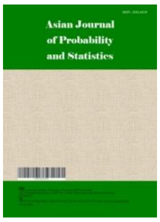

Asian Journal of Probability and Statistics

14(4): 52-64, 2021; Article no.AJPAS.74092
ISSN: 2582-0230

Modified Class of Estimator for Finite Population Mean Under Two-Phase Sampling Using Regression Estimation Approach

A. Y. Erinola[1*], R. V. K. Singh[1], A. Audu[2] and T. James[1]

[1]Department of Mathematics, Kebbi State University of Science and Technology, Aliero, Nigeria.
[2]Department of Mathematics, Usmanu Danfodiyo University, Sokoto, Nigeria.

Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/AJPAS/2021/v14i430338
Editor(s):
(1) Dr. Manuel Alberto M. Ferreira, Lisbon University, Portugal.
Reviewers:
(1) Waqar Hafeez, Universiti Utara Malaysia, Malaysia.
(2) Susanna Spektor, Canada.
Complete Peer review History: https://www.sdiarticle4.com/review-history/74092

Original Research Article

Received 06 July 2021
Accepted 16 September 2021
Published 30 September 2021

Abstract

This study proposed modified a class of estimator in simple random sampling for the estimation of population mean of the study variable using as axillary information. The biases and MSE of suggested estimators were derived up to the first order approximation using Taylor's series expansion approach. Theoretically, the suggested estimators were compared with the existing estimators in the literature. The mean square errors (MSE) and percentage relative efficiency (PRE) of proposed estimators and that of some existing estimators were computed numerically and the results revealed that the members of the proposed class of estimator were more efficient compared to their counterparts and can produce better estimates than other estimators considered in the study.

Keywords: Auxiliary attribute; ratio-exponential estimators; mean square error; efficiency; two-phase sampling.

**Corresponding author: Email: princessfathyar@yahoo.com, princessfaithyar@yahoo.com;*

1 Introduction

In the determination of information about a particular population characteristic (for example, the mean) a random sample from that population is usually used because it is infeasible to measure the entire population. Using that sample, the corresponding sample characteristic is calculated, which is used to summarize information about the unknown population characteristic. The population characteristic of interest is called a parameter and the corresponding sample characteristic is the sample statistic or parameter estimate. Because the statistic is a summary of information about a parameter obtained from the sample, the value of a statistic depends on the particular sample that was drawn from the population. Its values change randomly from one random sample to the next one, therefore a statistic is a random quantity (variable). The sampling distribution of a (sample) statistic is important because it enable us to draw conclusions about the corresponding population parameter based on a random sample Siraj et al. [1].

Several authors like Hafeez et al. [2], Hafeez and Shabbir [3], Ahmed et al. [4,5,6], Audu and Adewara [7], Singh and Audu [8], Muili et al. [9], Hafeez et al. [10] Audu et al. [11], Audu and Singh [12] e.t.c have suggested estimators of population parameters using auxiliary variables while many researchers have proposed estimators based on auxiliary attribute. Bahl and Tuteja [13], Jhajj et al. [14], Singh et al. [15], Shabbir and Gupta [16], Koyuncu [17], Malik and Singh [18], Solanki and Singh [19], Zaman [20], Zaman and Kadilar [21] have suggested estimators by using Auxiliary attributes. In this paper, modified estimators for finite population mean under two-phase sampling using regression estimation approach has been suggested.

2 Some Existing Estimators of Population Mean with Auxiliary Attribute

Consider a sample of size n drawn by simple random sampling without replacement (SRSWOR) from a population of size N. Let y_i and ϕ_i denote the observations on variable y and ϕ respectively for ith unit (i=1,2,…N).

Let ϕ_i = 1; if the ith unit of the population possesses attribute ϕ_i , = 0; otherwise.

Let A = $\sum_{i=1}^{N}\phi_i$ and $a=\sum_{i=1}^{n}\phi_i$, denote the total number of units in the population and sample respectively possessing attribute ϕ . P denotes the proportion of unit in the population. When P is not known, double sampling or two-phase sampling is used to estimate the population mean of the study variable. Under the double sampling scheme, two cases are used for the selection of the required sample as follows:

Case- I. The first phase sample $S'\left(S'\subset\varsigma\right)$ of a fixed size n' is drawn to measure only on the auxiliary attribute p in order to formulate a good estimate of a population proportion P.

Case- II. Given S' , the second phase sampling $S\left(S\subset S'\right)$ of a fixed size n is drawn to measure the study variable y. $\bar{y}=\frac{1}{n}\sum_{i\in S}y_{i,}$ $p=\frac{1}{n}\sum_{i\in S}a_{i,}$ $p'=\frac{1}{n}\sum_{i\in S}a_{i,}$ Where p' denote the proportion of unit possessing attribute ϕ in the first phase sample of size n'; p denote the proportion of unit possessing attribute ϕ in the second phase sample of size $n'>n$ and $\bar{y}$ denote the mean of the study variable y in the second phase sample.

Kumar and Bahl [22] considered ratio estimator of population mean in the two-phase sampling using information of the auxiliary attribute. Their proposed estimator as well as its MSE’s is given as in (2.1) (2.2) and (2.3) respectively.

$$t_{NG}^{d} = \bar{y}\frac{P'}{P} \tag{2.1}$$

$$MSE\left(t_{NGI}^{d}\right)_{I} = \bar{Y}^{2}\left[\theta C_{y}^{2} + \left(\theta - \theta'\right)\left(C_{p}^{2} - 2\theta\rho C_{y} C_{p}\right)\right] \tag{2.2}$$

$$MSE\left(t_{NGI}^{d}\right)_{II} = \bar{Y}^{2}\left[\theta C_{y}^{2} + \left(\theta + \theta'\right)C_{p}^{2} - 2\theta\rho C_{y} C_{p}\right] \tag{2.3}$$

where $\theta = \frac{1}{n} - \frac{1}{N}$, $\theta' = \frac{1}{n'} - \frac{1}{N}$, ρ is the population coefficient of the correlation between the auxiliary attribute and study variable, C_P is the population coefficient of variation for the form of attribute and C_y is the population coefficient of variation of the study variable.

Kumar and Bahl [22] suggested dual to ratio estimator of the population mean under the two- phase sampling as well as its MSE's is given as in (2.4) (2.5) and (2.6) respectively.

$$t_{NG2}^{*d} = \bar{y}\frac{p'*}{p} \tag{2.4}$$

were $p'* = \dfrac{n'p' - np}{n' - n}$

$$MSE\left(t_{NGI}^{*d}\right)_{I} = \bar{Y}^{2}\left\{\theta C_{y}^{2} + \frac{n}{n'-n}\left(\theta - \theta'\right)\left(\frac{n}{n'-n}C_{p}^{2} - 2\rho C_{y} C_{p}\right)\right\} \tag{2.5}$$

$$MSE\left(t_{NGI}^{*d}\right)_{II} = \bar{Y}^{2}\left\{\theta C_{y}^{2} + \frac{n}{n'-n}\left\{\frac{n}{n'-n}\left(\theta - \theta'\right)C_{p}^{2} - 2\rho C_{y} C_{p}\right\}\right\} \tag{2.6}$$

Singh et al. [23] considered the ratio type exponential estimator of population mean in the two-phase sampling using auxiliary attribute. Their proposed estimator is given as well as its MSE's is given as in (2.7) (2.8) and (2.9) respectively.

$$t_{SI} = \bar{y}\exp\left(\frac{p' - p}{p' + p}\right) \tag{2.7}$$

$$MSE\left(t_{SI}\right)_{I} = \bar{Y}^{2}\left\{\theta C_{y}^{2} + \left(\theta - \theta'\right)\left(\frac{C_{p}^{2}}{4} - \rho C_{y} C_{p}\right)\right\} \tag{2.8}$$

$$MSE\left(t_{SI}\right)_{II} = \bar{Y}^{2}\left\{\theta C_{y}^{2} + \left(\theta + \theta'\right)\frac{C_{p}^{2}}{4} - \rho\theta C_{y} C_{p}\right\} \tag{2.9}$$

Kalita and Singh [24], suggested exponential dual to ratio in the two-phase sampling using information of the auxiliary attribute. Their proposed estimator and MSE's are given in (2.10), (2.11) and (2.12) respectively.

$$t_{S}^{*} = \bar{y}\exp\left(\frac{p'* - p}{p'* + p}\right) \tag{2.10}$$

$$MSE(t_{SI}^{*})_I = \bar{Y}^2 \left\{ \theta C_y^2 + \frac{n}{n'-n}(\theta-\theta')\left\{ \frac{n}{4(n'-n)} C_p^2 - \rho C_y C_p \right\} \right\} \quad (2.11)$$

$$\text{MSE}\,(t_{SI}^{*})_{II} = \bar{Y}^2 \left\{ \theta C_y^2 + \frac{n^2}{(n'-n)^2}(\theta-\theta') C_p^2 - \theta \frac{n}{n'-n} \rho C_y C_p \right\} \quad (2.12)$$

Zaman and Kadilar [21], proposed class of ratio estimator to estimate the population mean of the study variable using the information about the population proportion possessing certain attributes in the two-phase sampling as well as its MSE's is given as in (2.13) (2.14) and (2.15) respectively.

$$\bar{y}_{ZKI} = \bar{y} \exp\left\{ \frac{(kp'+l)-(kp+l)}{(kp'+l)+(kp+l)} \right\} \quad (2.13)$$

The MSEs of these estimators are

$$MSE(\bar{y}_{ZKI})_I = \bar{Y}^2 \left\{ \theta C_y^2 + (\theta-\theta')\left(\lambda^2 C_p^2 - 2\lambda\rho C_y C_p\right) \right\} \quad (2.14)$$

$$MSE(\bar{y}_{ZKI})_{II} = \bar{Y}^2 \left\{ \theta C_y^2 + (\theta+\theta')\lambda^2 C_p^2 - 2\lambda\rho C_y C_p \right\} \quad (2.15)$$

3 Suggested Estimator

Having studied the estimator proposed by Zaman and Kadilar [21] we observed that their estimator can only be apply if the correlation between the study variable and auxiliary information is positive. Therefore, there is need to propose estimator that can be applied for all direction of correlation between the study and auxiliary information.

$$t_1 = \bar{y} + b_\phi (p'-p) \exp\left(\frac{2((Ap'+B)-(Ap+B))}{(Ap+B)+(Ap+B)} \right) \quad (3.1)$$

To derive the biases and MSEs of the proposed estimator, the following error term are defined $e_0 = \frac{\bar{y}-\bar{Y}}{\bar{Y}}$, $e_1 = \frac{p-P}{P}$, $e_2 = \frac{p'-P}{P}$ such that $|e_0|<1, |e_1|<1, |e_2|<1$ and $\bar{y} = \bar{Y}(1+e_0)$, $p = P(1+e_1)$, $p' = P(1+e_2)$.

The expectations of the error terms are given as;

Under case I:

$$\left\{ \begin{array}{l} E(e_0) = E(e_1) = E(e_2) = 0, E(e_0^2) = \theta C_y^2 \\ E(e_1^2) = \theta C_p^2, E(e_2^2) = \theta' C_p^2, E(e_0 e_1) = \theta\rho C_y C_p \\ E(e_0 e_2) = \theta'\rho C_y C_p, E(e_1 e_2) = \theta' C_p^2 \end{array} \right\} \quad (3.2)$$

Under case II:

$$\begin{cases} E(e_0)=E(e_1)=E(e_2)=0, E(e_0^2)=\theta C_y^2, E(e_1^2)=\theta C_p^2, \\ E(e_2^2)=\theta' C_p^2, E(e_0e_1)=\theta\rho C_y C_p, E(e_0e_2)=0, E(e_1e_2)=0 \end{cases} \tag{3.3}$$

Expressing the proposed estimator t_1 in terms of errors, we obtained (3.4)

$$t_1=\bar{Y}+\bar{Y}e_0+b_\phi\left(P+Pe_2-P-Pe_1\right)\exp\left(\frac{2\left(AP+APe_2+B-AP-APe_1-B\right)}{AP+APe_2+B+AP+APe_1+B}\right) \tag{3.4}$$

Simplifying (3.4), we have,

$$t_1=\left(\bar{Y}+\bar{Y}e_0+b_\phi Pe_2-b_\phi Pe_1\right)\exp\left(\left(\lambda e_2-\lambda e_1\right)\left(1+\frac{\lambda}{2}e_2+\frac{\lambda}{2}e_1\right)^{-1}\right) \tag{3.5}$$

were $\lambda=\dfrac{AP}{(AP+B)}$

Taking the expansion (3.5)

$$\begin{aligned} t_1-\bar{Y}=\bar{Y}&\left(\frac{\lambda}{2}e_2-\frac{\lambda}{2}e_1+\frac{\lambda^2}{2}e_1^2-\frac{\lambda^2}{2}e_1e_2+e_0+\frac{\lambda}{2}e_0e_2-\frac{\lambda}{2}e_0e_1\right)+ \\ &\left(Pb_\phi e_2+\frac{\lambda^2}{2}Pb_\phi e_2^2-\frac{\lambda}{2}Pb_\phi e_1e_2-\frac{\lambda}{2}Pb_\phi e_1-\frac{\lambda}{2}Pb_\phi e_1e_2+\frac{\lambda}{2}Pb_\phi e_1^2\right) \end{aligned} \tag{3.6}$$

Take expectation of (3.6) and apply the results of (3.2 and 3.3), the bias of proposed estimator $(t_1)_I$ under case I and $(t_1)_{II}$ under case II were obtained respectfully as:

$$Bias(t_I)_I=\bar{Y}\left\{\lambda^2\theta C_p^2-\lambda\theta' C_p^2+\lambda\theta'\rho C_y C_p-\lambda\theta\rho C_y\right\}-Pb_\phi\theta C_p^2 \tag{3.7}$$

$$Bias(t_I)_{II}=\bar{Y}\left\{\lambda^2\theta C_p^2-\lambda\theta\rho C_y C_p\right\}-Pb_\phi\theta C_p^2 \tag{3.8}$$

Square (3.6), take expectation of the result and apply the results of (3.2 and 3.3), the MSE of proposed estimator $(t_1)_I$ and $(t_1)_{II}$ under case I and II were obtained respectfully as:

$$MSE(t_1)_I=\bar{Y}^2\theta C_y^2+\left(\theta-\theta'\right)\left(L^2C_p^2-2\bar{Y}L\rho C_y C_p\right) \tag{3.9}$$

$$MSE(t_1)_{II}=\bar{Y}^2\theta C_y^2+\left(\theta+\theta'\right)L^2C_p^2-2\bar{Y}L\theta\rho C_y C_p \tag{3.10}$$

3.1 Theoretical efficiency comparisons

(i) Theoretical Comparison of Proposed Estimator and Sample Mean

$$\theta\bar{Y}^2C_y^2-\ \bar{Y}^2\theta C_y^2+\left(\theta-\theta'\right)\left(L_i^2C_p^2-2\bar{Y}L_i\rho C_y C_p\right)\ >0$$

$$\left(\theta-\theta'\right)\left(L_i^2C_p^2-2\bar{Y}L_i\rho C_y C_p\right)<0 \tag{3.11}$$

(ii) Theoretical Comparison of Proposed Ratio Estimator and Estimator of Zaman and Kadilar [21] under study for case I

$$\bar{Y}^2\theta C_y^2+(\theta-\theta')\left(L_i^2C_p^2-2\bar{Y}L_i\rho C_yC_p\right)-\bar{Y}^2\left[\theta C_y^2+(\theta-\theta')\left(\lambda_i^2C_p^2-2\lambda_i\rho C_yC_p\right)\right]<0 \qquad (3.12)$$

where L_i and λ_i i = 1,2,3,…..,9

Table 3.1. Members of the Proposed Ratio Estimator t_1

Estimator	Known Function of Auxiliary Attribute	
	A	**B**
$t_1=\left[\bar{y}+b_\phi(p'-p)\right]\exp\left[\frac{2(p'-p)}{p'+p+2\beta_2(\phi)}\right]$	1	$\beta_2(\phi)$
$t_2=\left[\bar{y}+b_\phi(p'-p)\right]\exp\left[\frac{2(p'-p)}{p'+p+2C_p}\right]$	1	C_p
$t_3=\left[\bar{y}+b_\phi(p'-p)\right]\exp\left[\frac{2(p'-p)}{p'+p+2\rho}\right]$	1	ρ_{pb}
$t_4=\left[\bar{y}+b_\phi(p'-p)\right]\exp\left[\frac{2(\beta_2(\phi)(p'-p))}{\beta_2(\phi)(p'+p)+2C_p}\right]$	$\beta_2(\phi)$	C_p
$t_5=\left[\bar{y}+b_\phi(p'-p)\right]\exp\left[\frac{2(C_p(p'-p))}{C_p(p'+p)+2\beta_2(\phi)}\right]$	C_p	$\beta_2(\phi)$
$t_6=\left[\bar{y}+b_\phi(p'-p)\right]\exp\left[\frac{2(C_p(p'-p))}{C_p(p'+p)+2\rho}\right]$	C_p	ρ_{pb}
$t_7=\left[\bar{y}+b_\phi(p'-p)\right]\exp\left[\frac{2(\rho(p'-p))}{\rho(p'+p)+2C_p}\right]$	ρ_{pb}	C_p
$t_8=\left[\bar{y}+b_\phi(p'-p)\right]\exp\left[\frac{2(\beta_2(\phi)(p'-p))}{\beta_2(\phi)(p'+p)+2\rho}\right]$	$\beta_2(\phi)$	ρ_{pb}
$t_9=\left[\bar{y}+b_\phi(p'-p)\right]\exp\left[\frac{2(\rho(p'-p))}{\rho(p'+p)+2(\beta_2(\phi))}\right]$	ρ_{pb}	$\beta_2(\phi)$

Table 3.2. Efficiency Comparison of Proposed Ratio and Zaman and Kadilar [21] Estimators under Case I

S/N	Estimators compared	Condition obtained	Numerical outcome	Decisions
1.	$MSE(t_I)_I<MSE(ZK_I)_I$	$\rho>\frac{C_p\left(L_1+\bar{Y}\lambda_1\right)}{\left(2\bar{Y}C_y\right)}$	0.776 > 0.1271419	Satisfied

S/N	Estimators compared	Condition obtained	Numerical outcome	Decisions
2.	$MSE(t_I)_I < MSE(ZK_I)_I$	$\rho > \dfrac{C_p(L_2 + \bar{Y}\lambda_2)}{(2\bar{Y}C_y)}$	0.776 > 0.1600486	Satisfied
3.	$MSE(t_I)_I < MSE(ZK_I)_I$	$\rho > \dfrac{C_p(L_3 + \bar{Y}\lambda_3)}{(2\bar{Y}C_y)}$	0.776 > 0.4744658	Satisfied
4.	$MSE(t_I)_I < MSE(ZK_I)_I$	$\rho > \dfrac{C_p(L_4 + \bar{Y}\lambda_4)}{(2\bar{Y}C_y)}$	0.776 > 0.4740168	Satisfied
5	$MSE(t_I)_I < MSE(ZK_I)_I$	$\rho > \dfrac{C_p(L_5 + \bar{Y}\lambda_5)}{(2\bar{Y}C_y)}$	0.776 > 0.4031618	Satisfied
6	$MSE(t_I)_I < MSE(ZK_I)_I$	$\rho > \dfrac{C_p(L_6 + \bar{Y}\lambda_6)}{(2\bar{Y}C_y)}$	0.776 > 0.411415	Satisfied
7	$MSE(t_I)_I < MSE(ZK_I)_I$	$\rho > \dfrac{C_p(L_7 + \bar{Y}\lambda_7)}{(2\bar{Y}C_y)}$	0.776 > 0.1582934	Satisfied
8	$MSE(t_I)_I < MSE(ZK_I)_I$	$\rho > \dfrac{C_p(L_8 + \bar{Y}\lambda_8)}{(2\bar{Y}C_y)}$	0.776 > 1.367272	Not satisfied
9	$MSE(t_I)_I < MSE(ZK_I)_I$	$\rho > \dfrac{C_p(L_9 + \bar{Y}\lambda_9)}{(2\bar{Y}C_y)}$	0.776 > 0.1247811	Satisfied

(iii) Theoretical Comparison of Proposed Ratio Estimator and Estimator of Zaman and Kadilar [21] under study for case II

$$\bar{Y}^2\theta C_y^2 + (\theta+\theta')L_i^2C_p^2 - 2\bar{Y}L_i\theta\rho C_yC_p - \bar{Y}^2\left\{\theta C_y^2 + (\theta+\theta')\lambda_i^2C_p^2 - 2\lambda_i\rho C_yC_p\right\} < 0 \quad (3.13)$$

where L_i and λ_i i = 1,2,3,…..,9

Table 3.3. Efficiency Comparison of Proposed Ratio and Zaman and Kadilar [21] Estimators under Case II

S/N	Estimators compared	Condition obtained	Numerical outcome	Decisions
1.	$MSE(t_I)_{II} < MSE(ZK_I)_{II}$	$\rho > \dfrac{C_p(\theta+\theta')(L_1 + \bar{Y}\lambda_1)}{(2\bar{Y}\theta C_y)}$	0.797 > 0.09673137	Satisfied
2.	$MSE(t_I)_{II} < MSE(ZK_I)_{II}$	$\rho > \dfrac{C_p(\theta+\theta')(L_2 + \bar{Y}\lambda_2)}{(2\bar{Y}\theta C_y)}$	0.797 > 0.1342345	Satisfied
3.	$MSE(t_I)_{II} < MSE(ZK_I)_{II}$	$\rho > \dfrac{C_p(\theta+\theta')(L_3 + \bar{Y}\lambda_3)}{(2\bar{Y}\theta C_y)}$	0.797 > 0.4178153	Satisfied
4.	$MSE(t_I)_{II} < MSE(ZK_I)_{II}$	$\rho > \dfrac{C_p(\theta+\theta')(L_4 + \bar{Y}\lambda_4)}{(2\bar{Y}\theta C_y)}$	0.797 > 0.4629537	Satisfied

S/N	Estimators compared	Condition obtained	Numerical outcome	Decisions
5.	$MSE(t_I)_{II} < MSE(ZK_I)_{II}$	$\rho > \frac{C_p(\theta+\theta')(L_5+\bar{Y}\lambda_5)}{(2\bar{Y}\theta C_y)}$	0.797 > 0.2503294	Satisfied
6.	$MSE(t_I)_{II} < MSE(ZK_I)_{II}$	$\rho > \frac{C_p(\theta+\theta')(L_6+\bar{Y}\lambda_6)}{(2\bar{Y}\theta C_y)}$	0.797 >0.9389652	Not Satisfied
7.	$MSE(t_I)_{II} < MSE(ZK_I)_{II}$	$\rho > \frac{C_p(\theta+\theta')(L_7+\bar{Y}\lambda_7)}{(2\bar{Y}\theta C_y)}$	0.797 >0.1358819	Satisfied
8.	$MSE(t_I)_{II} < MSE(ZK_I)_{II}$	$\rho > \frac{C_p(\theta+\theta')(L_8+\bar{Y}\lambda_8)}{(2\bar{Y}\theta C_y)}$	0.797 >1.185871	Not Satisfied
9.	$MSE(t_I)_{II} < MSE(ZK_I)_{II}$	$\rho > \frac{C_p(\theta+\theta')(L_9+\bar{Y}\lambda_9)}{(2\bar{Y}\theta C_y)}$	0.797 > 0.09784492	Satisfied

Efficiency conditions for which the proposed estimator outperformed other related estimators considered in the study have established in (3.11), (3.12) and (3.13). These conditions have been tested numerically as shown in Tables 3.2 and 3.3 and results revealed that all the efficiency conditions were satisfied with exception of few cases. This implies that the class of suggested estimators is more efficient and can produce better estimates of population means than their counterparts.

4 Empirical Study

Real life data were used to determine the superiority of the proposed estimators over the already existence estimators. The real life data were obtained in the previous research worked on Zaman and Kadilar for empirical study.

Population 1: The data is defined as follows; y = the number of villages in the circles.

$$\phi_i = \begin{cases} 1, \text{if a circle of villages} > 5 \\ 0, \text{if a circle of villages} < 5 \end{cases}$$

Population of statistic given in Table 4.1

Population 2: The data is defined as follows; y = the number of teachers

$$\phi_i = \begin{cases} 1, \text{if number of teachers} > 60 \\ 0, \text{if number of teachers} < 60 \end{cases}$$

Population of statistic given in Table 4.2

Table 4.1. Descriptive statistic of population 1

N=89	$\bar{Y}$ =3.3596	λ_1=0.0171	λ_5=0.0433	λ_9=0.0132
N=20	n' =45	λ_2 =0.0221	λ_6 =0.1508	
$\beta_2(\phi)$=3.492	C_y =0.6008	λ_3=0.0695	λ_7 =0.0171	
ρ_{pb}=0.766	C_p =2.6779	λ_4 =0.0694	λ_8=0.1802	

Table 4.2. Descriptive statistic of population 2

N=111	$\bar{Y}$ =29.279	λ_1=0.0146	λ_5=0.0382	λ_9=0.0117
N=30	n' =55	λ_2 =0.0203	λ_6 =0.1441	
$\beta_2(\phi)$=3.898	C_y =0.872	λ_3=0.0640	λ_7 =0.0164	
ρ_{pb}=0.797	C_p =2.758	λ_4 =0.0709	λ_8=0.1819	

Table 4.3. MSE and PRE of population 1

Estimators	**MSE**		**PRE**	
	CASE I	**CASEII**	**CASE I**	**CASE II**
$\bar{y}$	0.15793	15.85573	100	100
Kumar and Bahl [22] t_{NGI}^{d}	1.633285	64.86284	9.669469811	24.44501351
Kumar and Bahl [22] t_{NGI}^{*d}	0.97848	98.37626	16.14034012	16.11743524
Singh et al. [23] t_{SI}	0.333523	15.65928	47.35205668	101.2545277
Kalita and Singh [24] t_{SI}^{*}	0.208471	21.54799	75.75634021	72.67165058
Zaman and Kadilar [21] λ_1	0.145373	15.1512	108.6377801	104.6499947
Proposed Estimator $(t_{1,1})$	**0.1411044**	**14.98685**	**111.9242**	**105.797616**
Zaman and Kadilar [21] λ_2	0.141972	14.88346	111.2402446	106.5325536
Proposed Estimator $(t_{1,2})$	**0.1349351**	**14.48042**	**117.0415**	**109.497722**
Zaman and Kadilar [21] λ_3	0.115089	13.07351	137.2242352	121.2813544
Proposed Estimator $(t_{1,3})$	**0.1122884**	**12.83639**	**140.6468**	**123.521722**
Zaman and Kadilar [21] λ_4	0.115115	12.82033	137.1932415	123.6764576
Proposed Estimator $(t_{1,4})$	**0.1123316**	**12.83998**	**140.5927**	**123.487186**
Zaman and Kadilar [21] λ_5	0.128679	14.09668	122.7317589	112.4784701
Proposed Estimator $(t_{1,5})$	**0.125153**	**13.8438**	**126.1895**	**114.533076**
Zaman and Kadilar [21] λ_6	0.09249	10.73221	170.753595	147.7396547
Proposed Estimator $(t_{1,6})$	**0.09201862**	**10.57384**	**171.6283**	**149.95243**
Zaman and Kadilar [21] λ_7	0.145384	15.06792	108.6295603	105.2283925
Proposed Estimator $(t_{1,7})$	**0.1411044**	**15.01925**	**111.9242**	**105.569386**

Estimators	**MSE**		**PRE**	
	CASE I	**CASEII**	**CASE I**	**CASE II**
Zaman and Kadilar [21] λ_8	0.091645	10.06635	172.3280048	157.5122065
Proposed Estimator $\left(t_{1,8}\right)$	**0.09200599**	**10.0537**	**171.6519**	**157.710395**
Zaman and Kadilar [21] λ_9	0.148116	15.28755	106.6258878	103.7166191
Proposed Estimator $\left(t_{1,9}\right)$	**0.1437427**	**15.2393**	**109.8699**	**104.045002**

Table 4.4. MSE and PRE estimators using population 2

Estimators	**MSE**		**PRE**	
	CASE I	**CASE II**	**CASE I**	**CASE II**
$\bar{y}$	0.15793	15.85573	100	100
Kumar and Bahl [22] t_{NGI}^{d}	3.106026	154.3461	5.08463226	10.27284
Kumar and Bahl [22] t_{NGI}^{*d}	1.87212	234.4669	8.435890862	6.76246
Singh et al. [23] t_{SI}	0.625279	30.49386	25.25752504	51.99647
Kalita and Singh [24] t_{SI}^{*}	0.370737	46.52718	42.59893132	34.07842
Zaman and Kadilar [21] λ_1	0.140667	14.737378	112.2722458	107.5885
Proposed Estimator $\left(t_{2,1}\right)$	**0.137821**	**14.83997**	**114.5899164**	**106.8448**
Zaman and Kadilar [21] λ_2	0.136094	14.319605	116.0447926	110.7274
Proposed Estimator $\left(t_{2,2}\right)$	**0.1305843**	**14.25215**	**120.941032**	**111.2515**
Zaman and Kadilar [21] λ_3	0.102435	11.634163	154.1758188	136.286
Proposed Estimator $\left(t_{2,3}\right)$	**0.0993408**	**11.30576**	**158.9779829**	**140.2447**
Zaman and Kadilar [21] λ_4	0.102464	11.283763	154.132183	140.5181
Proposed Estimator $\left(t_{2,4}\right)$	**0.09938767**	**11.31067**	**158.9030108**	**140.1838**
Zaman and Kadilar [21] λ_5	0.118777	13.119154	132.9634525	120.8594
Proposed Estimator $\left(t_{2,5}\right)$	**0.1143738**	**12.74315**	**138.082323**	**124.4255**
Zaman and Kadilar [21] λ_6	0.086839	8.8723655	181.8652909	178.7092
Proposed Estimator $\left(t_{2,6}\right)$	**0.08788584**	**8.754461**	**179.6990277**	**181.116**
Zaman and Kadilar [21] λ_7	0.140682	14.606969	112.2602749	108.5491
Proposed Estimator $\left(t_{2,7}\right)$	**0.1349351**	**14.53094**	**117.0414518**	**109.117**
Zaman and Kadilar [21] λ_8	0.09429	8.5422689	167.4939018	185.615
Proposed Estimator $\left(t_{2,8}\right)$	**0.09684899**	**8.54205**	**163.0682984**	**185.6197**
Zaman and Kadilar [21] λ_9	0.14439	14.951729	109.3773807	106.0461
Proposed Estimator $\left(t_{2,9}\right)$	**0.1384684**	**14.87575**	**114.0549035**	**106.5878**

Table 4.3 shows MSE and PRE of some exiting, Zaman and Kadilar [21] and proposed estimators $t_i,\ i=1,2,...,9$ for population I. The result shows that the proposed estimators have minimum MSE and higher PRE than Kumar and Bahl [22] estimators, Singh et al. [23] and Kalita and Singh [24] estimator. Proposed estimators $t_{11}, t_{12}, t_{13}, t_{14}, t_{15}, t_{16}, t_{17}$ and t_{19} also have minimum MSE and higher PRE than Zaman and Kadilar [21] estimators $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7$ and λ_9 while Zaman and Kadilar [21] estimators λ_8 has minimum MSE and higher PRE than proposed estimator t_{18}.

Table 4.4 shows MSE and PRE of some existing, Zaman and Kadilar [21] and proposed estimators $t_i,\ i=1,2,...,9$ for population II. The results show that the proposed estimators have minimum MSE and higher PRE than Kumar and Bahl [22] estimators, Singh et al. [23] and Kalita and Singh [24] estimator. Proposed estimators $t_{2,1}\ t_{22}, t_{23}, t_{24}, t_{25}, t_{27}$ and λ_{29} also have minimum MSE and higher PRE than Zaman and Kadilar [21] estimators $\lambda_1\ \ \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_7$ and λ_9 while Zaman and Kadilar [21] estimators λ_6, and λ_8 has MSE and higher PRE than proposed estimators t_{26} and t_{28}.

5 Conclusion

By comparing the results obtained from the empirical study on the efficiency of the proposed estimators over the existing related estimators considered in the study, it is observed that the proposed estimator have minimum MSE and higher PRE compared to other estimators considered in all the numerical computation carried out in the study, hence, the proposed estimators demonstrated high level of efficiency over other estimators considered. Conclusively, the proposed estimators have higher chance of producing estimate that is better than exiting estimators, considered in the study.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Siraj M, Javid S, Alamgir K. Estimation of finite population mean in simple random sampling and stratified random sampling using two auxiliary variables. Communications in Statistics - Theory and Methods. 2017;46:5:2181-2192.
DOI: 10.1080/03610926.2015.1035394

[2] Hafeez W, Shabbir J, Shah MT, Ahmed S. Some median type estimators to estimate the finite population mean. Asian Journal of Probability and Statistics. 2020;7(4):48-58.

[3] Hafeez, Javid Shabbir. Estimation of the finite population mean, using median based estimators in stratified random sampling. J. Stat. Appl. Pro. 2015;4(3):367-374.

[4] Ahmed S, Shabbir J, Hafeez W. Hansen and Hurwitz estimator with scrambled response on second call in stratified random sampling. Journal of Statistics Applications & Probability. 2016a;5(2).
DOI: http://dx.doi.org/10.18576/jsap/050217

[5] Ahmed A, Adewara AA, Singh RVK. Class of ratio estimators with known functions of auxiliary variable for estimating finite population variance. Asian Journal of Mathematics and Computer Research. 2016b; 12(1):63-70.

[6] Ahmed A, Singh RVK, Adewara AA. Ratio and product type exponential estimators of population variance under transformed sample information of study and supplementary variables. Asian Journal of Mathematics and Computer Research. 2016c;11(3):175-183.

[7] Audu A, Adewara AA. Modified factor-type estimators under two-phase sampling. Punjab Journal of Mathematics. 2017;49(2):59-73. ISSN: 1016-2526.

[8] Singh RVK, Audu A. Efficiency of ratio estimators in stratified random sampling using information on auxiliary attribute. International Journal of Engineering Science and Innovative Tecnology. 2013; 2(1):166-172.

[9] Muili JO, Agwamba EN, Erinola YA, Yunusa MA, Audu A, Hamzat MA. Modified ratio-cum-product estimators of finite population variance. International Journal of Advances in Engineering and Management. 2020;2(4):309-319.
DOI: 10.35629/5252- 0204309319

[10] Hafeez W, Shabbir J, Aziz N. Estimation of finite population mean by using median type estimators in two-stage sampling. Thailand Statistician, Article in Press; 2021.

[11] Audu A, Singh R, Khare S, Dauran NS. Almost unbiased estimators for population mean in the presence of non-response and measurement error. Journal of Statistics & Management Systems. 2021;24(3):573-589.
DOI : 10.1080/09720510.2020.1759209

[12] Audu A, Singh RVK. Exponential-type regression compromised imputation class of estimators. Journal of Statistics and Management Systems; 2021.
DOI: 10.1080/09720510.2020.1814501

[13] Bahl S, Tuteja RK. Ratio and product type estimator. Information and Optimization Science. 1991;XII: 159−163.

[14] Jhajj HS, Sharma MK, Grove LK. A family of estimators of population mean using information on auxiliary attribute. Pakistan Journal of Statistics. 2006;22(1):43−50.

[15] Singh R, Chauhan P, Sawan N. On linear combination of Ratio-product type exponential estimator for estimating finite population mean. Statistics in Transition. 2008;9(1):105−115.

[16] Shabbir J, Gupta S. Estimation of the finite population mean in two phase sampling when auxiliary variables are attributes. Hacettepe Journal of Mathematics and Statistics. 2010;39(1).

[17] Koyuncu N. Efficient estimators of population mean using auxiliary attributes. Applied Mathematics and Computation. 2012;218(22):10900−10905.

[18] Malik S, Singh R. An improved estimator using two auxiliary attributes. Applied Mathematics and Computation. 2013;219(23):10983−10986.

[19] Solanki RS, Singh HP. Improved estimation of population mean using population proportion of an auxiliary character. Chilean Journal of Statistics. 2013;4(1):3−17.

[20] Zaman T. New family of estimators using two auxiliary attributes. International Journal of Advanced Research I Engineering & Management (IJAREM). 2018;4(11):11−16.

[21] Zaman T, Kadilar C. New class of exponential estimators for finite population mean in two-phase sampling. Communications in Statistics - Theory and Methods; 2019.
DOI: 10.1080/03610926.2019.1643480

[22] Kumar M, Bahl S. Class of dual to ratio estimators for double sampling. Statistical Papers. 2006;47(2):319–326.
DOI:10.1007/s00362-005-0291-6

[23] Singh R, Chauhan P, Sawan N, Smarandache F. Ratio-product type exponential estimator for estimating finite population mean using information on auxiliary attribute. Auxiliary Information and a Priori Values in Construction of Improved Estimators. 2007;1:18.

[24] Kalita D, Singh BK. Exponential dual to ratio and dual to product-type estimators for finite population mean in double sampling. Elixir Statistics. 2013;59:15458–15470.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
https://www.sdiarticle4.com/review-history/74092