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Interval-Valued 2-Tuple Prioritized Aggregation Operators and Their Application to Multiple Attribute Group Decision Making

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Author's contribution

Author ZZ designed the study, performed the statistical analysis, wrote the protocol, and wrote the overall draft of the manuscript. The author read and approved the final manuscript.

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ABSTRACT

This paper presents interval-valued 2-tuple prioritized aggregation operators for handling problems involving multiple attribute group decision making, where a prioritization relationship exists between attributes and decision makers. We first develop two interval-valued 2-tuple prioritized aggregation operators called (i) the interval-valued 2-tuple prioritized weighted average (IV2TPWA) operator and (ii) the interval-valued 2-tuple prioritized weighted geometric (IV2TPWG) operator and examine their desirable properties. The significant feature of these operators is that they not only deal with the linguistic and interval linguistic information but also account for the priority level of the arguments. Next, based on the IV2TPWA and the IV2TPWG operators, we develop an approach to multiple attribute group decision making with interval linguistic information. Finally, a practical example is presented to illustrate our method.

Keywords: Multiple attribute group decision making; interval-valued 2-tuple prioritized weighted average (IV2TPWA) operator; interval-valued 2-tuple prioritized weighted geometric (IV2TPWG) operator.

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1. INTRODUCTION

Multiple attribute group decision making (MAGDM) consists of finding the most desirable alternative(s) from a given alternative set according to the preferences provided by a group of experts [1]. For some MAGDM problems, the decision information about alternatives is uncertain or fuzzy because of the increasing complexity of the socio-economic environment and the vagueness inherent to the subjective nature of human thinking [2-4]. As a result, the decision information cannot be precisely assessed in a quantitative form. However, it may be appropriate and sufficient to assess the information in a qualitative form rather than a quantitative form. For example, when evaluating the cost of a house, linguistic terms such as "high," "medium," and "low" are usually used, and when evaluating the design of a house, linguistic terms such as "good," "medium," and "bad" are frequently used. To date, some methods have been developed for coping with linguistic information. These methods can be summarized as follows [5]. (1) The approximate computational model based on the Extension Principle [6]: This model transforms linguistic assessment information into fuzzy numbers and uses fuzzy arithmetic to make computations over these fuzzy numbers. The use of fuzzy arithmetic increases the vagueness. The results obtained by fuzzy arithmetic are fuzzy numbers that usually do not match any linguistic term in the initial term set. (2) The ordinal linguistic computational model [7]: This model is also called the symbolic model and makes direct computations on labels by using the ordinal structure of the linguistic-term sets. However, the symbolic method easily results in a loss of information, which is caused by the use of the round operator. (3) The 2-tuple linguistic computational model [8-11]. This model uses the 2-tuple linguistic representation and the computational model to make linguistic computations. (4) The model xxx computes directly with words [12-20].

The 2-tuple fuzzy linguistic representation model represents linguistic information by using a pair of values called a "2-tuple," which is composed of a linguistic term and a number. Meanwhile, the model also provides a computational technique to deal with 2-tuples without losing information. Since its introduction, the 2-tuple fuzzy linguistic representation model has received increasing attention. In a MAGDM problem involving the 2-tuple fuzzy linguistic representation model, 2-tuple aggregation operators are widely used to aggregate individual linguistic preference information into overall linguistic preference information. In the past few decades, many scholars have developed a variety of 2-tuple aggregation operators, such as the 2-tuple arithmetic mean operator [8,10], 2-tuple weighted averaging operator [8], 2-tuple ordered weighted average (OWA) operator [8], 2-tuple weighted geometric averaging (TWGA) operator [21], 2-tuple ordered weighted geometric averaging (TOWGA) operator [21], 2-tuple hybrid geometric averaging (THGA) operator [21], 2-tuple arithmetic average (TAA) operator [8], 2-tuple weighted average (TWA) operator [8], 2-tuple ordered weighted average (TOWA) operator [12], extended 2-tuple weighted average (ET-WA) operator [8], 2-tuple ordered weighted geometric (TOWG) operator [22], extended 2-tuple weighted geometric (ET-WG) operator [23], extended 2-tuple ordered weighted geometric (ET-OWG) operator [23], generalized 2-tuple weighted average (G-2TWA) operator [5], generalized 2-tuple ordered weighted average (G-2TOWA) operator [5], induced generalized 2-tuple ordered weighted average (IG-2TOWA) operator [5], 2-tuple linguistic power average (2TLPA) operator [24], 2-tuple linguistic weighted power average (2TLWPA) operator [24], and 2-tuple linguistic power ordered weighted average (2TLPOWA) operator [24].

The MAGDMs mentioned above assume that the attributes and the decision makers are at the same priority level. In this situation, we can counterbalance the attributes. For instance, if C_i and C_j are two attributes with weights w_i and w_j, respectively, we can compensate for a decrease of θ in satisfaction for attribute C_i by a gain $\frac{w_j}{w_i}\theta$ in satisfaction for attribute C_j. However, in some MAGDM problems, this kind of compensation between attributes should not be allowed. For example, consider the situation in which you are buying a car based on the safety and cost of cars. A benefit with respect to cost should not compensate for a loss in safety. In this case, we have a kind of prioritization of the attributes, i.e., safety has a higher priority than cost. As another example, consider decision making in a company: the general manager has a higher priority than the manager. Yager [25] first investigated criterion-aggregation problems involving prioritized criteria. Following this, Yager [26] and Yager et al. [27] provided much deeper insights into this issue. Motivated by the ideas of Yager [25,26] and Yager et al. [27], Wei [28] generalized prioritized aggregation operators to a hesitant fuzzy environment, proposed some hesitant fuzzy prioritized aggregation operators, and applied these operators to develop models for hesitant fuzzy multiple attribute decision making problems in which the attributes are at different priority levels. Yu and Xu [29] investigated the prioritization relationship between attributes in multi-attribute decision making with intuitionistic fuzzy information and developed some prioritized intuitionistic fuzzy aggregation operators by extending the prioritized aggregation operators. Yu et al. [30] proposed some interval-valued intuitionistic fuzzy prioritized aggregation operators and investigated the application of these operators in group decision making under an interval-valued intuitionistic fuzzy environment in which the attributes and experts are at different priority levels. However, the existing interval-valued 2-tuple aggregation operators have difficulty within the framework of MAGDMs where the attributes and the decision makers are at different priority levels. Moreover, rather little work has been done on using prioritized aggregation operators to solve MAGDMs with interval linguistic preference information. Thus, prioritized aggregation operators should be extended to the interval linguistic environment. To do this, we develop in the current paper some interval-valued 2-tuple prioritized aggregation operators. The prominent characteristic of the proposed operators is that they account for prioritization between the attributes and the decision makers. Next, we use these operators to develop approaches to MAGDMs where the attributes and the decision makers are at different priority levels. Finally, we present some numerical examples to verify the practicality and effectiveness of the proposed operators and approaches.

The rest of this paper is organized as follows: Section 2 introduces some basic concepts of the 2-tuple fuzzy linguistic approach and the prioritized average operator. In Section 3, we propose the interval-valued 2-tuple prioritized weighted average (IV2TPWA) operator and the interval-valued 2-tuple prioritized weighted geometric (IV2TPWG) operator. Furthermore, we develop a method for MAGDM based on the proposed operators under the interval linguistic environment. In Section 4, an example concerning talent introduction is provided to demonstrate the practicality and effectiveness of the proposed approach. The final section offers some concluding remarks.

2. PRELIMINARIES

In this section, we introduce the basic notions of the 2-tuple fuzzy linguistic approach, the interval-valued 2-tuple fuzzy linguistic approach, and the prioritized average operator.

2.1 2-Tuple Fuzzy Linguistic Representation Model

Let $S=\{s_i \mid i=0,1,2,\ldots,g\}$ be a finite and totally ordered discrete linguistic-term set with odd cardinality, where s_i represents a possible value for a linguistic variable. This set should satisfy the following characteristics [8-10]:

(1) The set is ordered: $s_i \geq s_j$ if $i \geq j$;
(2) There is the negation operator: $neg(s_i) = s_j$ such that $j = g - i$;
(3) The max operator is defined as $\max(s_i, s_j) = s_i$ if $s_i \geq s_j$;
(4) The min operator is defined as $\min(s_i, s_j) = s_i$ if $s_i \leq s_j$.

For example, a set S of seven terms could be [31-37].

$$S = \{s_0 = nothing,\ s_1 = very\,low,\ s_2 = low,\ s_3 = medium,\ s_4 = high,\ s_5 = very\,high,\ s_6 = perfect\}.$$

Based on the concept of symbolic translation, Herrera and Martinez [8,9] introduced a 2-tuple fuzzy linguistic representation model for dealing with linguistic information. This model represents the linguistic assessment information by a 2-tuple (s_i, α), where $s_i \in S$ represents a linguistic label from the predefined linguistic-term set S and $\alpha \in [-0.5, 0.5)$ is the value of symbolic translation.

Definition 2.1 [8,9]. Let β be the result of an aggregation of indices of a set of labels assessed in a linguistic-term set S, i.e., the result of a symbolic aggregation operation. Let $\beta \in [0, g]$, with $g+1$ being the cardinality of S. If $i = \text{round}(\beta)$ and $\alpha = \beta - i$ are two values such that $i \in [0, g]$ and $\alpha \in [-0.5, 0.5)$, then α is called a symbolic translation, where $\text{round}(\bullet)$ is the usual round operation.

Definition 2.2 [8,9]. If $S = \{s_i | i = 0,1,2,\ldots,g\}$ is a linguistic-term set and $\beta \in [0, g]$ is a value representing the result of a symbolic aggregation operation, then the 2-tuple that expresses the information equivalent to β is obtained from

$$\Delta : [0, g] \rightarrow S \times [-0.5, 0.5), \tag{1}$$

$$\Delta(\beta) = (s_i, \alpha), \quad \text{with} \begin{cases} s_i, & i = \text{round}(\beta) \\ \alpha = \beta - i, & \alpha \in [-0.5, 0.5), \end{cases} \tag{2}$$

where s_i has the index label closest to β and α is the value of the symbolic translation.

Theorem 2.1 [8,9]. Let $S = \{s_i | i = 0,1,2,\ldots,g\}$ be a linguistic-term set and (s_i, α) be a 2-tuple. There is always a Δ^{-1} function such that, from a 2-tuple, it returns its equivalent numerical value $\beta \in [0, g] \subset R$, where

$$\Delta^{-1} : S \times [-0.5, 0.5) \rightarrow [0, g], \tag{3}$$

$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta. \tag{4}$$

The conversion of a linguistic term into a linguistic 2-tuple consists of adding a zero value as a symbolic translation: $s_i \in S \Rightarrow (s_i, 0)$.

Definition 2.3 [8,9]. Linguistic information represented by 2-tuples is compared according to an ordinary lexicographic order. Let (s_k, α_k) and (s_l, α_l) be two 2-tuples, with each representing the following information ordering:

(1) If $k < l$ then (s_k, α_k) is smaller than (s_l, α_l).

(2) If $k = l$ then

- if $\alpha_k = \alpha_l$ then (s_k, α_k), (s_l, α_l) represents the same information;
- if $\alpha_k < \alpha_l$ then (s_k, α_k) is smaller than (s_l, α_l);
- if $\alpha_k > \alpha_l$ then (s_k, α_k) is bigger than (s_l, α_l).

2.2 Interval-Valued 2-Tuple Fuzzy Linguistic Representation Model

Chen and Tai [38] proposed the following generalized 2-tuple linguistic variable and translation function.

Definition 2.4 [38]. Suppose a linguistic-term set, $S = \{s_i | i = 0,1,2,\ldots,g\}$ and the crisp value β ($\beta \in [0,1]$) can be transformed into the 2-tuple linguistic variable by the following function:

$$\Delta(\beta) = (s_i, \alpha), \quad \text{with} \begin{cases} s_i, & i = \text{round}(\beta * g) \\ \alpha = \beta - \dfrac{i}{g}, & \alpha \in \left[\dfrac{-0.5}{g}, \dfrac{0.5}{g}\right). \end{cases} \tag{5}$$

Then the 2-tuple linguistic variable can be converted into the crisp value β ($\beta \in [0,1]$) as follows:

$$\Delta^{-1}(s_i, \alpha) = \frac{i}{g} + \alpha = \beta. \tag{6}$$

Unlike the value of β in Definition 2.2, the value of β in Definition 2.4 ranges from 0 to 1. That is, the 2-tuple linguistic variable of Definition 2.4 is standardized, which makes it very convenient for comparing 2-tuples from different multigranularity linguistic-term sets [39]. In this section, unless stated otherwise, both the linguistic-term set and the 2-tuple linguistic variable satisfy the requirements of Definition 2.4.

The existing aggregation operators involving 2-tuples mainly focus on the usual 2-tuples. If the 2-tuples are from different linguistic-term sets, they cannot be aggregated directly and must be tediously transformed before any aggregation operation. To avoid this tedious calculation, Zhang [39] introduced the concept of an interval-valued 2-tuple linguistic variable and developed some aggregation operators with interval-valued 2-tuple linguistic information.

Definition 2.5 [39]. Consider a linguistic-term set $S=\{s_i | i=0,1,2,\ldots,g\}$. An interval-valued 2-tuple is composed of two linguistic terms and two numbers, denoted $\left[(s_i,\alpha_1),(s_j,\alpha_2)\right]$, where $i \le j$, $s_i(s_j)$ and α_1 and α_2 represent the linguistic label of the predefined linguistic-term set S and the symbolic translation, respectively. An interval-valued 2-tuple linguistic variable can be converted into an interval value $[\beta_1,\beta_2]$ ($\beta_1,\beta_2 \in [0,1]$, $\beta_1 \le \beta_2$) as follows:

$$\Delta^{-1}\left(\left[(s_i,\alpha_1),(s_j,\alpha_2)\right]\right)=\left[i/g+\alpha_1,\ j/g+\alpha_2\right]=\left[\beta_1,\beta_2\right]. \tag{7}$$

Conversely, the interval value $[\beta_1,\beta_2]$ ($\beta_1,\beta_2 \in [0,1]$, $\beta_1 \le \beta_2$) can be transformed into the interval-valued 2-tuple linguistic variable by the following function:

$$\Delta([\beta_1,\beta_2])=\left[(s_i,\alpha_1),(s_j,\alpha_2)\right] \quad \text{with} \begin{cases} s_i, & i=\text{round}(\beta_1 * g) \\ s_j, & j=\text{round}(\beta_2 * g) \\ \alpha_1=\beta_1-i/g, & \alpha_1 \in [-0.5/g, 0.5/g) \\ \alpha_2=\beta_2-j/g, & \alpha_2 \in [-0.5/g, 0.5/g). \end{cases} \tag{8}$$

In particular, if $s_i=s_j$ and $\alpha_1=\alpha_2$, then the interval-valued 2-tuple linguistic variable reduces to a 2-tuple linguistic variable.

Zhang [39] extended the comparison of two 2-tuples to the interval-valued case and proposed the score and accuracy functions [4,17] to compare two interval-valued 2-tuples.

Definition 2.6 [39]. The score function of an interval-valued 2-tuple $A=\left[(s_i,\alpha_1),(s_j,\alpha_2)\right]$ is

$$S(A)=\frac{i+j}{2g}+\frac{\alpha_1+\alpha_2}{2}, \tag{9}$$

where $g+1$ is the cardinality of S, $S=\{s_i | i=0,1,2,\ldots,g\}$.

It is easy to prove that $0 \le S(A) \le 1$. The score function is regarded as a basis for comparing two interval-valued 2-tuples. For two interval-valued 2-tuples, the one with a larger score function corresponds to the larger interval-valued 2-tuple. However, it is possible that two different interval-valued 2-tuples may have identical score values. In that event, an accuracy function should be considered [39].

Definition 2.7 [39]. For an interval-valued 2-tuple $A=\left[(s_i,\alpha_1),(s_j,\alpha_2)\right]$, its accuracy function is

$$H(A)=\frac{j-i}{g}+\alpha_2-\alpha_1. \tag{10}$$

Note that $0 \le H(A) \le 1$. For two interval-valued 2-tuples with the same score function, the one with the smaller accuracy function is the larger of the two interval-valued 2-tuples.

Theorem 2.2 [39]. Let $A = \left[(s_i, \alpha_1), (s_j, \alpha_2) \right]$ and $B = \left[(s_i', \alpha_1'), (s_j', \alpha_2') \right]$ be two interval-valued 2-tuples:

If $S(A) > S(B)$, then $A > B$;
If $S(A) < S(B)$, then $A < B$;
If $S(A) = S(B)$, then:
(1) If $H(A) > H(B)$, then $A < B$;
(2) If $H(A) < H(B)$, then $A > B$;
(3) If $H(A) = H(B)$, then $A = B$.

2.3. Prioritized Average (PA) Operators

The prioritized average (PA) operator was originally introduced by Yager [25,26] and is defined as follows:

Definition 2.8 [25,26]. Let $C = \{C_1, C_2, \ldots, C_n\}$ be a collection of criteria with a prioritization between the criteria expressed by the linear ordering $C_1 \succ C_2 \succ C_3 \cdots \succ C_n$, which indicates criterion C_j has a higher priority than C_k if $j < k$. The value $C_j(x)$ is the performance of any alternative x under criterion C_j and satisfies $C_j(x) \in [0,1]$. If

$$PA(C_i(x)) = \sum_{j=1}^{n} w_j C_j(x), \tag{11}$$

where $w_j = \frac{T_j}{\sum_{j=1}^{n} T_j}$, $T_j = \prod_{k=1}^{j-1} C_k(x) \ (j = 2, \cdots, n)$, $T_1 = 1$, then PA is called the prioritized average (PA) operator.

3. INTERVAL-VALUED 2-TUPLE PRIORITIZED AGGREGATION OPERATORS

3.1 Interval-Valued 2-Tuple Prioritized Weighted Average Operators

In the following, we extend the PA operator to the interval-valued 2-tuple linguistic environment and define an IV2TPWA operator as follows:

Definition 3.1. Let $\{[(s_1, \alpha_1), (s_1', \alpha_1')], [(s_2, \alpha_2), (s_2', \alpha_2')], \ldots, [(s_n, \alpha_n), (s_n', \alpha_n')]\}$ be a set of interval-valued 2-tuples. If

$$\begin{aligned}&\mathrm{IV2TPWA}\left(\left[(s_1,\alpha_1),(s_1',\alpha_1')\right],\left[(s_2,\alpha_2),(s_2',\alpha_2')\right],\ldots,\left[(s_n,\alpha_n),(s_n',\alpha_n')\right]\right)\\&=\Delta\left(\left[\sum_{i=1}^{n}\left(\frac{T_i}{\sum_{j=1}^{n}T_j}\Delta^{-1}(s_i,\alpha_i)\right),\sum_{i=1}^{n}\left(\frac{T_i'}{\sum_{j=1}^{n}T_j'}\Delta^{-1}(s_i',\alpha_i')\right)\right]\right),\end{aligned} \tag{12}$$

where $T_1=T_1'=1$, $T_j=\prod_{k=1}^{j-1}\Delta^{-1}(s_k,\alpha_k)\ (j=2,\ldots,n)$, and $T_j'=\prod_{k=1}^{j-1}\Delta^{-1}(s_k',\alpha_k')\ (j=2,\ldots,n)$, then IV2TPWA is called an interval-valued 2-tuple prioritized weighted average operator.

Theorem 3.1 (Boundedness). If $\left\{\left[(s_1,\alpha_1),(s_1',\alpha_1')\right],\left[(s_2,\alpha_2),(s_2',\alpha_2')\right],\ldots,\left[(s_n,\alpha_n),(s_n',\alpha_n')\right]\right\}$ is a set of interval-valued 2-tuples, then

$$\begin{aligned}&\left[\min_{1\le i\le n}\{(s_i,\alpha_i)\},\min_{1\le i\le n}\{(s_i',\alpha_i')\}\right]\\&\le \mathrm{IV2TPWA}\left(\left[(s_1,\alpha_1),(s_1',\alpha_1')\right],\left[(s_2,\alpha_2),(s_2',\alpha_2')\right],\ldots,\left[(s_n,\alpha_n),(s_n',\alpha_n')\right]\right)\\&\le\left[\max_{1\le i\le n}\{(s_i,\alpha_i)\},\max_{1\le i\le n}\{(s_i',\alpha_i')\}\right].\end{aligned} \tag{13}$$

Theorem 3.2 (Idempotency). Let $\left\{\left[(s_1,\alpha_1),(s_1',\alpha_1')\right],\left[(s_2,\alpha_2),(s_2',\alpha_2')\right],\ldots,\left[(s_n,\alpha_n),(s_n',\alpha_n')\right]\right\}$ be a set of interval-valued 2-tuples. If all $\left[(s_j,\alpha_j),(s_j',\alpha_j')\right]\ (j=1,2,\ldots,n)$ are equal, i.e., if $\left[(s_j,\alpha_j),(s_j',\alpha_j')\right]=\left[(s,\alpha),(s',\alpha')\right]$ for all j, then

$$\mathrm{IV2TPWA}\left(\left[(s_1,\alpha_1),(s_1',\alpha_1')\right],\left[(s_2,\alpha_2),(s_2',\alpha_2')\right],\ldots,\left[(s_n,\alpha_n),(s_n',\alpha_n')\right]\right)=\left[(s,\alpha),(s',\alpha')\right]. \tag{14}$$

Theorem 3.3 (Monotonicity). Let $\left\{\left[(s_1,\alpha_1),(s_1',\alpha_1')\right],\left[(s_2,\alpha_2),(s_2',\alpha_2')\right],\ldots,\left[(s_n,\alpha_n),(s_n',\alpha_n')\right]\right\}$ and $\left\{\left[(s_1^*,\alpha_1^*),(s_1'^*,\alpha_1'^*)\right],\left[(s_2^*,\alpha_2^*),(s_2'^*,\alpha_2'^*)\right],\ldots,\left[(s_n^*,\alpha_n^*),(s_n'^*,\alpha_n'^*)\right]\right\}$ be two sets of interval-valued 2-tuples. If $(s_j^*,\alpha_j^*)\ge(s_j,\alpha_j)$ and $(s_j'^*,\alpha_j'^*)\ge(s_j',\alpha_j')$ for all j, then

$$\begin{aligned}&\mathrm{IV2TPWA}\left(\left[(s_1^*,\alpha_1^*),(s_1'^*,\alpha_1'^*)\right],\left[(s_2^*,\alpha_2^*),(s_2'^*,\alpha_2'^*)\right],\ldots,\left[(s_n^*,\alpha_n^*),(s_n'^*,\alpha_n'^*)\right]\right)\\&\quad\ge \mathrm{IV2TPWA}\left(\left[(s_1,\alpha_1),(s_1',\alpha_1')\right],\left[(s_2,\alpha_2),(s_2',\alpha_2')\right],\ldots,\left[(s_n,\alpha_n),(s_n',\alpha_n')\right]\right).\end{aligned} \tag{15}$$

3.2 Interval-Valued 2-Tuple Prioritized Weighted Geometric Operators

Based on the IV2TPWA operator and the geometric mean, we now define an interval-valued 2-tuple prioritized weighted geometric (IV2TPWG) operator.

Definition 3.2. Let $\left\{\left[(s_1,\alpha_1),(s_1',\alpha_1')\right],\left[(s_2,\alpha_2),(s_2',\alpha_2')\right],\ldots,\left[(s_n,\alpha_n),(s_n',\alpha_n')\right]\right\}$ be a set of interval-valued 2-tuples. If

$$\begin{aligned}&\mathrm{IV2TPWG}\left(\left[(s_1,\alpha_1),(s_1',\alpha_1')\right],\left[(s_2,\alpha_2),(s_2',\alpha_2')\right],\ldots,\left[(s_n,\alpha_n),(s_n',\alpha_n')\right]\right)\\&=\Delta\left(\left[\prod_{i=1}^{n}\left(\left(\Delta^{-1}(s_i,\alpha_i)\right)^{T_i/\sum_{j=1}^{n}T_j}\right),\prod_{i=1}^{n}\left(\left(\Delta^{-1}(s_i',\alpha_i')\right)^{T_i'/\sum_{j=1}^{n}T_j'}\right)\right]\right),\end{aligned} \tag{16}$$

where $T_1=T_1'=1$, $T_j=\prod_{k=1}^{j-1}\Delta^{-1}(s_k,\alpha_k)$ $(j=2,\ldots,n)$, and $T_j'=\prod_{k=1}^{j-1}\Delta^{-1}(s_k',\alpha_k')$ $(j=2,\ldots,n)$, then IV2TPWG is called an interval-valued 2-tuple prioritized weighted geometric (IV2TPWG) operator.

Theorem 3.4 (Boundedness). Let $\left\{\left[(s_1,\alpha_1),(s_1',\alpha_1')\right],\left[(s_2,\alpha_2),(s_2',\alpha_2')\right],\ldots,\left[(s_n,\alpha_n),(s_n',\alpha_n')\right]\right\}$ be a set of interval-valued 2-tuples, then

$$\begin{aligned}&\left[\min_{1\le i\le n}\{(s_i,\alpha_i)\},\min_{1\le i\le n}\{(s_i',\alpha_i')\}\right]\\&\le \mathrm{IV2TPWG}\left(\left[(s_1,\alpha_1),(s_1',\alpha_1')\right],\left[(s_2,\alpha_2),(s_2',\alpha_2')\right],\ldots,\left[(s_n,\alpha_n),(s_n',\alpha_n')\right]\right)\\&\le\left[\max_{1\le i\le n}\{(s_i,\alpha_i)\},\max_{1\le i\le n}\{(s_i',\alpha_i')\}\right].\end{aligned} \tag{17}$$

Theorem 3.5 (Idempotency). Let $\left\{\left[(s_1,\alpha_1),(s_1',\alpha_1')\right],\left[(s_2,\alpha_2),(s_2',\alpha_2')\right],\ldots,\left[(s_n,\alpha_n),(s_n',\alpha_n')\right]\right\}$ be a set of interval-valued 2-tuples. If all $\left[(s_j,\alpha_j),(s_j',\alpha_j')\right]$ $(j=1,2,\ldots,n)$ are equal, i.e., if $\left[(s_j,\alpha_j),(s_j',\alpha_j')\right]=\left[(s,\alpha),(s',\alpha')\right]$ for all j, then

$$\mathrm{IV2TPWG}\left(\left[(s_1,\alpha_1),(s_1',\alpha_1')\right],\left[(s_2,\alpha_2),(s_2',\alpha_2')\right],\ldots,\left[(s_n,\alpha_n),(s_n',\alpha_n')\right]\right)=\left[(s,\alpha),(s',\alpha')\right]. \tag{18}$$

Theorem 3.6 (Monotonicity). Let $\left\{\left[(s_1,\alpha_1),(s_1',\alpha_1')\right],\left[(s_2,\alpha_2),(s_2',\alpha_2')\right],\ldots,\left[(s_n,\alpha_n),(s_n',\alpha_n')\right]\right\}$ and $\left\{\left[(s_1^*,\alpha_1^*),(s_1'^*,\alpha_1'^*)\right],\left[(s_2^*,\alpha_2^*),(s_2'^*,\alpha_2'^*)\right],\ldots,\left[(s_n^*,\alpha_n^*),(s_n'^*,\alpha_n'^*)\right]\right\}$ be two sets of interval-valued 2-tuples. If $(s_j^*,\alpha_j^*)\ge(s_j,\alpha_j)$ and $(s_j'^*,\alpha_j'^*)\ge(s_j',\alpha_j')$ for all j, then

$$\begin{aligned}&\mathrm{IV2TPWG}\left(\left[(s_1^*,\alpha_1^*),(s_1'^*,\alpha_1'^*)\right],\left[(s_2^*,\alpha_2^*),(s_2'^*,\alpha_2'^*)\right],\ldots,\left[(s_n^*,\alpha_n^*),(s_n'^*,\alpha_n'^*)\right]\right)\\&\ge \mathrm{IV2TPWG}\left(\left[(s_1,\alpha_1),(s_1',\alpha_1')\right],\left[(s_2,\alpha_2),(s_2',\alpha_2')\right],\ldots,\left[(s_n,\alpha_n),(s_n',\alpha_n')\right]\right).\end{aligned} \tag{19}$$

Theorem 3.7. If $\left\{\left[(s_1,\alpha_1),(s_1',\alpha_1')\right],\left[(s_2,\alpha_2),(s_2',\alpha_2')\right],\ldots,\left[(s_n,\alpha_n),(s_n',\alpha_n')\right]\right\}$ is a set of interval-valued 2-tuples, then

$$\begin{aligned}&\mathrm{IV2TPWG}\left(\left[(s_1,\alpha_1),(s_1',\alpha_1')\right],\left[(s_2,\alpha_2),(s_2',\alpha_2')\right],\ldots,\left[(s_n,\alpha_n),(s_n',\alpha_n')\right]\right)\\&\le \mathrm{IV2TPWA}\left(\left[(s_1,\alpha_1),(s_1',\alpha_1')\right],\left[(s_2,\alpha_2),(s_2',\alpha_2')\right],\ldots,\left[(s_n,\alpha_n),(s_n',\alpha_n')\right]\right).\end{aligned} \tag{20}$$

Theorem 3.7 shows that the values obtained by the IV2TPWG operator are not larger than those obtained by the IV2TPWA operator.

3.3 Approach to Multiple Attribute Group Decision Making Based on Interval-Valued 2-Tuple Prioritized Aggregation Operators

Here, we solve a multiple attribute group decision making problem, where the attribute values are represented by interval linguistic variables and prioritization relationships exist between the attributes and decision makers.

A group decision making problem with interval linguistic preference information in which the attributes and decision makers are at different priority levels can be described as follows: Let $X=\{x_1,x_2,\cdots,x_m\}$ be the set of alternatives. Let $C=\{c_1,c_2,\ldots,c_n\}$ be a collection of attributes where a prioritization between the attributes is expressed by the linear ordering $c_1 \succ c_2 \succ c_3 \succ \cdots \succ c_n$, which indicates attribute c_j has a higher priority than c_k if $j<k$. Let $D=\{d_1,d_2,\ldots,d_l\}$ be the set of decision makers where a prioritization between the decision makers is expressed by the linear ordering $d_1 \succ d_2 \succ d_3 \succ \cdots \succ d_l$, which indicates decision-maker d_p has a higher priority than d_q if $p<q$. The decision makers $d_k \in D$ $(k=1,2,\ldots,l)$ provide their preferences for each alternative on each attribute and construct the interval linguistic decision matrices $\tilde{R}^{(k)}=\left(\tilde{r}_{ij}^{(k)}\right)_{m\times n}$ $(k=1,2,\ldots,l)$, where $\tilde{r}_{ij}^{(k)}$ takes the form of interval linguistic variable $\left[s_{ij}^{(k)},s_{ij}'^{(k)}\right]$, with $s_{ij}^{(k)},s_{ij}'^{(k)}\in S$, $S=\left\{s_i \middle| i=0,1,2,\ldots,g\right\}$, and $s_{ij}^{(k)}\le s_{ij}'^{(k)}$.

To get the best alternative(s), we now develop an approach based on applying interval-valued 2-tuple prioritized aggregation operators to multiple attribute group decision making with interval linguistic preference information. The proposed approach involves the following steps:

Step 1. Transform the interval linguistic decision matrix $\tilde{R}^{(k)}=\left(\tilde{r}_{ij}^{(k)}\right)_{m\times n}=\left(\left[s_{ij}^{(k)},s_{ij}'^{(k)}\right]\right)_{m\times n}$ $(k=1,2,\ldots,l)$ into the interval-valued 2-tuple linguistic decision matrix $\bar{\tilde{R}}^{(k)}=\left(\bar{\tilde{r}}_{ij}^{(k)}\right)_{m\times n}=\left(\left[\left(s_{ij}^{(k)},0\right),\left(s_{ij}'^{(k)},0\right)\right]\right)_{m\times n}$ $(k=1,2,\ldots,l)$.

Step 2. Calculate the matrices $T^{(p)}=\left(T_{ij}^{(p)}\right)_{m\times n}$ $(p=1,2,\ldots,l)$ and $T'^{(p)}=\left(T_{ij}'^{(p)}\right)_{m\times n}$ $(p=1,2,\ldots,l)$ based on the following equations:

$$T_{ij}^{(p)}=\prod_{k=1}^{p-1}\left(\Delta^{-1}\left(s_{ij}^{(k)},0\right)\right),\ p=2,\ldots,l,\ i=1,2,\ldots,m,\ \ j=1,2,\cdots,n, \tag{21}$$

$$T_{ij}'^{(p)}=\prod_{k=1}^{p-1}\left(\Delta^{-1}\left(s_{ij}'^{(k)},0\right)\right),\ p=2,\ldots,l,\ i=1,2,\ldots,m,\ \ j=1,2,\ldots,n, \tag{22}$$

$$T_{ij}^{(1)}=T_{ij}'^{(1)}=1,\ i=1,2,\ldots,m,\ \ j=1,2,\ldots,n. \tag{23}$$

Step 3. Use the IV2TPWA operator

$$\begin{aligned}&\mathrm{IV2TPWA}\left(\left[\left(s_{ij}^{(1)},0\right),\left(s_{ij}^{\prime(1)},0\right)\right],\left[\left(s_{ij}^{(2)},0\right),\left(s_{ij}^{\prime(2)},0\right)\right],\ldots,\left[\left(s_{ij}^{(l)},0\right),\left(s_{ij}^{\prime(l)},0\right)\right]\right)\\&=\Delta\left(\left[\sum_{k=1}^{l}\left(\frac{T_{ij}^{(k)}}{\sum_{p=1}^{l}T_{ij}^{(p)}}\Delta^{-1}\left(s_{ij}^{(k)},0\right)\right),\sum_{k=1}^{l}\left(\frac{T_{ij}^{\prime(k)}}{\sum_{p=1}^{l}T_{ij}^{\prime(p)}}\Delta^{-1}\left(s_{ij}^{\prime(k)},0\right)\right)\right]\right)\end{aligned} \tag{24}$$

or the IV2TPWG operator

$$\begin{aligned}&\mathrm{IV2TPWG}\left(\left[\left(s_{ij}^{(1)},0\right),\left(s_{ij}^{\prime(1)},0\right)\right],\left[\left(s_{ij}^{(2)},0\right),\left(s_{ij}^{\prime(2)},0\right)\right],\ldots,\left[\left(s_{ij}^{(l)},0\right),\left(s_{ij}^{\prime(l)},0\right)\right]\right)\\&=\Delta\left(\left[\prod_{k=1}^{l}\left(\left(\Delta^{-1}\left(s_{ij}^{(k)},0\right)\right)^{T_{ij}^{(k)}/\sum_{p=1}^{l}T_{ij}^{(p)}}\right),\prod_{k=1}^{l}\left(\left(\Delta^{-1}\left(s_{ij}^{\prime(k)},0\right)\right)^{T_{ij}^{\prime(k)}/\sum_{p=1}^{l}T_{ij}^{\prime(p)}}\right)\right]\right)\end{aligned} \tag{25}$$

to aggregate all the individual interval-valued 2-tuple linguistic decision matrices $\tilde{\bar{R}}^{(k)}=\left(\tilde{\bar{r}}_{ij}^{(k)}\right)_{m\times n}=\left(\left[\left(s_{ij}^{(k)},0\right),\left(s_{ij}^{\prime(k)},0\right)\right]\right)_{m\times n}$ $(k=1,2,\ldots,l)$ into the collective interval-valued 2-tuple linguistic decision matrix $\tilde{\bar{R}}=\left(\tilde{\bar{r}}_{ij}\right)_{m\times n}=\left(\left[\left(s_{ij},\alpha_{ij}\right),\left(s_{ij}',\alpha_{ij}'\right)\right]\right)_{m\times n}$.

Step 4. Calculate the matrix $T=\left(T_{ij}\right)_{m\times n}$ and $T'=\left(T_{ij}'\right)_{m\times n}$ based on the following equations:

$$T_{ij}=\prod_{k=1}^{j-1}\left(\Delta^{-1}\left(s_{ik},\alpha_{ik}\right)\right)\ \left(i=1,2,\ldots,m,\ j=2,\cdots,n\right), \tag{26}$$

$$T_{ij}'=\prod_{k=1}^{j-1}\left(\Delta^{-1}\left(s_{ik}',\alpha_{ik}'\right)\right)\ \left(i=1,2,\ldots,m,\ j=2,\cdots,n\right), \tag{27}$$

$$T_{i1}=T_{i1}'=1\ \left(i=1,2,\ldots,m\right). \tag{28}$$

Step 5. Use the IV2TPWA operator

$$\begin{aligned}&\tilde{\bar{r}}_i=\left[\left(s_i,\alpha_i\right),\left(s_i',\alpha_i'\right)\right]\\&=\mathrm{IV2TPWA}\left(\left[\left(s_{i1},\alpha_{i1}\right),\left(s_{i1}',\alpha_{i1}'\right)\right],\left[\left(s_{i2},\alpha_{i2}\right),\left(s_{i2}',\alpha_{i2}'\right)\right],\ldots,\left[\left(s_{in},\alpha_{in}\right),\left(s_{in}',\alpha_{in}'\right)\right]\right)\\&=\Delta\left(\left[\sum_{k=1}^{n}\frac{T_{ik}}{\sum_{j=1}^{n}T_{ij}}\Delta^{-1}\left(s_{ik},\alpha_{ik}\right),\sum_{k=1}^{n}\frac{T_{ik}'}{\sum_{j=1}^{n}T_{ij}'}\Delta^{-1}\left(s_{ik}',\alpha_{ik}'\right)\right]\right)\end{aligned} \tag{29}$$

or the IV2TPWG operator

$$\begin{aligned}\bar{\tilde{r}}_i &= \left[(s_i,\alpha_i),(s_i',\alpha_i')\right] \\ &= \text{IV2TPWG}\left(\left[(s_{i1},\alpha_{i1}),(s_{i1}',\alpha_{i1}')\right],\left[(s_{i2},\alpha_{i2}),(s_{i2}',\alpha_{i2}')\right],\ldots,\left[(s_{in},\alpha_{in}),(s_{in}',\alpha_{in}')\right]\right) \\ &= \Delta\left(\left[\prod_{k=1}^{n}\left(\left(\Delta^{-1}(s_{ik},\alpha_{ik})\right)^{T_{ik}\Big/\sum_{j=1}^{n}T_{ij}}\right),\prod_{k=1}^{n}\left(\left(\Delta^{-1}(s_{ik}',\alpha_{ik}')\right)^{T_{ik}'\Big/\sum_{j=1}^{n}T_{ij}'}\right)\right]\right)\end{aligned} \quad (30)$$

to derive the collective overall preference $\bar{\tilde{r}}_i = \left[(s_i,\alpha_i),(s_i',\alpha_i')\right]$ of the alternative x_i.

Step 6. According to Theorem 2.2, rank all alternatives x_i $(i=1,2,\ldots,m)$ and select the best one(s) in accordance with the collective overall preferences $\bar{\tilde{r}}_i = \left[(s_i,\alpha_i),(s_i',\alpha_i')\right]$ $(i=1,2,\ldots,m)$.

Step 7. End.

4. ILLUSTRATIVE EXAMPLES

We now consider an illustrative example adapted from Herrera et al. [11] and Herrera and Herrera-Viedma [9].

Example 4.1 [9,11]. Suppose an investment company wants to invest a sum of money in the best option. There are four alternatives in which to invest the money: (1) x_1 is in the car industry; (2) x_2 is a food company; (3) x_3 is a computer company; and (4) x_4 is in the arms industry. The investment company decides according to the following four attributes: (1) c_1 is the risk analysis; (2) c_2 is the growth analysis; (3) c_3 is the socio-political impact analysis; and (4) c_4 is the environmental-impact analysis. The four possible alternatives x_i $(i=1,2,3,4)$ are to be evaluated by using the linguistic-term set

$$S=\left\{\begin{array}{l} s_0=\text{extremely poor},\ s_1=\text{very poor},\ s_2=\text{poor},\ s_3=\text{slightly poor},\ s_4=\text{fair}, \\ s_5=\text{slightly good},\ s_6=\text{good},\ s_7=\text{very good},\ s_8=\text{extremely good}\end{array}\right\}$$

by three decision makers d_k $(k=1,2,3)$ under the four attributes listed above. Construct, respectively, the interval linguistic decision matrices $\tilde{R}^{(k)}=\left(\tilde{r}_{ij}^{(k)}\right)_{m\times n}=\left(\left[s_{ij}^{(k)},s_{ij}'^{(k)}\right]\right)_{m\times n}$ $(k=1,2,3)$ as shown in Tables 1–3. Decision maker d_1 has absolute priority for decision making, decision maker d_2 has the next priority. In other words, there is a prioritization between the three decision makers that is expressed by the linear ordering $d_1 \succ d_2 \succ d_3$. In the opinion of the three decision makers, the attributes have the following prioritization: the risk analysis of the candidate is the most important and the environmental-impact analysis of the candidate is not as important as the other attributes. Therefore, the prioritization relationship can be written as $c_1 \succ c_2 \succ c_3 \succ c_4$.

Table 1. Decision matrix $\tilde{R}^{(1)}$ provided by d_1

1	c_1	c_2	c_3	c_4
x_1	$[s_1, s_4]$	$[s_2, s_3]$	$[s_2, s_3]$	$[s_5, s_7]$
x_2	$[s_1, s_2]$	$[s_3, s_4]$	$[s_3, s_6]$	$[s_6, s_8]$
x_3	$[s_5, s_8]$	$[s_1, s_2]$	$[s_1, s_3]$	$[s_5, s_6]$
x_4	$[s_3, s_4]$	$[s_1, s_2]$	$[s_1, s_3]$	$[s_7, s_8]$

Table 2. Decision matrix $\tilde{R}^{(2)}$ provided by d_2

2	c_1	c_2	c_3	c_4
x_1	$[s_7, s_8]$	$[s_5, s_6]$	$[s_3, s_4]$	$[s_3, s_5]$
x_2	$[s_3, s_6]$	$[s_1, s_2]$	$[s_1, s_2]$	$[s_6, s_8]$
x_3	$[s_1, s_3]$	$[s_2, s_3]$	$[s_3, s_4]$	$[s_4, s_5]$
x_4	$[s_4, s_5]$	$[s_1, s_3]$	$[s_1, s_2]$	$[s_6, s_8]$

Table 3. Decision matrix $\tilde{R}^{(3)}$ provided by d_3

3	c_1	c_2	c_3	c_4
x_1	$[s_1, s_3]$	$[s_1, s_2]$	$[s_1, s_3]$	$[s_5, s_7]$
x_2	$[s_6, s_7]$	$[s_6, s_8]$	$[s_5, s_6]$	$[s_3, s_4]$
x_3	$[s_7, s_8]$	$[s_1, s_2]$	$[s_7, s_8]$	$[s_2, s_3]$
x_4	$[s_3, s_5]$	$[s_5, s_8]$	$[s_5, s_7]$	$[s_6, s_8]$

Step 1. Transform the interval linguistic decision matrices $\tilde{R}^{(k)} = \left(\tilde{r}_{ij}^{(k)}\right)_{4\times4} = \left(\left[s_{ij}^{(k)}, s_{ij}'^{(k)}\right]\right)_{4\times4}$ $(k = 1,2,3)$ given in Tables 1–3 into the interval-valued 2-tuple linguistic decision matrices $\bar{\tilde{R}}^{(k)} = \left(\bar{\tilde{r}}_{ij}^{(k)}\right)_{4\times4} = \left(\left[\left(s_{ij}^{(k)},0\right),\left(s_{ij}'^{(k)},0\right)\right]\right)_{4\times4}$ $(k = 1,2,3)$ given in Tables 4–6.

Table 4. Interval-valued 2-tuple linguistic decision matrix $\bar{\tilde{R}}^{(1)}$

4	c_1	c_2	c_3	c_4
x_1	$[(s_1,0),(s_4,0)]$	$[(s_2,0),(s_3,0)]$	$[(s_2,0),(s_3,0)]$	$[(s_5,0),(s_7,0)]$
x_2	$[(s_1,0),(s_2,0)]$	$[(s_3,0),(s_4,0)]$	$[(s_3,0),(s_6,0)]$	$[(s_6,0),(s_8,0)]$
x_3	$[(s_5,0),(s_8,0)]$	$[(s_1,0),(s_2,0)]$	$[(s_1,0),(s_3,0)]$	$[(s_5,0),(s_6,0)]$
x_4	$[(s_3,0),(s_4,0)]$	$[(s_1,0),(s_2,0)]$	$[(s_1,0),(s_3,0)]$	$[(s_7,0),(s_8,0)]$

Table 5. Interval-valued 2-tuple linguistic decision matrix $\tilde{\bar{R}}^{(2)}$

5	c_1	c_2	c_3	c_4
x_1	$[(s_7,0),(s_8,0)]$	$[(s_5,0),(s_6,0)]$	$[(s_3,0),(s_4,0)]$	$[(s_3,0),(s_5,0)]$
x_2	$[(s_3,0),(s_6,0)]$	$[(s_1,0),(s_2,0)]$	$[(s_1,0),(s_2,0)]$	$[(s_6,0),(s_8,0)]$
x_3	$[(s_1,0),(s_3,0)]$	$[(s_2,0),(s_3,0)]$	$[(s_3,0),(s_4,0)]$	$[(s_4,0),(s_5,0)]$
x_4	$[(s_4,0),(s_5,0)]$	$[(s_1,0),(s_3,0)]$	$[(s_1,0),(s_2,0)]$	$[(s_6,0),(s_8,0)]$

Table 6. Interval-valued 2-tuple linguistic decision matrix $\tilde{\bar{R}}^{(3)}$

6	c_1	c_2	c_3	c_4
x_1	$[(s_1,0),(s_3,0)]$	$[(s_1,0),(s_2,0)]$	$[(s_1,0),(s_3,0)]$	$[(s_5,0),(s_7,0)]$
x_2	$[(s_6,0),(s_7,0)]$	$[(s_6,0),(s_8,0)]$	$[(s_5,0),(s_6,0)]$	$[(s_3,0),(s_4,0)]$
x_3	$[(s_7,0),(s_8,0)]$	$[(s_1,0),(s_2,0)]$	$[(s_7,0),(s_8,0)]$	$[(s_2,0),(s_3,0)]$
x_4	$[(s_3,0),(s_5,0)]$	$[(s_5,0),(s_8,0)]$	$[(s_5,0),(s_7,0)]$	$[(s_6,0),(s_8,0)]$

Step 2. Use Eqs. (21)–(23) to calculate the matrices $T^{(1)}$, $T^{(2)}$, $T^{(3)}$, $T'^{(1)}$, $T'^{(2)}$, and $T'^{(3)}$ as follows:

$$T^{(1)}=\begin{pmatrix}1&1&1&1\\1&1&1&1\\1&1&1&1\\1&1&1&1\end{pmatrix},\qquad T^{(2)}=\begin{pmatrix}0.1250&0.2500&0.2500&0.6250\\0.1250&0.3750&0.3750&0.7500\\0.6250&0.1250&0.1250&0.6250\\0.3750&0.1250&0.1250&0.8750\end{pmatrix},$$

$$T^{(3)}=\begin{pmatrix}0.1094&0.1563&0.0938&0.2344\\0.0469&0.0469&0.0469&0.5625\\0.0781&0.0313&0.0469&0.3125\\0.1875&0.0156&0.0156&0.6563\end{pmatrix},$$

$$T'^{(1)}=\begin{pmatrix}1&1&1&1\\1&1&1&1\\1&1&1&1\\1&1&1&1\end{pmatrix},\qquad T'^{(2)}=\begin{pmatrix}0.5000&0.3750&0.3750&0.8750\\0.2500&0.5000&0.7500&1.0000\\1.0000&0.2500&0.3750&0.7500\\0.5000&0.2500&0.3750&1.0000\end{pmatrix},$$

$$T'^{(3)}=\begin{pmatrix}0.1094&0.1563&0.0938&0.2344\\0.0469&0.0469&0.0469&0.5625\\0.0781&0.0313&0.0469&0.3125\\0.1875&0.0156&0.0156&0.6563\end{pmatrix}.$$

Step 3. Use the IV2TPWA operator [Eq. (24)] to aggregate all the individual interval-valued 2-tuple linguistic decision matrices $\tilde{\bar{R}}^{(k)}=\left(\tilde{\bar{r}}_{ij}^{(k)}\right)_{4\times4}=\left(\left[\left(s_{ij}^{(k)},0\right),\left(s_{ij}'^{(k)},0\right)\right]\right)_{4\times4}$ $(k=1,2,3)$ into the collective interval-valued 2-tuple linguistic decision matrix $\tilde{\bar{R}} = (\tilde{\bar{r}}_{ij})_{4\times4} = ([(s_{ij}, \alpha_{ij}), (s'_{ij}, \alpha'_{ij})])_{4\times4}$ (see Table 7).

Step 4. Calculate the matrix $T = (T_{ij})_{4\times4}$ and $T' = (T'_{ij})_{4\times4}$ based on Eqs. (26)–(28):

$$T = \begin{pmatrix} 1 & 0.2009 & 0.0608 & 0.0161 \\ 1 & 0.1767 & 0.0568 & 0.0180 \\ 1 & 0.4530 & 0.0627 & 0.0114 \\ 1 & 0.4050 & 0.0534 & 0.0070 \end{pmatrix}, \qquad T' = \begin{pmatrix} 1 & 0.5938 & 0.2605 & 0.1055 \\ 1 & 0.4185 & 0.1931 & 0.1075 \\ 1 & 0.7368 & 0.2013 & 0.0966 \\ 1 & 0.5560 & 0.1810 & 0.0679 \end{pmatrix}.$$

Step 5. Use the IV2TPWA operator [Eq. (29)] to derive the collective overall preference $\tilde{\bar{r}}_i = [(s_i, \alpha_i), (s'_i, \alpha'_i)]$ of the alternative x_i :

$$\tilde{\bar{r}}_1 = [(s_2, -0.0257), (s_4, 0.0320)],\ \tilde{\bar{r}}_2 = [(s_2, -0.0396), (s_4, -0.0296)],$$
$$\tilde{\bar{r}}_3 = [(s_3, -0.0259), (s_4, 0.0384)],\ \tilde{\bar{r}}_4 = [(s_3, -0.0535), (s_4, -0.0164)].$$

Step 6. According to Eq. (9), calculate the score function $S(\tilde{\bar{r}}_i)$ $(i = 1,2,3,4)$ of $\tilde{\bar{r}}_i$ $(i = 1,2,3,4)$ as follows:

$$S(\tilde{\bar{r}}_1) = 0.3781,\quad S(\tilde{\bar{r}}_2) = 0.3404,\quad S(\tilde{\bar{r}}_3) = 0.4437,\quad S(\tilde{\bar{r}}_4) = 0.4025.$$

Because, by Theorem 2.2, $S(\tilde{\bar{r}}_3) > S(\tilde{\bar{r}}_4) > S(\tilde{\bar{r}}_1) > S(\tilde{\bar{r}}_2)$, we have $\tilde{\bar{r}}_3 > \tilde{\bar{r}}_4 > \tilde{\bar{r}}_1 > \tilde{\bar{r}}_2$, which implies that $x_3 \succ x_4 \succ x_1 \succ x_2$. Thus, the best candidate is x_3.

If we deal with Example 4.1 by using the IV2TPWG operator instead of the IV2TPWA operator, then the main steps are as follows:

Step 1. See Step 1.

Step 2. See Step 2.

Step 3. Use the IV2TPWG operator [Eq. (25)] to aggregate all the individual interval-valued 2-tuple linguistic decision matrices $\tilde{\bar{R}}^{(k)} = (\tilde{\bar{r}}_{ij}^{(k)})_{4\times4} = ([(s_{ij}^{(k)}, 0), (s_{ij}'^{(k)}, 0)])_{4\times4}$ $(k = 1,2,3)$ into the collective interval-valued 2-tuple linguistic decision matrix $\tilde{\bar{R}}^* = (\tilde{\bar{r}}_{ij}^*)_{4\times4} = ([(s_{ij}{}^*, \alpha_{ij}{}^*), (s_{ij}'^*, \alpha_{ij}'^*)])_{4\times4}$ (see Table 8).

Step 4. Calculate the matrix $T^* = (T^*{}_{ij})_{4\times4}$ and $T'^* = (T'^*_{ij})_{4\times4}$ based on Eqs. (26)–(28):

$$T^* = \begin{pmatrix} 1 & 0.1522 & 0.0415 & 0.0107 \\ 1 & 0.1510 & 0.0434 & 0.0124 \\ 1 & 0.3516 & 0.0474 & 0.0072 \\ 1 & 0.4018 & 0.0513 & 0.0066 \end{pmatrix}, \qquad T'^* = \begin{pmatrix} 1 & 0.5533 & 0.2266 & 0.0911 \\ 1 & 0.3564 & 0.1518 & 0.0744 \\ 1 & 0.6617 & 0.1784 & 0.0806 \\ 1 & 0.5526 & 0.1641 & 0.0586 \end{pmatrix}.$$

Table 7. Collective interval-valued 2-tuple linguistic decision matrix $\bar{\tilde{R}}$

7	c_1	c_2	c_3	c_4
x_1	$[(s_2,-0.0491),(s_5,-0.0313)]$	$[(s_2,0.0528),(s_4,-0.0613)]$	$[(s_2,0.0145),(s_3,0.0300)]$	$[(s_4,0.0410),(s_6,0.0347)]$
x_2	$[(s_1,0.0517),(s_3,0.0435)]$	$[(s_3,-0.0536),(s_4,-0.0385)]$	$[(s_3,-0.0577),(s_4,0.0565)]$	$[(s_5,0.0338),(s_7,-0.0417)]$
x_3	$[(s_4,-0.0470),(s_6,-0.0132)]$	$[(s_1,0.0135),(s_2,0.0233)]$	$[(s_1,0.0567),(s_4,-0.0200)]$	$[(s_4,0.0242),(s_5,0.0035)]$
x_4	$[(s_3,0.0300),(s_4,0.0560)]$	$[(s_1,0.0068),(s_3,-0.0494)]$	$[(s_1,0.0068),(s_3,-0.0000)]$	$[(s_6,0.0494),(s_8,0)]$

Table 8. Collective interval-valued 2-tuple linguistic decision matrix

8	c_1	c_2	c_3	c_4
x_1	$[(s_1,0.0272),(s_4,0.0533)]$	$[(s_2,0.0224),(s_3,0.0345)]$	$[(s_2,0.0069),(s_3,0.0268)]$	$[(s_4,0.0264),(s_6,0.0248)]$
x_2	$[(s_1,0.0260),(s_3,-0.0186)]$	$[(s_2,0.0372),(s_3,0.0511)]$	$[(s_2,0.0354),(s_4,-0.0098)]$	$[(s_5,0.0086),(s_6,0.0437)]$
x_3	$[(s_3,-0.0234),(s_5,0.0367)]$	$[(s_1,0.0097),(s_2,0.0196)]$	$[(s_1,0.0269),(s_4,-0.0480)]$	$[(s_4,0.0017),(s_5,-0.0159)]$
x_4	$[(s_3,0.0268),(s_4,0.0526)]$	$[(s_1,0.0028),(s_2,0.0470)]$	$[(s_1,0.0028),(s_3,-0.0181)]$	$[(s_6,0.0471),(s_8,0)]$

Step 5. Use the IV2TPWG operator [Eq. (30)] to derive the collective overall preference $\bar{\tilde{r}}_i^* = \left[(s_i^*, \alpha_i^*), (s_i^{\prime *}, \alpha_i^{\prime *}) \right]$ of the alternative x_i:

$$\bar{\tilde{r}}_1^* = \left[(s_1, 0.0437), (s_4, -0.0050) \right],\ \bar{\tilde{r}}_2^* = \left[(s_1, 0.0449), (s_3, 0.0222) \right],$$
$$\bar{\tilde{r}}_3^* = \left[(s_2, 0.0194), (s_4, -0.0329) \right],\ \bar{\tilde{r}}_4^* = \left[(s_2, 0.0324), (s_4, -0.0539) \right].$$

Step 6. According to Eq. (9), calculate the score function $S\left(\bar{\tilde{r}}_i^*\right)$ $(i = 1,2,3,4)$ of $\bar{\tilde{r}}_i^*$ $(i = 1,2,3,4)$ as follows:

$S\left(\bar{\tilde{r}}_1^*\right) = 0.3781$, $S\left(\bar{\tilde{r}}_2^*\right) = 0.3404$, $S\left(\bar{\tilde{r}}_3^*\right) = 0.4437$, $S\left(\bar{\tilde{r}}_4^*\right) = 0.4025$.

Because, by Theorem 2.2, $S\left(\bar{\tilde{r}}_3^*\right) > S\left(\bar{\tilde{r}}_4^*\right) > S\left(\bar{\tilde{r}}_1^*\right) > S\left(\bar{\tilde{r}}_2^*\right)$, we have $\bar{\tilde{r}}_3^* > \bar{\tilde{r}}_4^* > \bar{\tilde{r}}_1^* > \bar{\tilde{r}}_2^*$, which implies that $x_3 \succ x_4 \succ x_1 \succ x_2$. Thus, the best candidate is x_3.

From Example 4.1, we see that there are different priority levels among the four attributes and the three decision makers. For instance, if a candidate is immoral, then this candidate cannot be selected by the three decision makers, no matter how high the other three attributes are for him or her. If a candidate receives a bad evaluation from a university president, then he or she cannot be selected no matter how high the evaluations are from the other two decision makers. Clearly, the existing interval-valued 2-tuple linguistic aggregation operators have difficulty dealing with such cases because these operators are usually used to solve MAGDMs, where the attributes and the decision makers are at the same priority level. However, the operators proposed in this paper not only accommodate the interval-valued linguistic preference information but also account for prioritization among the attributes and the decision makers. Thus, the proposed operators and approaches can cope effectively with situations in which the attributes and the decision makers are at different priority levels.

5. CONCLUSIONS

This paper provides interval-valued 2-tuple prioritized aggregation operators to deal with situations in which a prioritization relationship exists between the attributes and decision makers in multiple attribute group decision making problems with interval-valued linguistic information. Furthermore, we apply the proposed operators to solve multiple attribute group decision making problems. Finally, an illustrative example is presented to show that the proposed approaches are not only more reasonable but more efficient in practical applications because these approaches consider the prioritization relationship between the attributes and decision makers.

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COMPETING INTERESTS

Author has declared that no competing interests exist.

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