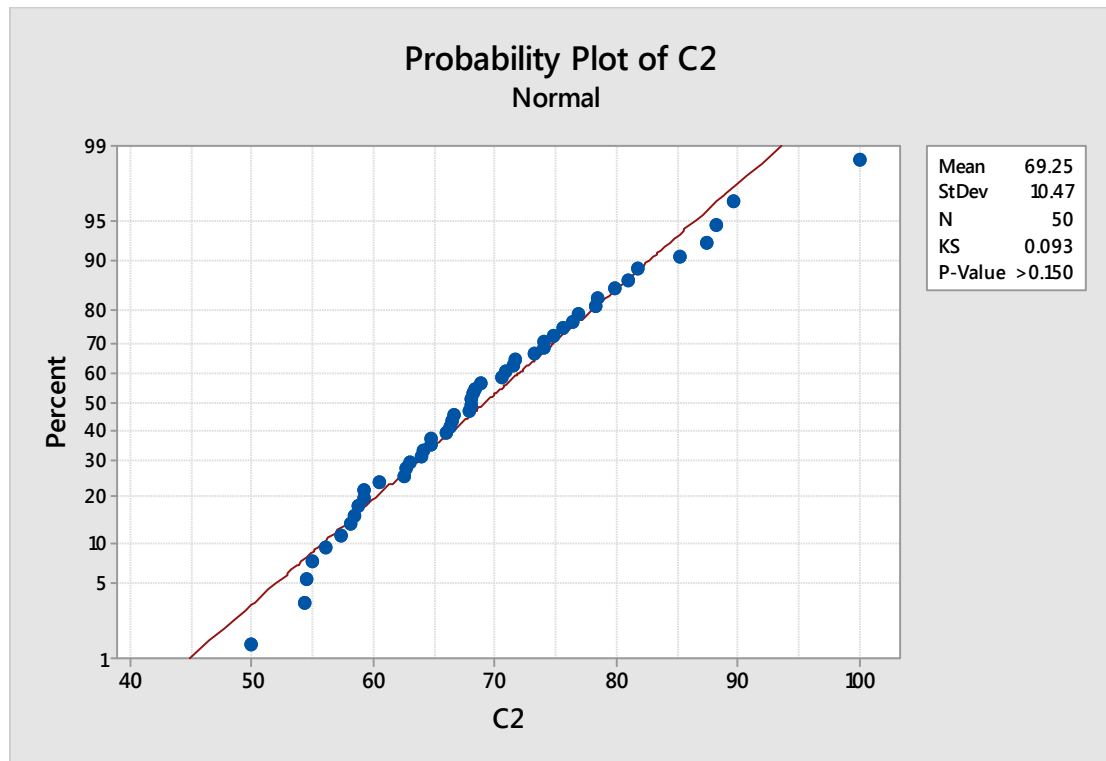
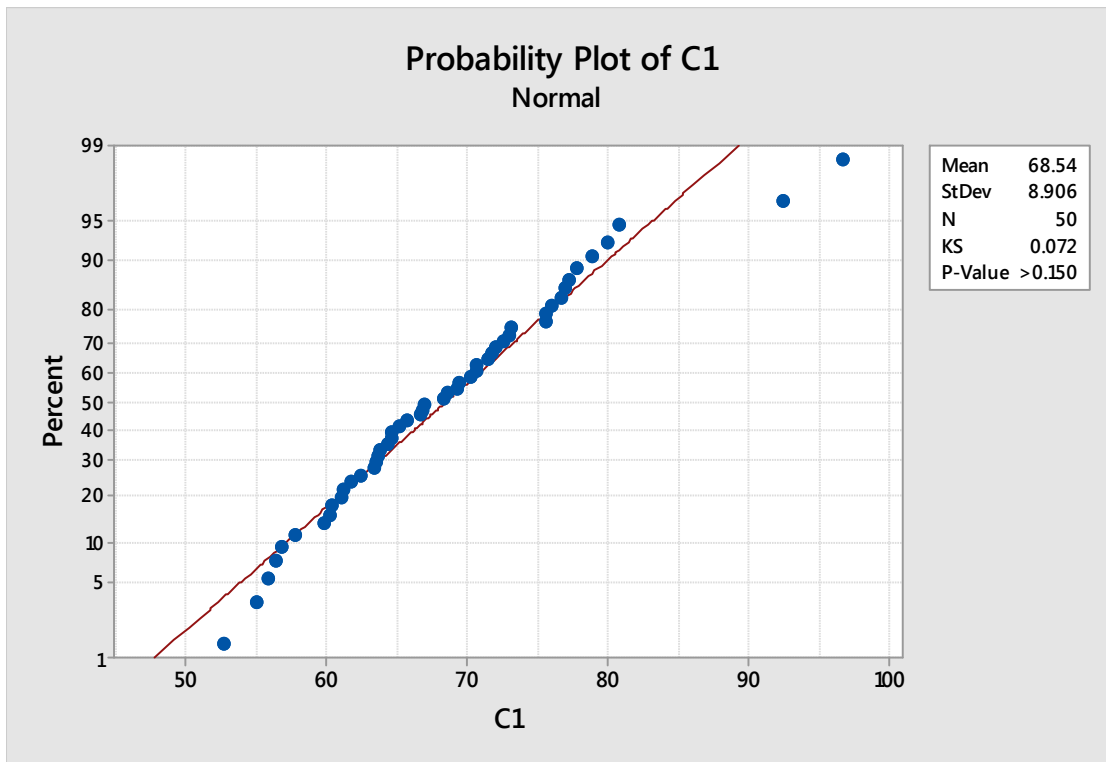
Category	Test		Contribution	
	Observed	Proportion	Expected	to Chi-Sq
1	59.4930	0.01	59.7851	0.00143
2	62.8864	0.01	59.7851	0.16088
3	55.6405	0.01	59.7851	0.28733
4	57.5830	0.01	59.7851	0.08112
5	67.7264	0.01	59.7851	1.05485
6	59.4992	0.01	59.7851	0.00137
7	54.9360	0.01	59.7851	0.39331
8	66.2719	0.01	59.7851	0.70382
9	58.9497	0.01	59.7851	0.01168
10	60.3593	0.01	59.7851	0.00551
11	64.2017	0.01	59.7851	0.32626
12	59.5158	0.01	59.7851	0.00121
13	57.0593	0.01	59.7851	0.12428
14	68.5422	0.01	59.7851	1.28269
15	60.0724	0.01	59.7851	0.00138
16	66.1049	0.01	59.7851	0.66804
17	55.2737	0.01	59.7851	0.34044
18	57.3175	0.01	59.7851	0.10185
19	53.3554	0.01	59.7851	0.69151
20	64.6407	0.01	59.7851	0.39435
21	63.6383	0.01	59.7851	0.24834
22	66.5680	0.01	59.7851	0.76955
23	59.7570	0.01	59.7851	0.00001
24	57.6484	0.01	59.7851	0.07637
25	63.4828	0.01	59.7851	0.22869
26	61.7392	0.01	59.7851	0.06387
27	59.5595	0.01	59.7851	0.00085
28	58.3292	0.01	59.7851	0.03546
29	58.9980	0.01	59.7851	0.01036
30	57.2427	0.01	59.7851	0.10812
31	61.3691	0.01	59.7851	0.04197
32	55.7004	0.01	59.7851	0.27909
33	63.2126	0.01	59.7851	0.19650
34	57.5454	0.01	59.7851	0.08391
35	57.3754	0.01	59.7851	0.09712
36	59.0969	0.01	59.7851	0.00792
37	65.4284	0.01	59.7851	0.53268
38	54.6226	0.01	59.7851	0.44579
39	57.5061	0.01	59.7851	0.08688
40	64.0901	0.01	59.7851	0.30999
41	60.7061	0.01	59.7851	0.01419
42	55.1579	0.01	59.7851	0.35813
43	60.3034	0.01	59.7851	0.00449
44	54.5095	0.01	59.7851	0.46555
45	61.7328	0.01	59.7851	0.06345
46	61.9034	0.01	59.7851	0.07506
47	58.3178	0.01	59.7851	0.03601
48	63.4352	0.01	59.7851	0.22284
49	51.6858	0.01	59.7851	1.09724
50	57.9216	0.01	59.7851	0.05808

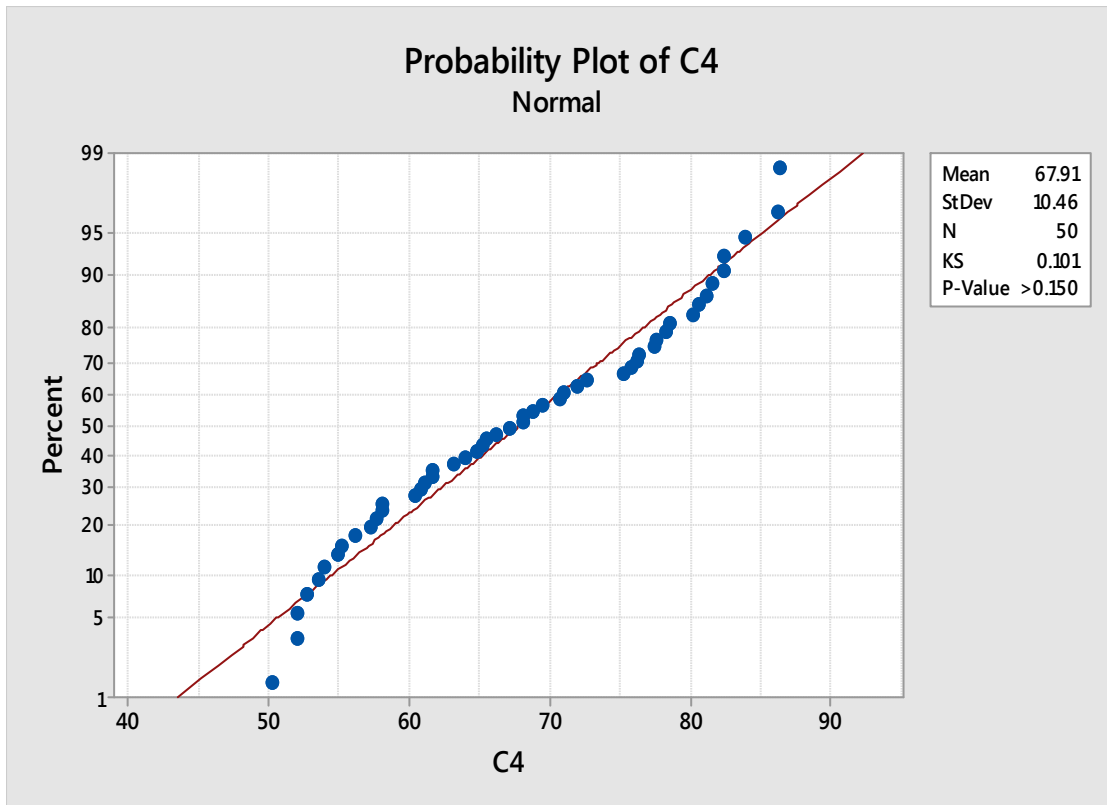
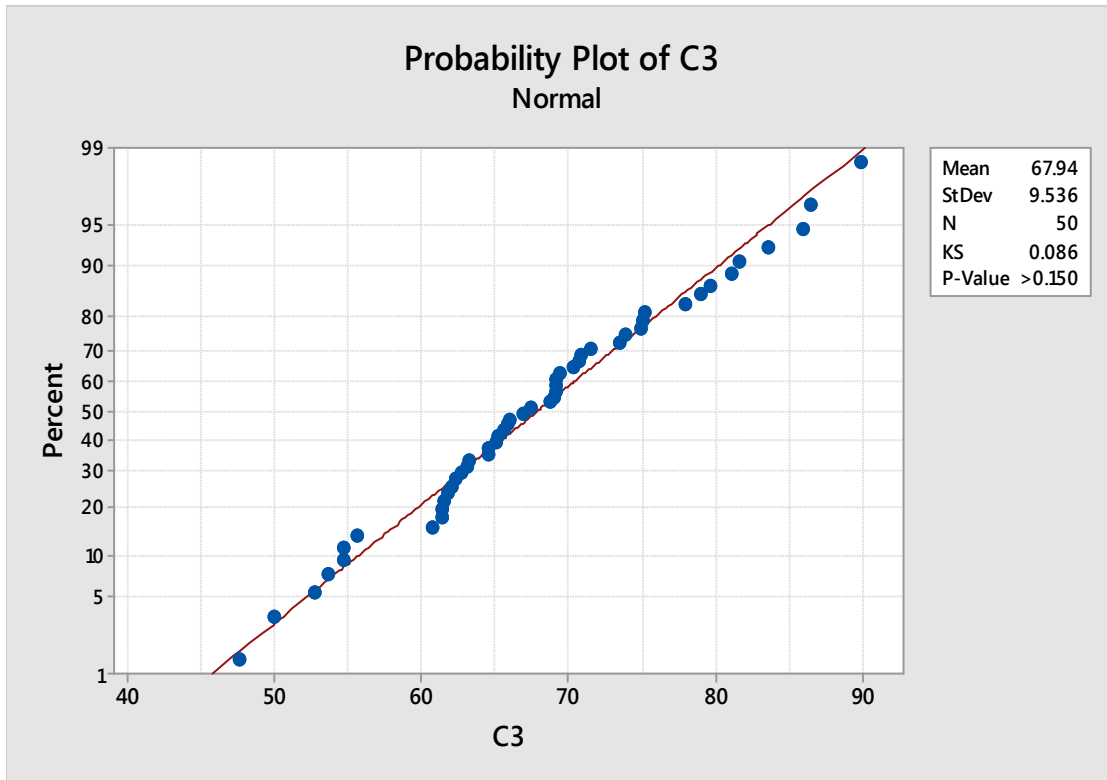
51	64.1005	0.01	59.7851	0.31148
52	51.7562	0.01	59.7851	1.07826
53	61.0102	0.01	59.7851	0.02510
54	59.9390	0.01	59.7851	0.00040
55	61.3549	0.01	59.7851	0.04121
56	54.3843	0.01	59.7851	0.48789
57	61.6892	0.01	59.7851	0.06064
58	54.4691	0.01	59.7851	0.47269
59	67.4029	0.01	59.7851	0.97064
60	59.2042	0.01	59.7851	0.00564
61	61.2783	0.01	59.7851	0.03729
62	56.1507	0.01	59.7851	0.22094
63	52.6425	0.01	59.7851	0.85335
64	60.8505	0.01	59.7851	0.01898
65	55.2669	0.01	59.7851	0.34146
66	60.7334	0.01	59.7851	0.01504
67	61.8386	0.01	59.7851	0.07053
68	60.4728	0.01	59.7851	0.00791
69	55.5273	0.01	59.7851	0.30323
70	53.8824	0.01	59.7851	0.58280
71	65.2531	0.01	59.7851	0.50010
72	64.9194	0.01	59.7851	0.44092
73	61.3720	0.01	59.7851	0.04212
74	54.8447	0.01	59.7851	0.40827
75	62.5947	0.01	59.7851	0.13203
76	63.9012	0.01	59.7851	0.28338
77	60.9116	0.01	59.7851	0.02122
78	59.0900	0.01	59.7851	0.00808
79	61.2680	0.01	59.7851	0.03678
80	61.8430	0.01	59.7851	0.07083
81	63.0178	0.01	59.7851	0.17480
82	56.3016	0.01	59.7851	0.20298
83	69.6645	0.01	59.7851	1.63255
84	61.6288	0.01	59.7851	0.05686
85	57.8346	0.01	59.7851	0.06363
86	54.8108	0.01	59.7851	0.41389
87	56.8491	0.01	59.7851	0.14419
88	63.2730	0.01	59.7851	0.20348
89	56.8695	0.01	59.7851	0.14219
90	55.1001	0.01	59.7851	0.36714
91	60.2948	0.01	59.7851	0.00434
92	62.1054	0.01	59.7851	0.09005
93	56.9528	0.01	59.7851	0.13418
94	54.4178	0.01	59.7851	0.48187
95	55.4058	0.01	59.7851	0.32079
96	66.5318	0.01	59.7851	0.76134
97	61.8379	0.01	59.7851	0.07048
98	56.7078	0.01	59.7851	0.15840
99	53.6795	0.01	59.7851	0.62355
100	67.2664	0.01	59.7851	0.93618

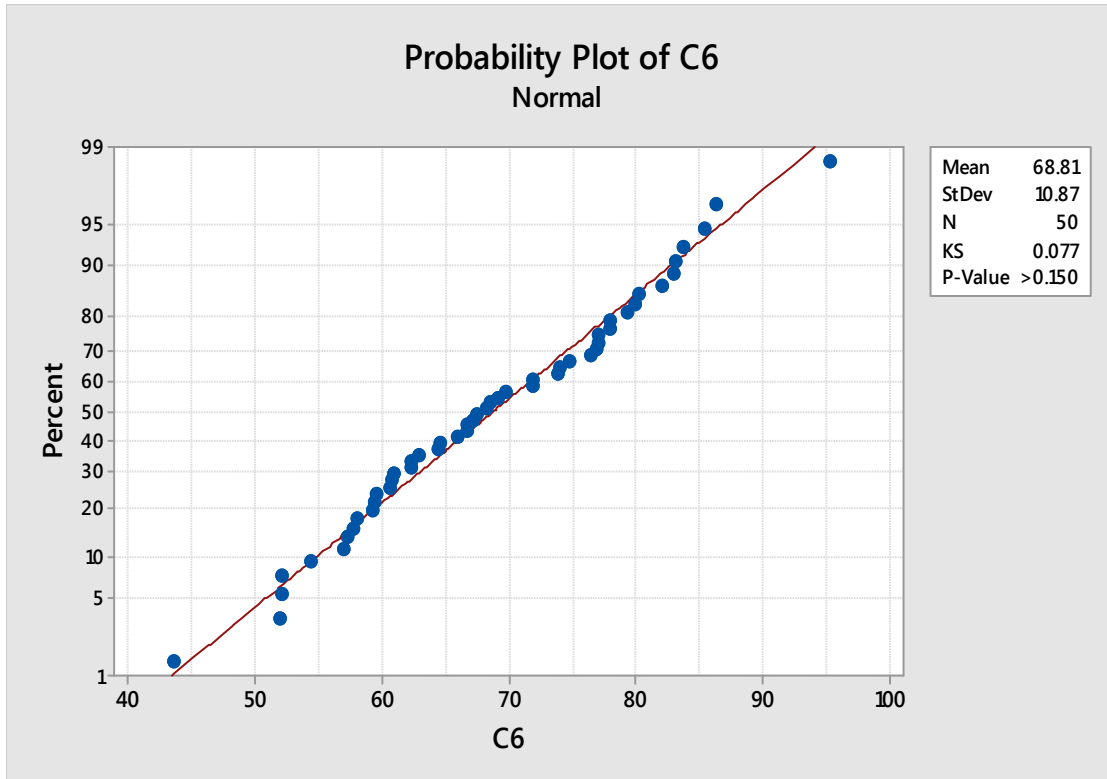
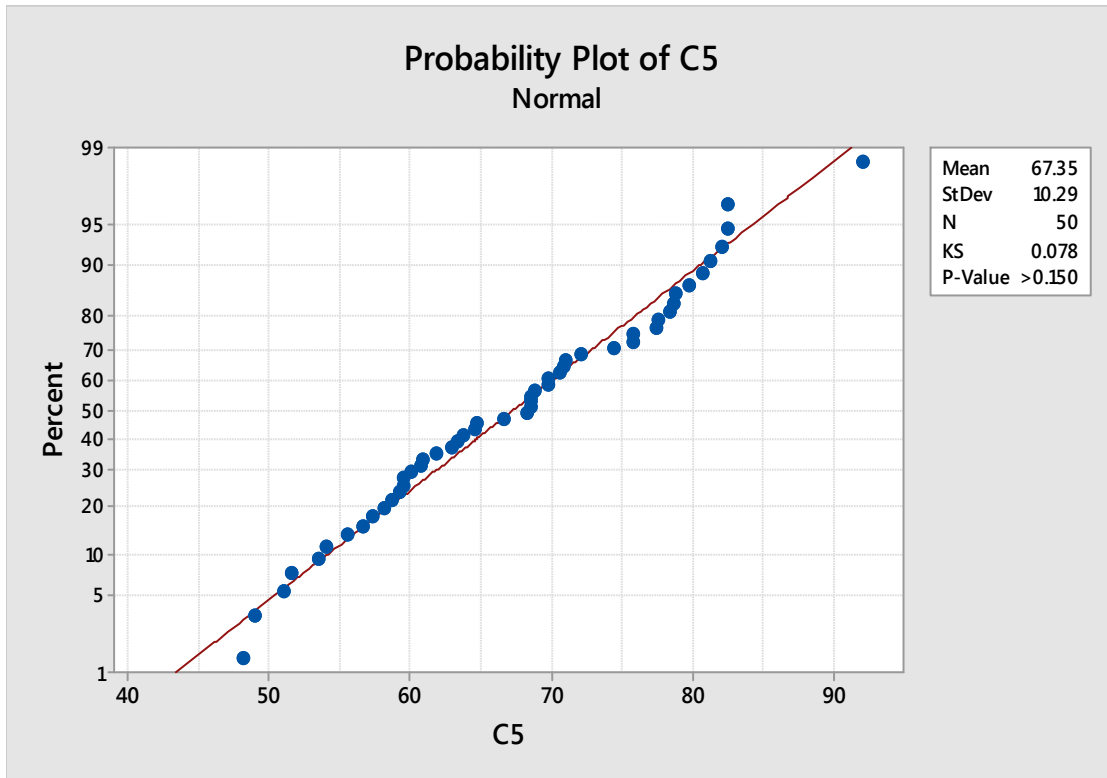
N	DF	Chi-Sq	P-Value
5978.51	99	27.4839	1.000

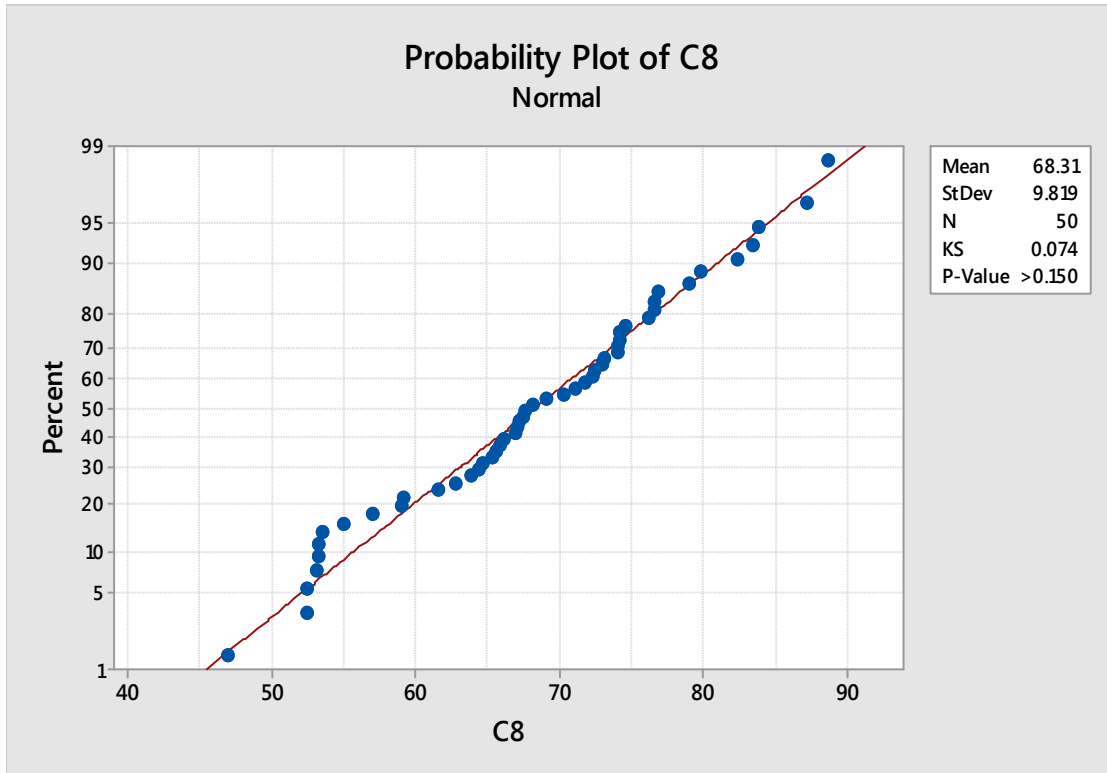
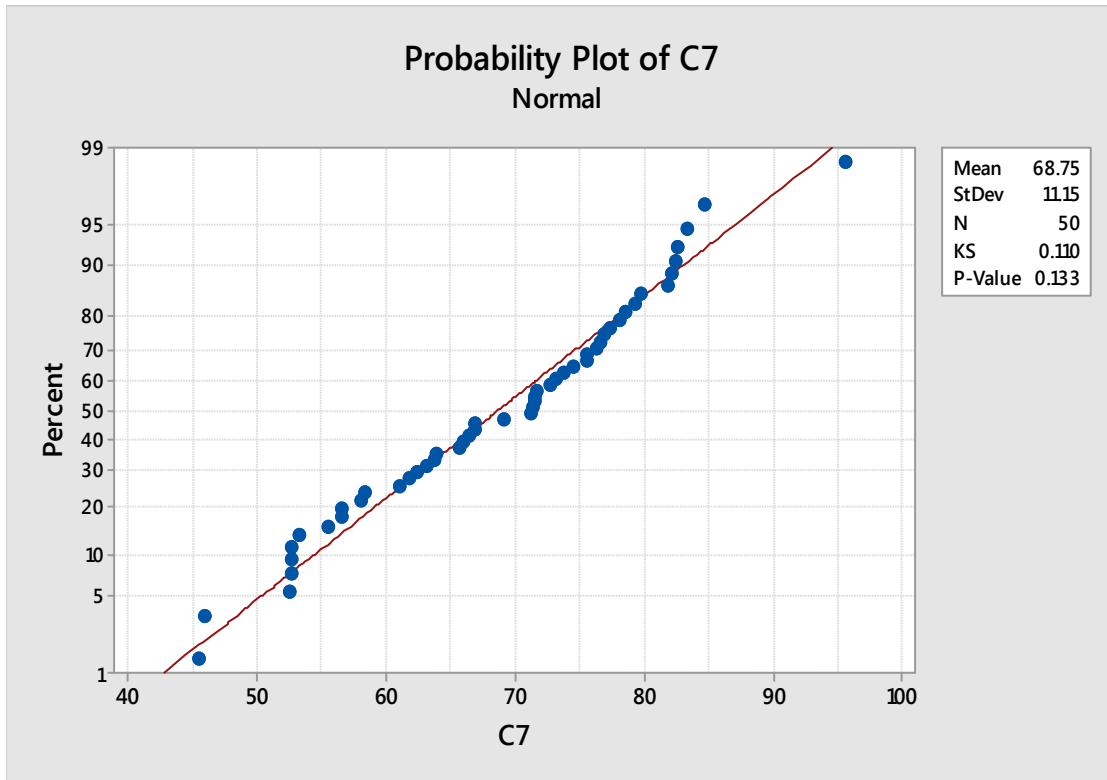
Decision Rule: Since the p-value is 1.000 and is greater than 0.05 which is above threshold of 5% (i.e. $0.003 \geq 0.05$) we accept the null hypothesis (H_0). Hence, we say that the distribution is normal (according to Chi-square goodness of fit test).

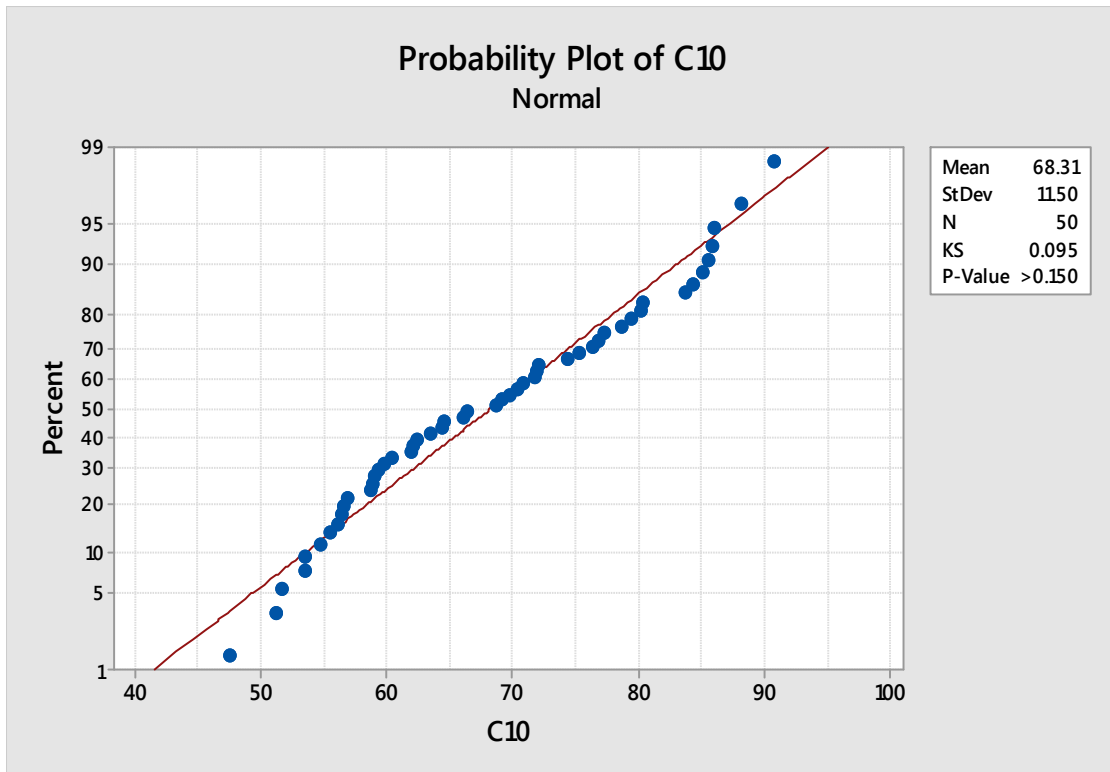
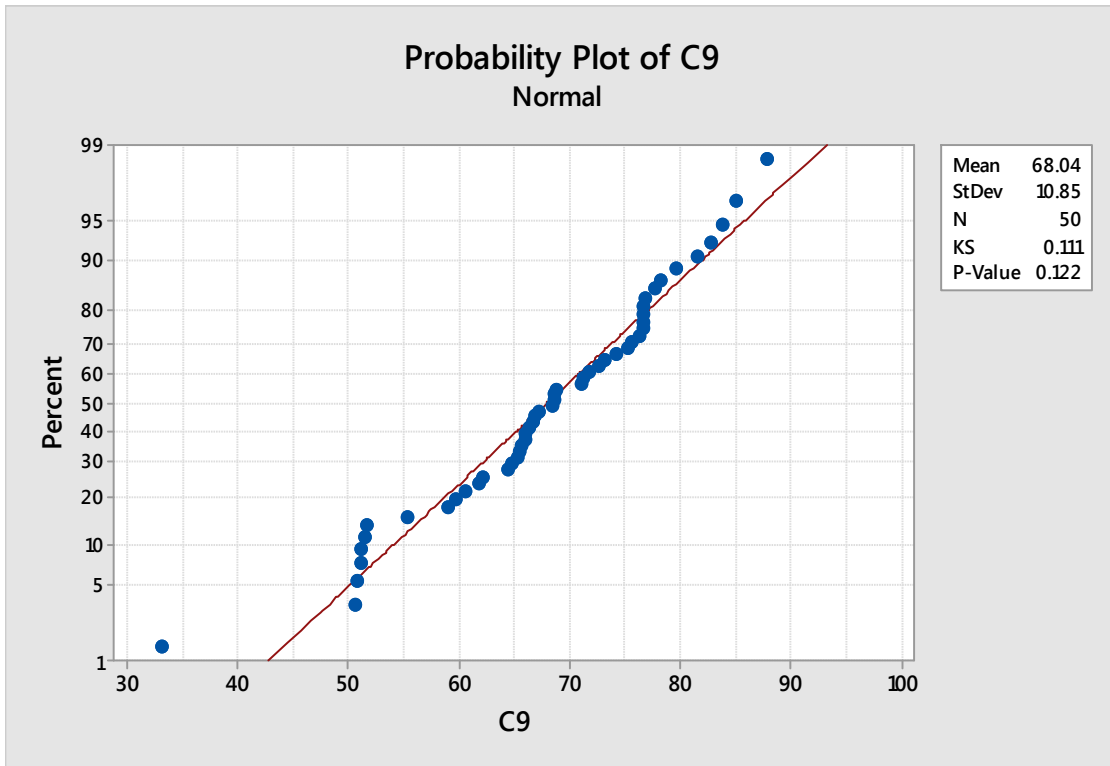
7.6 The Kolmogorov-Smirnov Test (Simulated 15 Data Set)

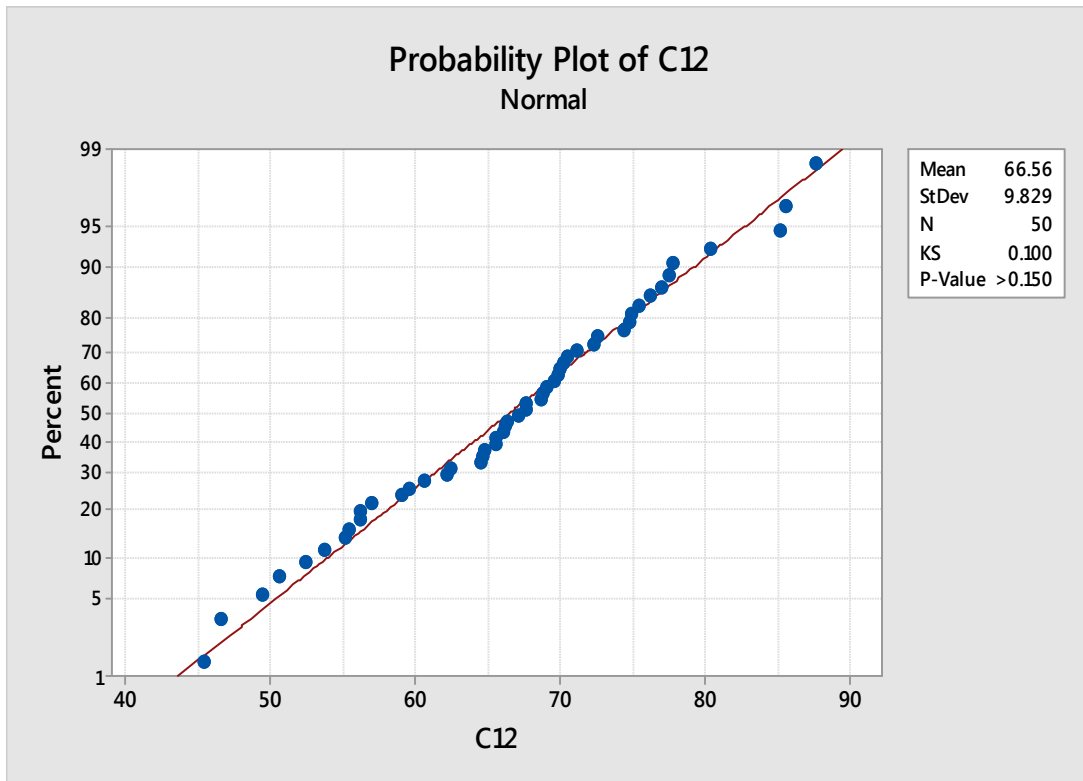
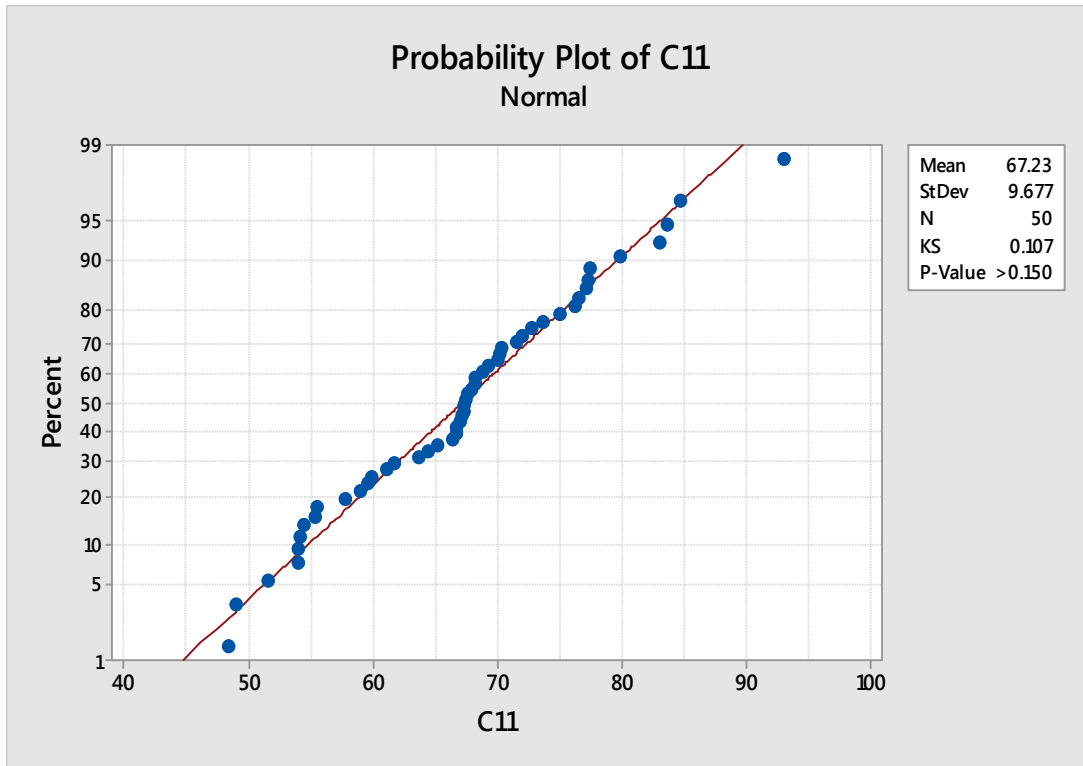


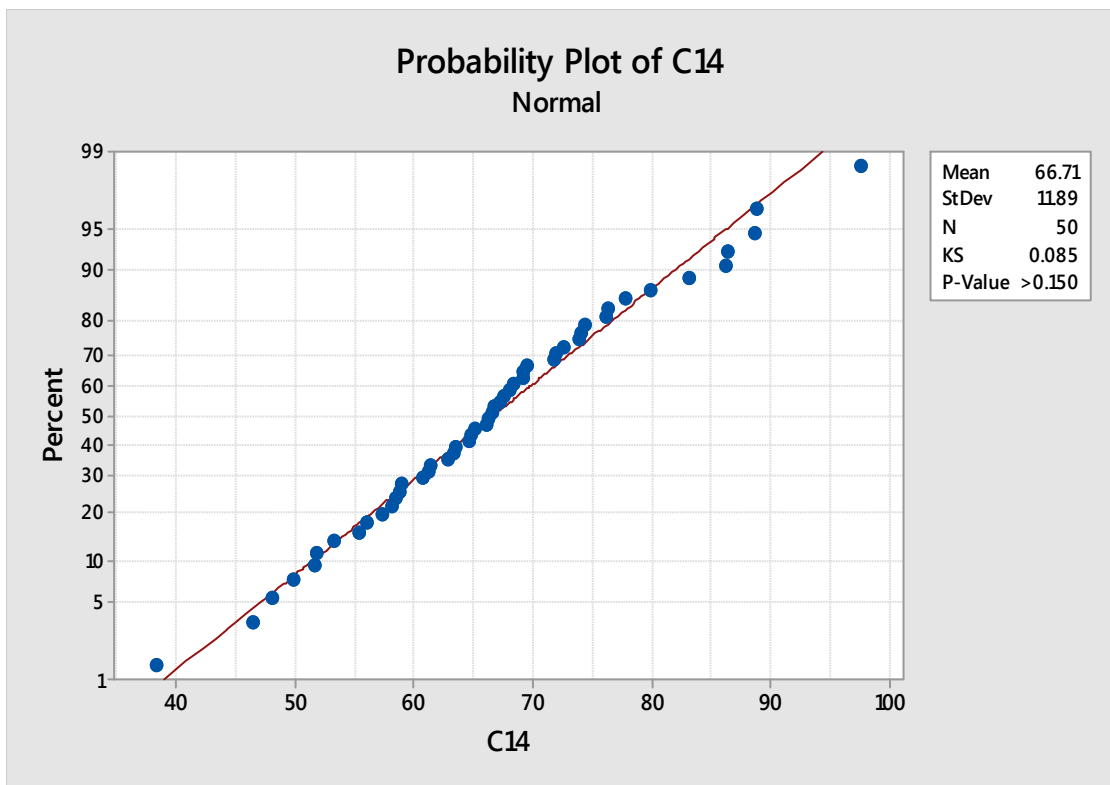
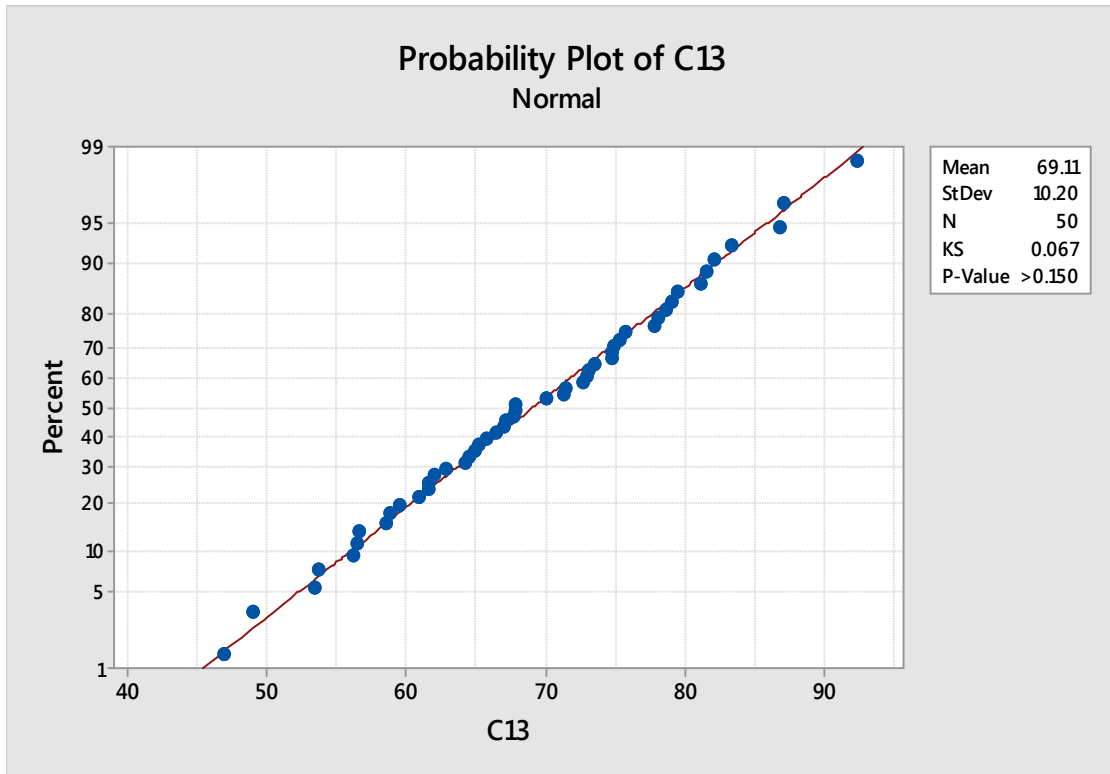


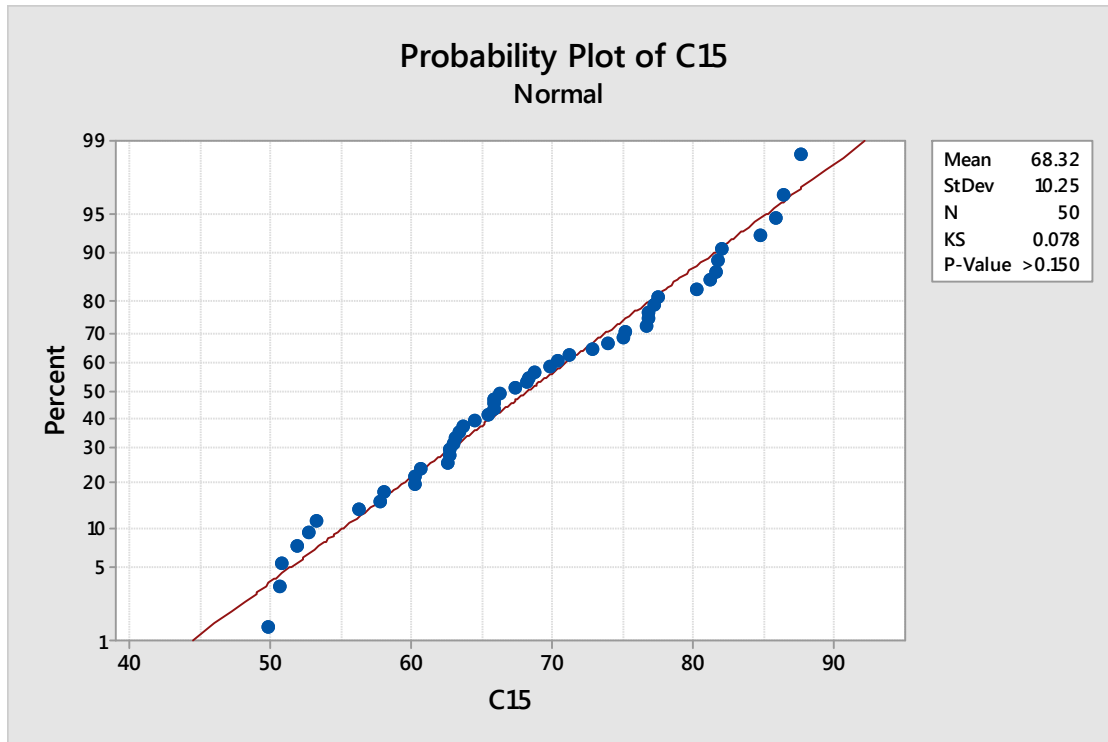












CHI-SQUARE GOODNESS OF FIT TEST (15 DATA SET)

CHI-SQUARE GOODNESS-OF-FIT TEST FOR C1

Category	Test Observed	Test Proportion	Contribution Expected	Contribution to Chi-Sq
1	65.1546	0.02	68.5389	0.1671
2	75.6317	0.02	68.5389	0.7340
3	61.0160	0.02	68.5389	0.8257
4	80.7941	0.02	68.5389	2.1913
5	69.4623	0.02	68.5389	0.0124
6	66.7966	0.02	68.5389	0.0443
7	79.9395	0.02	68.5389	1.8963
8	77.2697	0.02	68.5389	1.1122
9	75.9614	0.02	68.5389	0.8038
10	71.5181	0.02	68.5389	0.1295
11	72.6195	0.02	68.5389	0.2430
12	75.5633	0.02	68.5389	0.7199
13	77.7378	0.02	68.5389	1.2346
14	64.6661	0.02	68.5389	0.2188
15	68.3449	0.02	68.5389	0.0005
16	52.7234	0.02	68.5389	3.6494
17	60.2750	0.02	68.5389	0.9964
18	62.4100	0.02	68.5389	0.5481
19	55.0689	0.02	68.5389	2.6473
20	72.9448	0.02	68.5389	0.2832
21	65.6683	0.02	68.5389	0.1202
22	73.1260	0.02	68.5389	0.3070
23	56.3987	0.02	68.5389	2.1504
24	61.2097	0.02	68.5389	0.7837

25	76.9885	0.02	68.5389	1.0417
26	71.7900	0.02	68.5389	0.1542
27	63.4845	0.02	68.5389	0.3727
28	63.6969	0.02	68.5389	0.3421
29	96.7918	0.02	68.5389	11.6463
30	57.8415	0.02	68.5389	1.6696
31	72.0244	0.02	68.5389	0.1773
32	66.6847	0.02	68.5389	0.0502
33	78.9152	0.02	68.5389	1.5709
34	63.3895	0.02	68.5389	0.3869
35	64.6001	0.02	68.5389	0.2264
36	59.8641	0.02	68.5389	1.0979
37	70.6655	0.02	68.5389	0.0660
38	64.3156	0.02	68.5389	0.2602
39	60.3518	0.02	68.5389	0.9780
40	68.5568	0.02	68.5389	0.0000
41	92.5052	0.02	68.5389	8.3804
42	76.6716	0.02	68.5389	0.9650
43	69.3454	0.02	68.5389	0.0095
44	70.2106	0.02	68.5389	0.0408
45	66.9196	0.02	68.5389	0.0383
46	70.7146	0.02	68.5389	0.0691
47	63.8778	0.02	68.5389	0.3170
48	55.8182	0.02	68.5389	2.3609
49	56.8241	0.02	68.5389	2.0023
50	61.7974	0.02	68.5389	0.6631

N DF Chi-Sq P-Value
 3426.95 49 56.7060 0.210

CHI-SQUARE GOODNESS-OF-FIT TEST FOR C2

Category	Test		Contribution to Chi-Sq	
	Observed	Proportion Expected		
1	54.979	0.02	69.2502	2.9409
2	60.420	0.02	69.2502	1.1260
3	74.038	0.02	69.2502	0.3311
4	88.171	0.02	69.2502	5.1697
5	87.458	0.02	69.2502	4.7875
6	71.761	0.02	69.2502	0.0911
7	81.027	0.02	69.2502	2.0027
8	81.754	0.02	69.2502	2.2575
9	73.315	0.02	69.2502	0.2386
10	76.371	0.02	69.2502	0.7322
11	56.026	0.02	69.2502	2.5253
12	74.843	0.02	69.2502	0.4517
13	68.253	0.02	69.2502	0.0144
14	68.390	0.02	69.2502	0.0107
15	62.470	0.02	69.2502	0.6639
16	68.903	0.02	69.2502	0.0017
17	64.198	0.02	69.2502	0.3686
18	89.721	0.02	69.2502	6.0514
19	76.834	0.02	69.2502	0.8304
20	67.852	0.02	69.2502	0.0282
21	68.119	0.02	69.2502	0.0185
22	64.802	0.02	69.2502	0.2857
23	58.373	0.02	69.2502	1.7085

24	71.550	0.02	69.2502	0.0764
25	57.395	0.02	69.2502	2.0296
26	66.550	0.02	69.2502	0.1053
27	74.044	0.02	69.2502	0.3318
28	64.744	0.02	69.2502	0.2932
29	54.433	0.02	69.2502	3.1702
30	58.199	0.02	69.2502	1.7637
31	54.346	0.02	69.2502	3.2077
32	58.759	0.02	69.2502	1.5893
33	62.967	0.02	69.2502	0.5702
34	65.952	0.02	69.2502	0.1571
35	100.072	0.02	69.2502	13.7183
36	59.294	0.02	69.2502	1.4315
37	70.619	0.02	69.2502	0.0270
38	68.095	0.02	69.2502	0.0193
39	75.558	0.02	69.2502	0.5746
40	78.242	0.02	69.2502	1.1676
41	59.246	0.02	69.2502	1.4452
42	78.540	0.02	69.2502	1.2461
43	85.303	0.02	69.2502	3.7212
44	66.405	0.02	69.2502	0.1169
45	66.646	0.02	69.2502	0.0979
46	70.967	0.02	69.2502	0.0425
47	49.964	0.02	69.2502	5.3714
48	79.855	0.02	69.2502	1.6239
49	64.044	0.02	69.2502	0.3915
50	62.647	0.02	69.2502	0.6297

N DF Chi-Sq P-Value
 3462.51 49 77.5555 0.006

CHI-SQUARE GOODNESS-OF-FIT TEST FOR C3

Category	Test Observed	Contribution Proportion	Expected	to Chi-Sq
1	70.8227	0.02	67.9388	0.12242
2	71.5189	0.02	67.9388	0.18866
3	77.9724	0.02	67.9388	1.48184
4	73.8638	0.02	67.9388	0.51672
5	63.1454	0.02	67.9388	0.33819
6	69.4455	0.02	67.9388	0.03342
7	75.1361	0.02	67.9388	0.76248
8	70.7481	0.02	67.9388	0.11617
9	79.6661	0.02	67.9388	2.02434
10	50.0676	0.02	67.9388	4.70099
11	64.5797	0.02	67.9388	0.16608
12	62.3211	0.02	67.9388	0.46452
13	83.6186	0.02	67.9388	3.61882
14	69.1255	0.02	67.9388	0.02073
15	65.9991	0.02	67.9388	0.05538
16	68.9681	0.02	67.9388	0.01560
17	54.7138	0.02	67.9388	2.57437
18	65.2195	0.02	67.9388	0.10884
19	55.6536	0.02	67.9388	2.22147
20	70.2982	0.02	67.9388	0.08194
21	89.9244	0.02	67.9388	7.11478
22	47.6367	0.02	67.9388	6.06684

23	61.7723	0.02	67.9388	0.55971
24	63.3138	0.02	67.9388	0.31485
25	64.5781	0.02	67.9388	0.16624
26	52.7015	0.02	67.9388	3.41739
27	65.6691	0.02	67.9388	0.07582
28	65.0820	0.02	67.9388	0.12012
29	62.6935	0.02	67.9388	0.40497
30	66.8818	0.02	67.9388	0.01644
31	81.1064	0.02	67.9388	2.55212
32	62.0698	0.02	67.9388	0.50700
33	61.3664	0.02	67.9388	0.63580
34	69.1278	0.02	67.9388	0.02081
35	53.6567	0.02	67.9388	3.00238
36	68.7329	0.02	67.9388	0.00928
37	60.7577	0.02	67.9388	0.75903
38	61.5912	0.02	67.9388	0.59306
39	65.8544	0.02	67.9388	0.06395
40	75.1262	0.02	67.9388	0.76038
41	81.6331	0.02	67.9388	2.76036
42	61.4130	0.02	67.9388	0.62682
43	69.1910	0.02	67.9388	0.02308
44	79.0068	0.02	67.9388	1.80313
45	73.4903	0.02	67.9388	0.45363
46	85.9510	0.02	67.9388	4.77547
47	67.4218	0.02	67.9388	0.00393
48	54.7935	0.02	67.9388	2.54345
49	86.5256	0.02	67.9388	5.08504
50	74.9855	0.02	67.9388	0.73090

N DF Chi-Sq P-Value
 3396.94 49 65.5798 0.057

CHI-SQUARE GOODNESS-OF-FIT TEST FOR C4

Category	Test		Contribution	
	Observed	Proportion	Expected	to Chi-Sq
1	81.5681	0.02	67.9121	2.74598
2	55.1698	0.02	67.9121	2.39082
3	58.0913	0.02	67.9121	1.42020
4	77.6479	0.02	67.9121	1.39571
5	72.7100	0.02	67.9121	0.33897
6	65.4607	0.02	67.9121	0.08849
7	56.2104	0.02	67.9121	2.01628
8	78.3371	0.02	67.9121	1.60030
9	60.8741	0.02	67.9121	0.72939
10	50.2737	0.02	67.9121	4.58113
11	86.2262	0.02	67.9121	4.93881
12	66.1375	0.02	67.9121	0.04637
13	64.8289	0.02	67.9121	0.13997
14	52.1107	0.02	67.9121	3.67658
15	60.4853	0.02	67.9121	0.81220
16	70.7940	0.02	67.9121	0.12230
17	54.0296	0.02	67.9121	2.83786
18	72.0210	0.02	67.9121	0.24860
19	53.5729	0.02	67.9121	3.02762
20	80.5690	0.02	67.9121	2.35890
21	63.1328	0.02	67.9121	0.33634

22	76.3505	0.02	67.9121	1.04850
23	80.1919	0.02	67.9121	2.22041
24	82.3596	0.02	67.9121	3.07351
25	68.1848	0.02	67.9121	0.00110
26	82.3855	0.02	67.9121	3.08454
27	77.4198	0.02	67.9121	1.33106
28	52.7840	0.02	67.9121	3.36995
29	61.0889	0.02	67.9121	0.68554
30	68.8381	0.02	67.9121	0.01263
31	69.5618	0.02	67.9121	0.04007
32	54.9396	0.02	67.9121	2.47801
33	83.8854	0.02	67.9121	3.75698
34	76.2092	0.02	67.9121	1.01370
35	78.6241	0.02	67.9121	1.68963
36	75.3292	0.02	67.9121	0.81006
37	63.9439	0.02	67.9121	0.23187
38	57.6663	0.02	67.9121	1.54579
39	57.2451	0.02	67.9121	1.67548
40	65.2012	0.02	67.9121	0.10821
41	61.7121	0.02	67.9121	0.56603
42	52.0600	0.02	67.9121	3.70023
43	61.6849	0.02	67.9121	0.57101
44	71.0712	0.02	67.9121	0.14696
45	58.0540	0.02	67.9121	1.43102
46	67.1626	0.02	67.9121	0.00827
47	86.3399	0.02	67.9121	5.00031
48	81.2195	0.02	67.9121	2.60756
49	75.7672	0.02	67.9121	0.90855
50	68.0749	0.02	67.9121	0.00039

N DF Chi-Sq P-Value
 3395.61 49 78.9702 0.004

CHI-SQUARE GOODNESS-OF-FIT TEST FOR C4

Category	Test Observed	Contribution Proportion	Expected	to Chi-Sq
1	81.5681	0.02	67.9121	2.74598
2	55.1698	0.02	67.9121	2.39082
3	58.0913	0.02	67.9121	1.42020
4	77.6479	0.02	67.9121	1.39571
5	72.7100	0.02	67.9121	0.33897
6	65.4607	0.02	67.9121	0.08849
7	56.2104	0.02	67.9121	2.01628
8	78.3371	0.02	67.9121	1.60030
9	60.8741	0.02	67.9121	0.72939
10	50.2737	0.02	67.9121	4.58113
11	86.2262	0.02	67.9121	4.93881
12	66.1375	0.02	67.9121	0.04637
13	64.8289	0.02	67.9121	0.13997
14	52.1107	0.02	67.9121	3.67658
15	60.4853	0.02	67.9121	0.81220
16	70.7940	0.02	67.9121	0.12230
17	54.0296	0.02	67.9121	2.83786
18	72.0210	0.02	67.9121	0.24860
19	53.5729	0.02	67.9121	3.02762
20	80.5690	0.02	67.9121	2.35890

21	63.1328	0.02	67.9121	0.33634
22	76.3505	0.02	67.9121	1.04850
23	80.1919	0.02	67.9121	2.22041
24	82.3596	0.02	67.9121	3.07351
25	68.1848	0.02	67.9121	0.00110
26	82.3855	0.02	67.9121	3.08454
27	77.4198	0.02	67.9121	1.33106
28	52.7840	0.02	67.9121	3.36995
29	61.0889	0.02	67.9121	0.68554
30	68.8381	0.02	67.9121	0.01263
31	69.5618	0.02	67.9121	0.04007
32	54.9396	0.02	67.9121	2.47801
33	83.8854	0.02	67.9121	3.75698
34	76.2092	0.02	67.9121	1.01370
35	78.6241	0.02	67.9121	1.68963
36	75.3292	0.02	67.9121	0.81006
37	63.9439	0.02	67.9121	0.23187
38	57.6663	0.02	67.9121	1.54579
39	57.2451	0.02	67.9121	1.67548
40	65.2012	0.02	67.9121	0.10821
41	61.7121	0.02	67.9121	0.56603
42	52.0600	0.02	67.9121	3.70023
43	61.6849	0.02	67.9121	0.57101
44	71.0712	0.02	67.9121	0.14696
45	58.0540	0.02	67.9121	1.43102
46	67.1626	0.02	67.9121	0.00827
47	86.3399	0.02	67.9121	5.00031
48	81.2195	0.02	67.9121	2.60756
49	75.7672	0.02	67.9121	0.90855
50	68.0749	0.02	67.9121	0.00039

N DF Chi-Sq P-Value
 3395.61 49 78.9702 0.004

CHI-SQUARE GOODNESS-OF-FIT TEST FOR C5

Category	Test Observed	Contribution Proportion	Expected	to Chi-Sq
1	80.7548	0.02	67.3542	2.66615
2	92.0513	0.02	67.3542	9.05578
3	82.0900	0.02	67.3542	3.22391
4	59.5873	0.02	67.3542	0.89564
5	62.9956	0.02	67.3542	0.28206
6	68.8397	0.02	67.3542	0.03276
7	69.8471	0.02	67.3542	0.09227
8	82.4777	0.02	67.3542	3.39579
9	78.6699	0.02	67.3542	1.90105
10	60.9375	0.02	67.3542	0.61131
11	59.5997	0.02	67.3542	0.89278
12	68.6101	0.02	67.3542	0.02342
13	81.3533	0.02	67.3542	2.90960
14	61.9140	0.02	67.3542	0.43941
15	77.4902	0.02	67.3542	1.52535
16	54.0418	0.02	67.3542	2.63118
17	51.0629	0.02	67.3542	3.94048
18	59.2114	0.02	67.3542	0.98442
19	70.9773	0.02	67.3542	0.19489

20	60.8392	0.02	67.3542	0.63019
21	58.7667	0.02	67.3542	1.09489
22	58.1324	0.02	67.3542	1.26262
23	57.4031	0.02	67.3542	1.47022
24	60.0559	0.02	67.3542	0.79082
25	70.6908	0.02	67.3542	0.16529
26	51.6022	0.02	67.3542	3.68392
27	68.5520	0.02	67.3542	0.02130
28	49.0712	0.02	67.3542	4.96286
29	56.7361	0.02	67.3542	1.67390
30	69.7586	0.02	67.3542	0.08583
31	64.5950	0.02	67.3542	0.11303
32	70.8621	0.02	67.3542	0.18270
33	63.7530	0.02	67.3542	0.19255
34	63.4375	0.02	67.3542	0.22776
35	55.5432	0.02	67.3542	2.07113
36	78.4558	0.02	67.3542	1.82982
37	68.5305	0.02	67.3542	0.02054
38	53.5405	0.02	67.3542	2.83308
39	48.2435	0.02	67.3542	5.42238
40	77.5587	0.02	67.3542	1.54603
41	82.5406	0.02	67.3542	3.42407
42	75.7857	0.02	67.3542	1.05547
43	64.7641	0.02	67.3542	0.09960
44	79.8253	0.02	67.3542	2.30912
45	78.7766	0.02	67.3542	1.93707
46	72.1496	0.02	67.3542	0.34141
47	68.2852	0.02	67.3542	0.01287
48	74.3916	0.02	67.3542	0.73528
49	75.8344	0.02	67.3542	1.06768
50	66.7185	0.02	67.3542	0.00600

N DF Chi-Sq P-Value
 3367.71 49 76.9676 0.007

CHI-SQUARE GOODNESS-OF-FIT TEST FOR C6

Category	Test Observed	Contribution Proportion	Expected	to Chi-Sq
1	86.3219	0.02	68.8085	4.4576
2	76.9007	0.02	68.8085	0.9517
3	67.1474	0.02	68.8085	0.0401
4	66.7692	0.02	68.8085	0.0604
5	59.6605	0.02	68.8085	1.2162
6	62.3847	0.02	68.8085	0.5997
7	77.9749	0.02	68.8085	1.2211
8	69.7618	0.02	68.8085	0.0132
9	95.2474	0.02	68.8085	10.1589
10	83.1864	0.02	68.8085	3.0043
11	80.1919	0.02	68.8085	1.8832
12	74.7413	0.02	68.8085	0.5115
13	57.3286	0.02	68.8085	1.9153
14	79.8755	0.02	68.8085	1.7800
15	73.9052	0.02	68.8085	0.3775
16	60.9518	0.02	68.8085	0.8971
17	85.3886	0.02	68.8085	3.9952
18	82.0308	0.02	68.8085	2.5408

19	74.0551	0.02	68.8085	0.4000
20	76.9919	0.02	68.8085	0.9733
21	77.0952	0.02	68.8085	0.9980
22	54.5006	0.02	68.8085	2.9751
23	59.3561	0.02	68.8085	1.2985
24	59.3836	0.02	68.8085	1.2909
25	78.0340	0.02	68.8085	1.2369
26	64.5772	0.02	68.8085	0.2602
27	43.6943	0.02	68.8085	9.1664
28	82.9828	0.02	68.8085	2.9198
29	68.2868	0.02	68.8085	0.0040
30	57.7834	0.02	68.8085	1.7665
31	58.1362	0.02	68.8085	1.6553
32	76.3968	0.02	68.8085	0.8368
33	62.2694	0.02	68.8085	0.6214
34	52.0600	0.02	68.8085	4.0767
35	67.4489	0.02	68.8085	0.0269
36	69.2350	0.02	68.8085	0.0026
37	60.6018	0.02	68.8085	0.9788
38	65.9027	0.02	68.8085	0.1227
39	57.0319	0.02	68.8085	2.0156
40	68.6221	0.02	68.8085	0.0005
41	71.8541	0.02	68.8085	0.1348
42	83.7519	0.02	68.8085	3.2453
43	66.7716	0.02	68.8085	0.0603
44	60.8638	0.02	68.8085	0.9173
45	71.8905	0.02	68.8085	0.1380
46	64.5203	0.02	68.8085	0.2672
47	62.9284	0.02	68.8085	0.5025
48	52.1107	0.02	68.8085	4.0521
49	79.2898	0.02	68.8085	1.5966
50	52.2294	0.02	68.8085	3.9947

N DF Chi-Sq P-Value
 3440.42 49 84.1597 0.001

CHI-SQUARE GOODNESS-OF-FIT TEST FOR C7

Category	Test		Contribution	
	Observed	Proportion	Expected	to Chi-Sq
1	45.5489	0.02	68.7456	7.8273
2	56.5727	0.02	68.7456	2.1555
3	75.5888	0.02	68.7456	0.6812
4	63.9847	0.02	68.7456	0.3297
5	75.5888	0.02	68.7456	0.6812
6	69.1366	0.02	68.7456	0.0022
7	84.6959	0.02	68.7456	3.7008
8	71.7521	0.02	68.7456	0.1315
9	73.8389	0.02	68.7456	0.3774
10	79.8048	0.02	68.7456	1.7791
11	71.5013	0.02	68.7456	0.1105
12	52.6550	0.02	68.7456	3.7662
13	63.2074	0.02	68.7456	0.4462
14	52.7113	0.02	68.7456	3.7399
15	65.6880	0.02	68.7456	0.1360
16	61.0342	0.02	68.7456	0.8650
17	71.3754	0.02	68.7456	0.1006

18	58.3818	0.02	68.7456	1.5624
19	46.0333	0.02	68.7456	7.5037
20	66.0007	0.02	68.7456	0.1096
21	63.7993	0.02	68.7456	0.3559
22	66.4546	0.02	68.7456	0.0763
23	66.9846	0.02	68.7456	0.0451
24	71.2832	0.02	68.7456	0.0937
25	95.6557	0.02	68.7456	10.5338
26	66.8657	0.02	68.7456	0.0514
27	52.6254	0.02	68.7456	3.7800
28	76.6133	0.02	68.7456	0.9004
29	53.3736	0.02	68.7456	3.4373
30	62.4622	0.02	68.7456	0.5743
31	74.5933	0.02	68.7456	0.4974
32	82.6019	0.02	68.7456	2.7929
33	55.5936	0.02	68.7456	2.5162
34	71.5465	0.02	68.7456	0.1141
35	83.3525	0.02	68.7456	3.1036
36	56.6161	0.02	68.7456	2.1401
37	78.1420	0.02	68.7456	1.2843
38	78.5500	0.02	68.7456	1.3983
39	81.8667	0.02	68.7456	2.5043
40	79.2814	0.02	68.7456	1.6147
41	61.8129	0.02	68.7456	0.6991
42	58.1605	0.02	68.7456	1.6298
43	76.3731	0.02	68.7456	0.8463
44	82.4455	0.02	68.7456	2.7302
45	82.1498	0.02	68.7456	2.6136
46	77.3789	0.02	68.7456	1.0842
47	73.1616	0.02	68.7456	0.2837
48	72.7869	0.02	68.7456	0.2376
49	76.9355	0.02	68.7456	0.9757
50	52.7137	0.02	68.7456	3.7387

N DF Chi-Sq P-Value
 3437.28 49 88.6589 0.003

CHI-SQUARE GOODNESS-OF-FIT TEST FOR C8

Category	Test		Contribution	
	Observed	Proportion	Expected	to Chi-Sq
1	65.8422	0.02	68.3078	0.08900
2	83.8147	0.02	68.3078	3.52032
3	71.8474	0.02	68.3078	0.18342
4	78.9866	0.02	68.3078	1.66946
5	74.0560	0.02	68.3078	0.48373
6	74.1874	0.02	68.3078	0.50610
7	66.9782	0.02	68.3078	0.02588
8	83.4256	0.02	68.3078	3.34586
9	65.5630	0.02	68.3078	0.11029
10	53.1505	0.02	68.3078	3.36335
11	72.4835	0.02	68.3078	0.25527
12	53.5231	0.02	68.3078	3.20002
13	64.3951	0.02	68.3078	0.22412
14	88.6733	0.02	68.3078	6.07189
15	74.2352	0.02	68.3078	0.51436
16	62.7458	0.02	68.3078	0.45288

17	76.2082	0.02	68.3078	0.91376
18	55.0063	0.02	68.3078	2.59018
19	74.0597	0.02	68.3078	0.48435
20	71.1001	0.02	68.3078	0.11415
21	76.6639	0.02	68.3078	1.02221
22	67.5080	0.02	68.3078	0.00936
23	87.1950	0.02	68.3078	5.22239
24	73.0906	0.02	68.3078	0.33489
25	53.3179	0.02	68.3078	3.28945
26	52.4708	0.02	68.3078	3.67176
27	73.0225	0.02	68.3078	0.32542
28	67.1978	0.02	68.3078	0.01804
29	68.2207	0.02	68.3078	0.00011
30	67.0408	0.02	68.3078	0.02350
31	69.0767	0.02	68.3078	0.00866
32	76.9085	0.02	68.3078	1.08294
33	74.5780	0.02	68.3078	0.57557
34	66.1074	0.02	68.3078	0.07088
35	63.9361	0.02	68.3078	0.27978
36	52.4834	0.02	68.3078	3.66592
37	79.8386	0.02	68.3078	1.94650
38	70.3518	0.02	68.3078	0.06116
39	72.3268	0.02	68.3078	0.23647
40	76.5586	0.02	68.3078	0.99661
41	53.2234	0.02	68.3078	3.33107
42	46.9829	0.02	68.3078	6.65739
43	65.4020	0.02	68.3078	0.12361
44	61.6430	0.02	68.3078	0.65028
45	59.1543	0.02	68.3078	1.22660
46	82.4294	0.02	68.3078	2.91947
47	67.5855	0.02	68.3078	0.00764
48	59.1156	0.02	68.3078	1.23698
49	64.6213	0.02	68.3078	0.19895
50	57.0547	0.02	68.3078	1.85383

N DF Chi-Sq P-Value
 3415.39 49 69.1658 0.030

CHI-SQUARE GOODNESS-OF-FIT TEST FOR C9

Category	Test		Contribution	
	Observed	Proportion	Expected	to Chi-Sq
1	79.6792	0.02	68.0366	1.9923
2	65.4433	0.02	68.0366	0.0988
3	62.1656	0.02	68.0366	0.5066
4	50.7257	0.02	68.0366	4.4045
5	66.1147	0.02	68.0366	0.0543
6	61.9178	0.02	68.0366	0.5503
7	50.8206	0.02	68.0366	4.3563
8	65.6757	0.02	68.0366	0.0819
9	64.7388	0.02	68.0366	0.1599
10	77.7523	0.02	68.0366	1.3874
11	68.7321	0.02	68.0366	0.0071
12	68.6858	0.02	68.0366	0.0062
13	59.7075	0.02	68.0366	1.0197
14	76.6419	0.02	68.0366	1.0884
15	67.1994	0.02	68.0366	0.0103

16	51.7640	0.02	68.0366	3.8920
17	68.4532	0.02	68.0366	0.0026
18	76.8928	0.02	68.0366	1.1528
19	76.3893	0.02	68.0366	1.0254
20	71.3197	0.02	68.0366	0.1584
21	51.2325	0.02	68.0366	4.1504
22	66.6895	0.02	68.0366	0.0267
23	72.7467	0.02	68.0366	0.3261
24	78.2700	0.02	68.0366	1.5392
25	85.0973	0.02	68.0366	4.2781
26	66.4586	0.02	68.0366	0.0366
27	73.2114	0.02	68.0366	0.3936
28	68.7990	0.02	68.0366	0.0085
29	75.6163	0.02	68.0366	0.8444
30	74.2709	0.02	68.0366	0.5712
31	82.8522	0.02	68.0366	3.2262
32	76.7890	0.02	68.0366	1.1259
33	71.8220	0.02	68.0366	0.2106
34	71.1688	0.02	68.0366	0.1442
35	55.4382	0.02	68.0366	2.3329
36	59.0568	0.02	68.0366	1.1852
37	66.1115	0.02	68.0366	0.0545
38	81.7004	0.02	68.0366	2.7441
39	75.2892	0.02	68.0366	0.7731
40	76.6551	0.02	68.0366	1.0917
41	76.7567	0.02	68.0366	1.1176
42	33.1233	0.02	68.0366	17.9160
43	60.7132	0.02	68.0366	0.7883
44	64.4727	0.02	68.0366	0.1867
45	65.4318	0.02	68.0366	0.0997
46	51.1127	0.02	68.0366	4.2098
47	87.8685	0.02	68.0366	5.7807
48	66.9581	0.02	68.0366	0.0171
49	83.8196	0.02	68.0366	3.6613
50	51.4804	0.02	68.0366	4.0289

N DF Chi-Sq P-Value
 3401.83 49 84.8245 0.001

CHI-SQUARE GOODNESS-OF-FIT TEST FOR C10

Category	Test		Contribution	
	Observed	Proportion	Expected	to Chi-Sq
1	56.9674	0.02	68.3077	1.88268
2	53.4838	0.02	68.3077	3.21703
3	59.1074	0.02	68.3077	1.23917
4	77.3718	0.02	68.3077	1.20278
5	80.1996	0.02	68.3077	2.07029
6	58.7607	0.02	68.3077	1.33434
7	62.0002	0.02	68.3077	0.58244
8	71.7285	0.02	68.3077	0.17131
9	62.4696	0.02	68.3077	0.49897
10	70.4298	0.02	68.3077	0.06593
11	56.5502	0.02	68.3077	2.02377
12	75.2573	0.02	68.3077	0.70704
13	66.4659	0.02	68.3077	0.04966
14	51.7387	0.02	68.3077	4.01905

15	90.8331	0.02	68.3077	7.42803
16	59.2973	0.02	68.3077	1.18856
17	55.4977	0.02	68.3077	2.40231
18	76.7801	0.02	68.3077	1.05085
19	71.8922	0.02	68.3077	0.18810
20	60.3603	0.02	68.3077	0.92465
21	68.7608	0.02	68.3077	0.00301
22	47.5391	0.02	68.3077	6.31459
23	53.4773	0.02	68.3077	3.21987
24	85.1326	0.02	68.3077	4.14413
25	79.5195	0.02	68.3077	1.84028
26	72.1171	0.02	68.3077	0.21245
27	56.4353	0.02	68.3077	2.06351
28	85.9241	0.02	68.3077	4.54324
29	88.1467	0.02	68.3077	5.76193
30	78.7449	0.02	68.3077	1.59476
31	51.2051	0.02	68.3077	4.28206
32	84.3989	0.02	68.3077	3.79060
33	85.6011	0.02	68.3077	4.37817
34	74.4579	0.02	68.3077	0.55375
35	58.8690	0.02	68.3077	1.30423
36	63.4526	0.02	68.3077	0.34509
37	76.4065	0.02	68.3077	0.96023
38	83.7664	0.02	68.3077	3.49844
39	69.8318	0.02	68.3077	0.03401
40	80.3231	0.02	68.3077	2.11353
41	85.9982	0.02	68.3077	4.58154
42	62.1372	0.02	68.3077	0.55741
43	69.2102	0.02	68.3077	0.01192
44	64.4727	0.02	68.3077	0.21531
45	54.8284	0.02	68.3077	2.65989
46	64.6467	0.02	68.3077	0.19622
47	66.0610	0.02	68.3077	0.07390
48	56.1069	0.02	68.3077	2.17925
49	70.8301	0.02	68.3077	0.09315
50	59.7922	0.02	68.3077	1.06159

N DF Chi-Sq P-Value
 3415.38 49 94.8350 0.004

CHI-SQUARE GOODNESS-OF-FIT TEST FOR C11

Category	Test		Contribution	
	Observed	Proportion	Expected	to Chi-Sq
1	61.0950	0.02	67.2314	0.56010
2	63.7801	0.02	67.2314	0.17717
3	67.1914	0.02	67.2314	0.00002
4	65.1829	0.02	67.2314	0.06242
5	67.6716	0.02	67.2314	0.00288
6	70.2033	0.02	67.2314	0.13137
7	83.0653	0.02	67.2314	3.72906
8	61.7906	0.02	67.2314	0.44030
9	70.3648	0.02	67.2314	0.14603
10	68.0160	0.02	67.2314	0.00916
11	83.6352	0.02	67.2314	4.00234
12	49.0405	0.02	67.2314	4.92196
13	77.5386	0.02	67.2314	1.58018

14	72.7817	0.02	67.2314	0.45819
15	59.9225	0.02	67.2314	0.79458
16	68.2916	0.02	67.2314	0.01672
17	57.8678	0.02	67.2314	1.30411
18	66.7676	0.02	67.2314	0.00320
19	76.2743	0.02	67.2314	1.21631
20	77.2489	0.02	67.2314	1.49259
21	66.7716	0.02	67.2314	0.00314
22	48.4428	0.02	67.2314	5.25071
23	51.5531	0.02	67.2314	3.65617
24	68.8421	0.02	67.2314	0.03859
25	67.2762	0.02	67.2314	0.00003
26	72.0465	0.02	67.2314	0.34485
27	93.1191	0.02	67.2314	9.96810
28	76.5641	0.02	67.2314	1.29549
29	55.4570	0.02	67.2314	2.06210
30	71.5565	0.02	67.2314	0.27824
31	59.6470	0.02	67.2314	0.85561
32	66.4328	0.02	67.2314	0.00949
33	68.2342	0.02	67.2314	0.01496
34	70.1062	0.02	67.2314	0.12292
35	54.0559	0.02	67.2314	2.58203
36	54.4430	0.02	67.2314	2.43254
37	54.2282	0.02	67.2314	2.51495
38	66.9998	0.02	67.2314	0.00080
39	84.7125	0.02	67.2314	4.54530
40	64.4292	0.02	67.2314	0.11680
41	77.4151	0.02	67.2314	1.54253
42	75.0178	0.02	67.2314	0.90178
43	55.5752	0.02	67.2314	2.02090
44	79.8741	0.02	67.2314	2.37741
45	59.0604	0.02	67.2314	0.99308
46	69.3398	0.02	67.2314	0.06612
47	54.0984	0.02	67.2314	2.56543
48	67.2954	0.02	67.2314	0.00006
49	67.4697	0.02	67.2314	0.00084
50	73.7784	0.02	67.2314	0.63755

N DF Chi-Sq P-Value
3361.57 49 68.2472 0.036

CHI-SQUARE GOODNESS-OF-FIT TEST FOR C12

Category	Test		Contribution to Chi-Sq
	Observed	Proportion Expected	
1	50.6826	0.02	66.5558 3.78567
2	71.1572	0.02	66.5558 0.31812
3	69.5642	0.02	66.5558 0.13598
4	76.2487	0.02	66.5558 1.41163
5	57.0176	0.02	66.5558 1.36692
6	87.6484	0.02	66.5558 6.68457
7	64.5730	0.02	66.5558 0.05907
8	52.4708	0.02	66.5558 2.98076
9	65.4995	0.02	66.5558 0.01676
10	53.7796	0.02	66.5558 2.45255
11	67.1281	0.02	66.5558 0.00492
12	80.3293	0.02	66.5558 2.85040

13	77.4867	0.02	66.5558	1.79525
14	67.6421	0.02	66.5558	0.01773
15	66.3429	0.02	66.5558	0.00068
16	49.4104	0.02	66.5558	4.41681
17	60.5819	0.02	66.5558	0.53620
18	66.2034	0.02	66.5558	0.00187
19	45.4085	0.02	66.5558	6.71927
20	66.1123	0.02	66.5558	0.00296
21	85.4846	0.02	66.5558	5.38347
22	69.8069	0.02	66.5558	0.15881
23	74.8858	0.02	66.5558	1.04258
24	70.4746	0.02	66.5558	0.23074
25	64.6737	0.02	66.5558	0.05322
26	68.8549	0.02	66.5558	0.07942
27	68.6356	0.02	66.5558	0.06499
28	65.6050	0.02	66.5558	0.01358
29	70.2139	0.02	66.5558	0.20106
30	70.0020	0.02	66.5558	0.17844
31	62.4296	0.02	66.5558	0.25581
32	72.5866	0.02	66.5558	0.54646
33	46.6356	0.02	66.5558	5.96214
34	59.0249	0.02	66.5558	0.85212
35	72.3276	0.02	66.5558	0.50054
36	74.4465	0.02	66.5558	0.93551
37	55.1416	0.02	66.5558	1.95750
38	75.3944	0.02	66.5558	1.17378
39	67.6437	0.02	66.5558	0.01778
40	56.1823	0.02	66.5558	1.61683
41	77.7679	0.02	66.5558	1.88883
42	56.2617	0.02	66.5558	1.59216
43	69.0584	0.02	66.5558	0.09410
44	59.5602	0.02	66.5558	0.73530
45	64.8121	0.02	66.5558	0.04568
46	62.2477	0.02	66.5558	0.27885
47	55.4451	0.02	66.5558	1.85481
48	74.7500	0.02	66.5558	1.00886
49	76.9423	0.02	66.5558	1.62088
50	85.2068	0.02	66.5558	5.22662

N DF Chi-Sq P-Value
 3327.79 49 71.1290 0.021

CHI-SQUARE GOODNESS-OF-FIT TEST FOR C13

Category	Test Observed	Contribution Proportion	Expected	to Chi-Sq
1	49.0537	0.02	69.1133	5.82218
2	60.9058	0.02	69.1133	0.97467
3	64.9959	0.02	69.1133	0.24529
4	71.4613	0.02	69.1133	0.07977
5	53.4991	0.02	69.1133	3.52760
6	74.8732	0.02	69.1133	0.48002
7	62.0831	0.02	69.1133	0.71512
8	46.9244	0.02	69.1133	7.12376
9	77.7631	0.02	69.1133	1.08255
10	56.4763	0.02	69.1133	2.31062
11	53.7922	0.02	69.1133	3.39642

12	78.0699	0.02	69.1133	1.16071
13	82.1324	0.02	69.1133	2.45244
14	92.3526	0.02	69.1133	7.81415
15	73.0083	0.02	69.1133	0.21951
16	74.7384	0.02	69.1133	0.45783
17	67.9013	0.02	69.1133	0.02126
18	72.6621	0.02	69.1133	0.18222
19	67.8750	0.02	69.1133	0.02219
20	61.6342	0.02	69.1133	0.80936
21	62.8456	0.02	69.1133	0.56841
22	65.2170	0.02	69.1133	0.21966
23	71.2716	0.02	69.1133	0.06740
24	81.1833	0.02	69.1133	2.10792
25	78.6830	0.02	69.1133	1.32505
26	65.8078	0.02	69.1133	0.15810
27	86.8527	0.02	69.1133	4.55317
28	58.8894	0.02	69.1133	1.51243
29	67.0961	0.02	69.1133	0.05888
30	56.2633	0.02	69.1133	2.38917
31	64.5305	0.02	69.1133	0.30388
32	81.5376	0.02	69.1133	2.23346
33	67.6533	0.02	69.1133	0.03084
34	59.5760	0.02	69.1133	1.31611
35	73.0711	0.02	69.1133	0.22664
36	56.6235	0.02	69.1133	2.25709
37	64.2907	0.02	69.1133	0.33651
38	73.5581	0.02	69.1133	0.28585
39	87.1496	0.02	69.1133	4.70686
40	66.4684	0.02	69.1133	0.10122
41	75.6757	0.02	69.1133	0.62310
42	58.5321	0.02	69.1133	1.61997
43	70.1086	0.02	69.1133	0.01433
44	83.3389	0.02	69.1133	2.92804
45	75.3332	0.02	69.1133	0.55976
46	79.4953	0.02	69.1133	1.55955
47	61.6596	0.02	69.1133	0.80388
48	66.9878	0.02	69.1133	0.06537
49	79.0530	0.02	69.1133	1.42950
50	74.7114	0.02	69.1133	0.45344

N DF Chi-Sq P-Value
 3455.67 49 73.7133 0.013

CHI-SQUARE GOODNESS-OF-FIT TEST FOR C14

Category	Test Observed	Contribution Proportion	Expected	to Chi-Sq
1	58.1029	0.02	66.7141	1.1115
2	74.3859	0.02	66.7141	0.8822
3	74.1500	0.02	66.7141	0.8288
4	64.8811	0.02	66.7141	0.0504
5	86.2898	0.02	66.7141	5.7441
6	49.8579	0.02	66.7141	4.2589
7	58.5000	0.02	66.7141	1.0113
8	58.8486	0.02	66.7141	0.9273
9	71.7555	0.02	66.7141	0.3810
10	73.9911	0.02	66.7141	0.7938

11	57.2744	0.02	66.7141	1.3357
12	63.5684	0.02	66.7141	0.1483
13	61.4051	0.02	66.7141	0.4225
14	69.2382	0.02	66.7141	0.0955
15	46.4524	0.02	66.7141	6.1537
16	61.3217	0.02	66.7141	0.4359
17	66.7901	0.02	66.7141	0.0001
18	48.0838	0.02	66.7141	5.2026
19	69.2126	0.02	66.7141	0.0936
20	83.1091	0.02	66.7141	4.0291
21	88.9119	0.02	66.7141	7.3859
22	38.3419	0.02	66.7141	12.0661
23	66.1871	0.02	66.7141	0.0042
24	66.5443	0.02	66.7141	0.0004
25	66.3016	0.02	66.7141	0.0026
26	60.8103	0.02	66.7141	0.5224
27	97.5394	0.02	66.7141	14.2429
28	72.0151	0.02	66.7141	0.4212
29	76.1445	0.02	66.7141	1.3330
30	53.2460	0.02	66.7141	2.7189
31	77.8636	0.02	66.7141	1.8634
32	76.3893	0.02	66.7141	1.4031
33	51.6508	0.02	66.7141	3.4011
34	51.8613	0.02	66.7141	3.3067
35	62.8063	0.02	66.7141	0.2289
36	55.4740	0.02	66.7141	1.8938
37	86.3470	0.02	66.7141	5.7777
38	69.5826	0.02	66.7141	0.1233
39	63.3548	0.02	66.7141	0.1691
40	68.4572	0.02	66.7141	0.0455
41	67.5088	0.02	66.7141	0.0095
42	64.6314	0.02	66.7141	0.0650
43	79.8460	0.02	66.7141	2.5849
44	55.9862	0.02	66.7141	1.7251
45	68.0542	0.02	66.7141	0.0269
46	65.1196	0.02	66.7141	0.0381
47	72.6761	0.02	66.7141	0.5328
48	88.6140	0.02	66.7141	7.1890
49	67.3138	0.02	66.7141	0.0054
50	58.9061	0.02	66.7141	0.9138

N DF Chi-Sq P-Value
 3335.70 49 103.907 0.008

CHI-SQUARE GOODNESS-OF-FIT TEST FOR C15

Category	Test		Contribution	
	Observed	Proportion	Expected	to Chi-Sq
1	60.2278	0.02	68.3234	0.95924
2	65.8299	0.02	68.3234	0.09100
3	65.5152	0.02	68.3234	0.11542
4	73.8950	0.02	68.3234	0.45436
5	62.9502	0.02	68.3234	0.42256
6	65.9190	0.02	68.3234	0.08461
7	50.6659	0.02	68.3234	4.56337
8	80.2241	0.02	68.3234	2.07291
9	64.4795	0.02	68.3234	0.21626

10	75.2473	0.02	68.3234	0.70167
11	87.6619	0.02	68.3234	5.47366
12	81.1765	0.02	68.3234	2.41795
13	52.7161	0.02	68.3234	3.56518
14	65.9264	0.02	68.3234	0.08409
15	81.7059	0.02	68.3234	2.62126
16	77.4949	0.02	68.3234	1.23117
17	68.2629	0.02	68.3234	0.00005
18	62.6631	0.02	68.3234	0.46892
19	68.7799	0.02	68.3234	0.00305
20	57.8019	0.02	68.3234	1.62024
21	60.2535	0.02	68.3234	0.95314
22	63.7539	0.02	68.3234	0.30561
23	69.9003	0.02	68.3234	0.03640
24	50.8563	0.02	68.3234	4.46551
25	66.2999	0.02	68.3234	0.05992
26	71.2143	0.02	68.3234	0.12232
27	86.3903	0.02	68.3234	4.77747
28	63.4721	0.02	68.3234	0.34446
29	51.9273	0.02	68.3234	3.93467
30	68.3975	0.02	68.3234	0.00008
31	85.8907	0.02	68.3234	4.51692
32	72.8000	0.02	68.3234	0.29332
33	84.8269	0.02	68.3234	3.98646
34	76.7812	0.02	68.3234	1.04700
35	60.6560	0.02	68.3234	0.86044
36	75.0120	0.02	68.3234	0.65479
37	82.0613	0.02	68.3234	2.76231
38	76.6760	0.02	68.3234	1.02113
39	49.9261	0.02	68.3234	4.95378
40	62.6345	0.02	68.3234	0.47367
41	81.6041	0.02	68.3234	2.58153
42	67.4385	0.02	68.3234	0.01146
43	63.2011	0.02	68.3234	0.38402
44	62.7916	0.02	68.3234	0.44787
45	56.3205	0.02	68.3234	2.10865
46	70.4168	0.02	68.3234	0.06414
47	77.2419	0.02	68.3234	1.16419
48	53.3224	0.02	68.3234	3.29358
49	58.1362	0.02	68.3234	1.51893
50	76.8213	0.02	68.3234	1.05695

N	DF	Chi-Sq	P-Value
3416.17	49	75.3677	0.009

The Mean and Standard Deviation of the K-S Test P-Values (15 Data Set)

	Mean	StDev.
K-S	0.1470	0.00819

The Mean and Standard Deviation of the Chi-square Test P-Values (15 Data Set)

	Mean	StDev.
CHI-SQUARE	0.0273	0.0529

8. DISCUSSION

From the mean and standard deviation of the simulated data p-values for the two goodness of fit tests, the K-S goodness of fit test has a higher mean value and standard deviation value than the Chi-square goodness of fit test. The result revealed that the K-S goodness of fit test is more precise to the Chi-square goodness of fit test with p-values greater than the chi-square test.

9. CONCLUSION

When comparing the K.S goodness of fit test and the chi-square goodness of fit test the null hypothesis is accepted when the $P\text{-value} < 0.05$ otherwise.

From the mean and standard deviation of the simulated data p-values for the two goodness of fit tests, the Kolmogorov-Smirnov test has a higher mean value and standard deviation which is greater than that of the Chi-square test. Also for the simulated data, the analysis showed that Kolmogorov-Smirnov test is more precise with p-values greater than the Chi-square test.

This study shows that in the test for goodness of fit for normality (i.e. in the first 50 simulated data), the Kolmogorov-Smirnov of fit test p-value is greater than the Chi-square goodness test p-value. But comparing the results to that of the simulated data, the Kolmogorov-Smirnov test tends to give a higher p-value than the Chi-square. This proves that chi-square goodness of fit test is not the most accurate of tests when taking goodness of fit test.

Comparing the results of the 50 British boys heights and the 15 data set shows that Kolmogorov-Smirnov is more precise in comparison to the Chi-square test.

10. RECOMMENDATION

Based on the result of the study, the following recommendations made:

- i. When testing goodness of fit test for small samples, it is important that other types of goodness of fit test be used, rather than the more common Pearson Chi-square.
- ii. The test statistic is recommended when testing goodness of fit for a normal distribution than the Kolmogorov-Smirnov test statistic and the Pearson's Chi-square test statistic, as it has a precise p-value.

- iii. The two statistics should be compare using simulated skew datasets in further studies.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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