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Progress in Reducing the Uncertainty of Measurement of Planck's Constant in Terms of the Information Approach

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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ABSTRACT

Aims: The purpose of this work is to prove that only by implementing a weighted and thorough theoretical information approach to the development of a physical and mathematical model for measuring Planck's constant, it is possible to prepare a reasonable justification for calculating the required relative uncertainty.
Place and Duration of Study: Mechanical & Refrigeration Consultation Expert, between June 2018 and February 2019.
Methodology: Using the principles of information theory and similarity theory, a dimensionless parameter (comparative uncertainty) was formulated to compare the experimental results of measurements of Planck's constant and the simulated data.
Results: Examples of the application of the proposed original method to measure Planck's constant using the Kibble balance and X-ray crystal density methods are given.
Conclusion: The proposed information-oriented approach is theoretically justified and does not include such concepts as a statistically significant trend, cumulative values of consensus or

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statistical control, which are characteristic of the statistical expert tool adopted in CODATA. We tried to show how the mathematical and, apparently, rather arbitrary expert formalism can be replaced by a simple, theoretically grounded postulate on the use of information in measurements.

Keywords: Planck constant; comparative uncertainty; information-based approach; relative uncertainty.

1. INTRODUCTION

The General Conference on Weights and Measures [1] voted on draft resolution A, in which the definitions of units are expressed as fundamental constants. Fundamental constants are crucial to the laws of physics. The evolution of nature and the state of dimensional constants (*c* is the speed of light, *h* is Planck's constant, *e* is the elementary charge, and ***k*** is the Boltzmann constant) associated with the progress of physics. The modification of the international system of units (SI) was made possible by highly accurate experiments. The most difficult (and most expensive) part was the definition of Planck's constant. Planck's constant is of fundamental importance in quantum mechanics, and in physical dimensions, it is the basis for determining a kilogram. The measurements included the moving-coil watt balance experiment (Kibble balance) [2] and the so-called "x-ray crystal density (XRCD)" method [3], which use two macroscopic quantum effects (the Josephson and Hall effects), as well as the famous "coarse objects in the world": silicon forms of unprecedented purity, which are almost ideal spheres [4]. However, the numerical value of the Planck constant is fixed only when special requirements are met for the relative uncertainty of measurement [5]:

- At least three independent experiments, including the results of experiments with watt balance and XRCD experiments, provide consistent values of Planck's constant with relative standard uncertainties not exceeding 5 parts per 10^8.
- At least one of these results should have a relative standard uncertainty of no more than 2 parts in 10^8.

Uncertainty of fundamental constants is a very important topic. Each experiment to measure them contains uncertainty. The desire is to reduce the value of uncertainty in the measurement of fundamental physical constants for several reasons. First, achieving an accurate quantitative description of the physical universe depends on the numerical values of the constants appearing in the theories. Second, the general consistency and validity of the basic theories of physics can be confirmed by careful consideration of the numeric values of these constants, determined from various experiments in different areas of physics.

Most researchers focused on analysing the data and calculating the value of uncertainty of the fundamental physical constant after the formulation of the mathematical model and the construction of the test bench. But the inevitable uncertainty that existed prior to the start of an experiment or computer simulation and caused only by a finite number of quantities recorded in the mathematical model of the fundamental physical constant is typically ignored. Of course, in addition to this uncertainty, the general ambiguity of measurement of Planck's constant includes a posteriori uncertainty associated with the internal structure of the model and its subsequent computerisation and characteristics of the test equipment: inaccurate input data, inaccurate physical assumptions, limited accuracy of solving integral differential equations, etc.

In the CODATA method, to determine the recommended value of the relative uncertainty of the fundamental physical constant, a detailed discussion of the input data is conducted, as well as the justification and construction of tables of values sufficient to directly use the relative uncertainty with modern advanced statistical methods and powerful computers. This, in turn, allows you to check the self-consistency of the input data and the output set of values. However, at each stage of data processing, an expert conclusion based on intuition, accumulated knowledge, and accumulated life experience of scientists (personal philosophical convictions [6]) is also used.

In this case, one cannot exclude the possibility of the presence of a biased statistical expert motivated by personal convictions or preferences. It should be noted that the method of relative uncertainty for determining the accuracy of measurement does not indicate the direction to which the true value of the

fundamental physical constant can be found. In addition, it includes an element of subjective judgement [7].

In this paper, the focus is on applying an information approach to analysing experimental data to calculate relative uncertainty when measuring Planck's constant. This method is theoretically justified in comparison with the statistical-expert tool, CODATA, and has already found numerous applications in quantum mechanics [8], experimental physics [9], cosmology [10], and engineering [11,12]. To provide a factual background, a clearly defined problem, proposed solutions, a brief literature survey, and the scope and justification of the work done is presented.

2. PRELIMINARY REMARKS

Henceforth, the term "comparative uncertainty" $\boldsymbol{\varepsilon}$ is used, which is the ratio between the dimensional absolute uncertainty ΔU in determining the dimensional quantity U and the dimensional considered range of changes S^* of U proposed by Brillouin [13]:

$$\boldsymbol{\varepsilon} = \boldsymbol{\Delta U}/S^* \tag{1}$$

Absolute and relative uncertainties are familiar to physicists. As for the comparative uncertainty, it is rarely mentioned. Nevertheless, the comparative uncertainty is of great importance for the application of Information Theory in physics and engineering [13].

In the theory of measurements, it is assumed that for each dimensional measured value U, there is a dimensional "presumed uncertainty" ΔU. The full result can be represented as $U \pm \Delta U$. This means that the "true value" probably lies between the maximum value of $U + \Delta U$ and the minimum value of $U - \Delta U$. The term "relative uncertainty", r, is widely used in measurements of the Planck constant:

$$\mathbf{r} = \boldsymbol{\Delta U}/\boldsymbol{U} \tag{2}$$

The choice of relative uncertainty is explained by the fact that absolute uncertainty does not always give an idea of how important uncertainty is. In addition, relative uncertainty is useful for comparing the accuracy of various measurements. It also greatly facilitates the calculation of uncertainty dispersion. In addition to the above types of uncertainties, to weigh the approximate uncertainty, the usual value of Planck's constant is given by applying the usual (i.e., accepted) values of the Josephson constant, $K_{J\text{-}90}$, and the von Klitzing constant, $R_{K\text{-}90}$ [14,15]. Thus, the international standard for the value of the Planck constant was chosen to improve the uniformity of the comparison of subsequent measurements.

However, these methods for determining the accuracy of measurements do not indicate the direction in which the true value of the Planck constant is found. At the same time, the evaluation of uncertainty due to possible systematic uncertainties in physical dimensions necessarily includes an element of subjective judgement. A study of historical measurements and recommended values of fundamental physical constants shows that the stated uncertainty has a constant bias toward underestimating the actual uncertainty. These data are consistent with the results of constant self-confidence in psychological studies to assess subjective probability distribution. Awareness of these deviations can help in interpreting measurement accuracy, and will also serve as the basis for improving the assessment of measurement uncertainty [16].

Thus, it is considered here that the comparative uncertainties of the dimensionless researched quantity $\boldsymbol{u}$ and the dimensional researched quantity $\boldsymbol{U}$ are equal:

$$(\boldsymbol{\Delta U}/\boldsymbol{S}^*) = (\boldsymbol{\Delta U}/\mathbf{r}^*)/(\boldsymbol{S}^*/\mathbf{r}^*) = (\boldsymbol{\Delta u}/\boldsymbol{S}) \tag{3}$$

$$(r/R) = (\Delta U/U)/(\Delta U/u) = (\Delta U/U)\cdot(a/\Delta U)\cdot(U/a) = 1$$

where S and Δu are the dimensionless quantities (respectively, the range of variations and the total absolute uncertainty in determining the dimensionless quantity u); S^* and ΔU are the dimensional quantities (respectively, the range of variations and the total absolute uncertainty in determining the dimensional quantity U); a is the dimensional scale parameter with the same dimension as that of U and S^*; r is the relative uncertainty of the dimensional quantity U; and R is the relative uncertainty of the dimensionless quantity u.

Therefore, due to (3), it is possible to apply an information-based approach for measurements of the *dimensional* value of the Planck constant.

3. MAIN THESIS OF INFORMATION APPROACH

In Menin [17], a formula was proposed that links the comparative uncertainty, $\boldsymbol{\varepsilon}$, of the measured quantity, $\boldsymbol{u}$, and the design of the model describing the measurement process. Moreover,

the design concept of the model includes only the quantitative and qualitative availability of certain quantities. Taking into account (3), it can be argued that the formula below applies to the measurement of the Planck constant, which is a dimensional quantity. Namely, it was proved that:

$$\boldsymbol{\varepsilon} = \boldsymbol{\Delta u}/\boldsymbol{S} = (\boldsymbol{z}' - \boldsymbol{\beta}')/\boldsymbol{\mu}_{SI} + (\boldsymbol{z}'' - \boldsymbol{\beta}'')/(\boldsymbol{z}' - \boldsymbol{\beta}') \quad (4)$$

where μ_{SI} is the number of the dimensionless possible criteria in the international system of units (SI) with $\xi = 7$ as the base quantities: meter, the length L, kilogram, the mass M, and second, the time T, Kelvin, the thermodynamic temperature, θ, ampere, the electrical current I, mole, the amount of substance F, and candela, the luminous intensity J [18], μ_{SI} = 38,265 (μ_{SI} corresponds to the maximum amount of information contained in SI). In spite of the fact that the set of dimensionless criteria μ_{SI} does not exist in physical reality, the actually existing and observed object may be expressed by this set.

SI is a set of the dimensional quantities, base and, calculated on their basis, used for descriptions of different classes of phenomena (*CoP*), which is dependent on the chosen base quantities. In other words, the limits of the description of the studied material object are caused due to the choice of *CoP* and the number of derived quantities considered in the mathematical model [19]. For example, in mechanics, SI uses the basis {the length L, weight M, time T} (i.e. $CoP_{SI} \equiv LMT$). In this case, β' is the number of base quantities of the chosen *CoP*, z' is the total number of the dimensional quantities of the chosen *CoP*, z'' is a given number of the dimensional physical quantities recorded in the model, and β'' is the number of the base quantities recorded in the model. The dimension of any derived quantity q can only express a unique combination of the dimensions of the base quantities in different degrees [19]:

$$q \ni L^l \cdot M^m \cdot T^t \cdot I^i \cdot \theta^\theta \cdot J^j \cdot F^f, \quad (5)$$

where l, m... f are the exponents of the base quantities, and the range of each has a maximum and minimum value. According to [18], the exponents of the base quantities change in the following ranges:

$$3 \le l \le +3,\ -1 \le m \le +1,\ -4 \le t \le +4,\ -2 \le i \le +2 \quad (6)$$
$$-4 \le \Theta \le +4,\quad -1 \le j \le +1,\quad -1 \le f \le +1$$

The exponents of the base quantities take only integer values [18], thus, the number of choices of dimensions for each base quantity e_l, …, e_f, according to (6), is the following:

$$\boldsymbol{e}_l = 7;\ \boldsymbol{e}_m = 3;\ \boldsymbol{e}_t = 9;\ \boldsymbol{e}_i = 5;\ \boldsymbol{e}_\theta = 9;\ \boldsymbol{e}_j = 3;\ \boldsymbol{e}_f = 3 \quad (7)$$

It was proven that, according to the proposed hypothesis, the minimum achievable comparative uncertainty is not constant and varies depending on the class of phenomena choice. Moreover, theory can predict its value. In particular, this means that when switching from a mechanistic model (*LMT*) to CoP_{SI} with a larger number of the base quantities, this uncertainty grows. This change is due to the potential effects of the interaction between the increased number of quantities that can be taken into account or not by the researcher. Below, Table 1 introduces different CoP and the corresponding achievable comparative uncertainties and recommended number of quantities. As can be seen from Table 1, the number of quantities required to achieve the minimum comparative uncertainty for the selected class of phenomena is growing very rapidly [9]. Since, as a rule, researchers take into account a small number of values in the model, the experimentally achieved relative and comparative uncertainties, as we will see later, differ significantly from those theoretically calculated.

Table 1. Comparative uncertainties and recommended number of dimensionless criteria

CoP_{SI}	Comparative uncertainty	Number of criteria
LMT	0.0048	0.2 < 1
LMTF	0.0146	≦2
LMTI	0.0245	≦6
LMTθ	0.0442	≦19
LMTIF	0.0738	≦52
LMTθF	0.1331	≦169
LMTθI	0.2220	≦471
LMTθFI	0.6665	≦4,249

The information approach will be applied to analyse the experimental results of the measurement of the Planck constant made by two methods: The Kibble balance (KB) and XRCD.

4. ANALYSING MEASUREMENT RESULTS

4.1 KB

The idea of the watt balance was based on the traditional ampere balance experiment, which was developed for the absolute measurement of the basic SI unit, the ampere. The ampere balance was achieved by balancing the electrical force and gravity of the test mass [20]. However, for the balance ampere, it was very difficult to measure mass at the kilogram level, and the typical uncertainty achieved was about one part out of 10^5. In 1975, the watt balance experiment was divided into two separate modes: the weighing and the speed mode [21]. The watt balance weighing mode works similarly to an ampere weight with a much stronger magnetic field. Further, the watt balance experiment was widely applied with various modifications in many national metrological institutes for precise determination of the Planck constant *h* [22].

The measurement data are summarised in Table 2. The noted scientific articles belong to $CoP_{SI} \equiv LMTI$ [23-29]. The values of absolute and relative uncertainties differ by more than a factor of 10. A similar situation exists in the spread of the values of comparative uncertainties.

Following the method, *IARU*, one can discuss the order of the desired value of the relative uncertainty belonging to $CoP_{SI} \equiv LMTI$. An estimated observation interval of *h* is chosen as the difference in its values obtained from the experimental results of two projects: $h_{max} = 6.626070341 \cdot 10^{-34}$ $m^2 \cdot kg/s$ [25] and $h_{min} = 6.626069120 \cdot 10^{-34}$ $m^2 \cdot kg/s$ [28]. In this case, the possible observed range $\boldsymbol{S}_h$ of *h* placing is equal to:

$$\boldsymbol{S}_h = h_{max} - h_{min} = 1.22 \cdot 10^{-40}\ \mathrm{m^2 \cdot kg/s}. \quad (8)$$

To achieve the minimum comparative uncertainty for a particular *CoP,* there was proved [17]:

$$(\boldsymbol{z}' - \boldsymbol{\beta}')^2/(\boldsymbol{\Psi} - \boldsymbol{\xi}) = (\boldsymbol{z}'' - \boldsymbol{\beta}'') \quad (9)$$

For this purpose, considering (6) and (7), one can arrive at the lowest comparative uncertainty ε_{LMTI} using the following conditions:

$$(z'-\beta')_{LMTI} = (e_l \cdot e_m \cdot e_t \cdot e_i - 1)/2 - 4 = (7 \cdot 9 \cdot 5 - 1)/2 - 4 = 468, \quad (10)$$

$$(z''-\beta'')_{LMTI} = (z'-\beta')^2 / \mu_{SI} = 468^2 / 38,265 \approx 6, \quad (11)$$

where '–1' corresponds to the case where all the base quantity exponents are zero in formula (1), 4 corresponds to the four base quantities *L*, *M*, *T* and *I*, and division by 2 indicates that there are direct and inverse quantities (e.g., L^1 is the length and L^{-1} is the run length). The object can be judged based on the knowledge of only one of its symmetrical parts, while the other parts that structurally duplicate this one may be regarded as information empty.

Therefore, the number of options of dimensions may be reduced by a factor of two. According to (10) and (11):

$$\varepsilon_{LMTI} = (\Delta \boldsymbol{u}/\boldsymbol{S})_{LMTI} = 468/38,265 + 6/468 = 0.0146. \quad (12)$$

Taking into account (12), the lowest possible absolute uncertainty for KB ($CoP_{SI} \equiv LMTI$) is given by the following:

$$\Delta_{LMTI} = \varepsilon_{LMTI} \cdot \boldsymbol{S}_h = 0.0146 \cdot 1.22 \cdot 10^{-40} = 3.0 \cdot 10^{-42}\ \mathrm{m^2 \cdot kg/s}. \quad (13)$$

In this case, the lowest possible relative uncertainty $(r_{min})_{LMTI}$ for KB and the achieved mentioned results is the following:

$$r_{LMTI} = \Delta_{LMTI} / \left((h_{max} + h_{min})/2 \right) = 3.0 \cdot 10^{-42} / 6.626069731 \cdot 10^{-34} = 4.5 \cdot 10^{-9}. \quad (14)$$

This value is much smaller than $1.3 \cdot 10^{-8}$ cited in [29]. This situation confirms the main principle of the information approach, meaning that any experimental values of the relative uncertainty must be greater than the relative uncertainty corresponding to the KB method ($CoP_{SI} \equiv LMTI$), i.e., $4.5 \cdot 10^{-9}$.

Guided by the *IACU* and *IARU* methods, one can calculate the achieved comparative uncertainty in each experiment (Table 2). There is a large gap between the comparative uncertainty calculated according to the information-oriented approach $\varepsilon_{LMT\theta I} = 0.0245$ and the experimental magnitudes achieved during measuring *h* by KB. At the same time, progress to achieving a higher accuracy has been realised during the last four years.

Significant differences between the values of the comparative uncertainties achieved in the experiments and calculated in accordance with the *IACU* can be explained as follows. Within the framework of the information approach, the concept of comparative uncertainty assumes an equally probable account of various quantities, regardless of their specific choice by scientists,

when formulating a model for measuring *h*. Based on their experience, intuition, and knowledge, the researchers built a model containing a small number of quantities, and which, in their opinion, reflected the fundamental essence of the process under investigation. In this case, many secondary phenomena are characterised by specific quantities, are not considered here.

4.2 XRCD

XRCD is an indirect approach for accurately measuring Planck's constant. In this approach, the Avogadro constant N_A is first measured by counting Si atoms in a purified silicon sphere, then the Planck constant is determined based on the product $N_A \cdot h$, the value of which can be determined much more accurately than the goal of determining the Planck constant [31]. The measurement data are summarised in Table 3. The noted scientific articles belong to $CoP_{SI} \equiv LMTF$ [32-38]. The values of absolute and relative uncertainties differ by more than a factor of three. On the other hand, guided by the *IARU* method, one can calculate the achieved comparative uncertainty in each experiment (Table 3). A look at the distribution of the values of comparative uncertainties indicates relative consistency. But there is a large gap between the comparative uncertainty calculated according to the information-oriented approach $\varepsilon_{LMTF} = 0.0146$ and the experimental magnitudes achieved during measuring *h* by XRCD. Unfortunately, there has been no progress to achieve higher accuracy during the last eight years.

Following the method *IARU*, one can discuss the order of the desired value of the relative uncertainty belonging to $CoP_{SI} \equiv LMTF$. An estimated observation interval of *h* is chosen as the difference in its values obtained from the experimental results of two projects: $h_{max} = 6.626070406 \cdot 10^{-34}$ m² kg/s [36] and $h_{min} = 6.626069942 \cdot 10^{-34}$ m² kg/s [33]. In this case, the possible observed range S_h of *h* placing is equal to:

$$\boldsymbol{S}_h = h_{\max} - h_{\min} = 4.64 \cdot 10^{-41}\,\mathrm{m^2 \cdot kg / s}. \quad (15)$$

Taking into account (7) and (9), one can arrive at the lowest comparative uncertainty ε_{LMTF} using the following conditions:

$$\left(z' - \boldsymbol{\beta}'\right)_{LMTF} = \left(\boldsymbol{e}_l \cdot \boldsymbol{e}_m \cdot \boldsymbol{e}_t \cdot \boldsymbol{e}_f - 1\right)/2 - 4 = (7 \cdot 3 \cdot 9 \cdot 3 - 1)/2 - 4 = 279, \quad (16)$$

$$\left(z'' - \boldsymbol{\beta}''\right)_{LMTF} = \left(z' - \boldsymbol{\beta}'\right)^2 / \boldsymbol{\mu}_{SI} = 279^2 / 38,265 \approx 2, \quad (17)$$

where '–1' corresponds to the case where all the base quantity exponents are zero in formula (1). Four corresponds to the four base quantities *L*, *M*, *T* and *F* and a division by two indicates that there are direct and inverse quantities (e.g., L^1 is the length and L^{-1} is the run length). The object can be judged based on the knowledge of only one of its symmetrical parts, while the other parts that structurally duplicate this one may be regarded as information empty. Therefore, the number of options of dimensions may be reduced by a factor of two. According to (16) and (17):

$$\varepsilon_{LMTF} = \left(\boldsymbol{\Delta u} / \boldsymbol{S}_h\right)_{LMTF} = 279 / 38,265 + 2 / 279 = 0.0146. \quad (18)$$

Taking into account (18), the lowest possible absolute uncertainty for XRCD ($CoP_{SI} \equiv LMTF$) is given by the following:

$$\boldsymbol{\Delta}_{LMTF} = \boldsymbol{\varepsilon}_{LMTF} \cdot \boldsymbol{S}_h = 0.0146 \cdot 4.64 \cdot 10^{-29} = 6.8 \cdot 10^{-43}\,\mathrm{m^2 \cdot kg / s}. \quad (19)$$

In this case, the lowest possible relative uncertainty $(r_{min})_{LMTF}$ for XRCD is the following:

$$r_{LMTF} = \boldsymbol{\Delta}_{LMTF} / \left(\left(h_{\max} + h_{\min}\right)/2\right) = 6.8 \cdot 10^{-43} / 6.626070174 \cdot 10^{-34} = 1.0 \cdot 10^{-9}. \quad (20)$$

This value is much smaller than $9.1 \cdot 10^{-9}$ cited in [35]. This means that experimenters must continue to improve the measuring stands and identify possible sources of uncertainty. Progress in reducing the relative uncertainty to a value recommended according to the information approach suggests that a high potential of this method is possible when measuring the Planck constant.

5. DISCUSSION

Under the proposed approach, for each mathematical model of a physical law, there is an uncertainty, which initially, before the full-scale experimental studies or computer simulations, describes its proximity to the examined physical phenomenon or process. This value is called the comparative uncertainty. It depends only on the number of selected quantities and the observation interval of the selected primary quantity. One of the interesting features of the proposed hypothesis is that the minimum achievable comparative uncertainty is not constant and varies depending on the class of phenomena choice.

Table 2. Determinations of the Planck constant and achieved relative and comparative uncertainties using KB

Year	*CoP*	Planck's constant	Achieved relative uncertainty	Absolute uncertainty	*h* possible interval of placing*	Calculated comparative uncertainty	Calculated comparative uncertainty	Ref.
		$h \cdot 10^{34}$ m² kg/s	$r_h \cdot 10^{8}$	$\Delta_h \cdot 10^{42}$ m² kg/s	$u_h \cdot 10^{41}$ m² kg/s	$\varepsilon_h' = \Delta_k/u_h$ *IACU*	$\varepsilon_h'' = \Delta_h/S_h$ *IARU*	
2014		6.626069120	29.0	192.16	40.1	0.4792	1.5735	[23]
2014		6.626069793	4.5	29.817	6.1	0.4888	0.2442	[24]
2014		**6.626070341**	1.4	9.5415	2.4	0.3976	0.0781	[25]
2015	***LMTI***	6.626069364	5.7	37.769	7.7	0.4905	0.3093	[26]
2016		6.626069832	3.4	2.2529	4.4	0.5120	0.1845	[27]
2017		**6.626069216**	24.0	159.03	3.1	5.1299	1.3022	[28]
2017		6.626069935	1.3	8.6139	1.8	0.4786	0.0705	[29]

** Data are introduced in [22, 29, 30]*

Table 3. Determinations of the Planck constant and achieved relative and comparative uncertainties using XRCD

Year	*CoP*	Planck's constant	Achieved relative uncertainty	Absolute uncertainty	*h* possible interval of placing*	Calculated comparative uncertainty	Calculated comparative uncertainty	Ref.
		$h \cdot 10^{34}$ m² kg/s	$r_h \cdot 10^{8}$	$\Delta_h \cdot 10^{42}$ m² kg/s	$u_h \cdot 10^{41}$ m² kg/s	$\varepsilon_h' = \Delta_k/u_h$ *IACU*	$\varepsilon_h'' = \Delta_h/S_h$ *IARU*	
2011		6.626070082	3.0	1.9878	4.0	0.4970	0.4286	[32]
2011		**6.626069942**	3.0	1.9878	4.1	0.4848	0.4286	[33]
2015		6.626070221	2.0	1.3252	2.6	0.5097	0.2857	[34]
2017	***LMTF***	6.626070134	0.91	6.0297	1.2	0.5025	0.1300	[35]
2017		**6.626070406**	1.2	7.9513	1.6	0.4970	0.1714	[36]
2017		6.626070132	2.4	15.903	3.2	0.4970	0.3429	[37]
2018		6.626070151	1.0	6.6261	1.4	0.4733	0.1429	[38]

** Data are introduced in [22, 29, 30]*

Table 4. Summarised data

Variable	KB	XRCD
CoP	*LMTI*	*LMTF*
Comparative uncertainty according to CoP_{SI}	0.0245	0.0146
$S_k = k_{max} - k_{min}$, m² kg/(s² K)	$1.2 \cdot 10^{-40}$	$4.6 \cdot 10^{-41}$
Relative uncertainty according to CoP_{SI} (*IARU*)	$4.5 \cdot 10^{-9}$	$1.0 \cdot 10^{-9}$
Achieved experimental lowest relative uncertainty	$1.3 \cdot 10^{-8}$	$1.2 \cdot 10^{-8}$

Moreover, theory can predict value. In particular, this means that when switching from a mechanistic model (*LMT*) to CoP_{SI} with a larger number of the base quantities, this uncertainty grows. This change is due to the potential effects of the interaction between the increased number of quantities that can or cannot be taken into account by the researcher. That is why, within the framework of the information approach, in contradiction to the concept approved by CODATA, *it is not recommended to determine and declare only one value of relative uncertainty when measuring the Planck constant by different methods*.

In addition to the comments made in Section 3 regarding the analysis of the measurement results of the Planck constant based on two different methods using *IACU* and *IARU* (summarised in Table 4), the following should be noted:

As can be seen in Table 4, the recent results for Planck's constant do not agree at the level of relative uncertainty according to CoP_{SI} (*IARU*). A disagreement of this magnitude is unacceptable. This situation has occurred over the past years. These are signs that the main methods have not yet reached the required level of consistency and stability by 2018.

While a common set of comparative uncertainty data calculated in accordance with the *IACU* is consistent (each group of scientists' studies from another group to identify sources of uncertainty), the set of comparative uncertainties calculated in accordance with the *IARU* is inconsistent. The difference between these results is due to certain systematic errors. That is, why further and detailed research of the current watt balance and the Avogadro project should be continued. The greatest success in achieving high accuracy in measuring *h* was achieved using KB, given the significant difference in the magnitude of the comparative uncertainties between $CoP_{SI} \equiv$ ***LMTF*** (XRCD – 0.0146) and $CoP_{SI} \equiv$ ***LMTI*** (KB – 0.0245). However, experimenters involved in KB and XRCD will have to carefully check all sources of error. This is due to the requirement of the information method, according to which, the experimental relative uncertainty must always be close to the relative uncertainty, theoretically calculated.

At the moment, these two methods seem very attractive (in terms of their physical acceptability for measuring *h*) regarding the possibility of achieving higher accuracy. This is explained by the fact that the values of relative uncertainty calculated by $CoP_{SI} \equiv$ ***LMTI*** and $CoP_{SI} \equiv$ ***LMTF*** and achieved in the experiment, are very far from each other, respectively ($4.5 \cdot 10^{-9}$, $1.0 \cdot 10^{-9}$ and $1.3 \cdot 10^{-8}$, $1.2 \cdot 10^{-8}$).

6. CONCLUSION

So far, the experimental results have been inadequate. It is assumed that these discrepancies may be caused by unknown systematic uncertainties that should be reduced to a satisfactory level. Therefore, for existing methods, there should be additional investments in both improving test benches and improving measurement results, as well as applying a universal metric (comparative uncertainty) that allows one to check the true target value of the Planck constant with a given achievable relative uncertainty. We reviewed and discussed experimental data and uncertainty estimates, which are analysed from the perspective of the information approach. These conditions relate to the accuracy and mutual consistency of the results of qualification measurements.

Although it was argued [30] that the last agreed value obtained from the best available measurements for *h* using the KB or XRCD method is reliable and has an uncertainty not exceeding the uncertainty associated with current implementations of the primary and secondary mass units, it is necessary to continue to improve the methods of measuring constant Planck to reduce the impact of sources of uncertainty. In contrast to the statement that, "after Planck's constant is fixed (exact number with *zero* uncertainty…" [22]), according to the

information approach, first, the uncertainty cannot be “zero”, and second, the relative uncertainty of measurement of Planck’s constant always changes depending on the chosen class of the phenomenon inherent in the selected model. The problem, which appeared only recently and was related to the convergence of the experimental results of measurement of Planck’s constant to the theoretically justified value, remains highly relevant.

The proposed approach is theoretically justified and does not include such concepts as a statistically significant trend, cumulative consensus values, or statistical control, which are characteristic of the statistical expert tool adopted in CODATA. We sought to show how the mathematical and, apparently, rather arbitrary, expert formalism can be replaced by a simple, theoretically substantiated postulate on the use of information in measurements. Perhaps the information approach seems strange, however, it may give us more chances to understand it. This means that in the post-revision age, efforts will be required to maintain and improve relevant experiments. The application of the information approach is a prerequisite for the successful implementation of SI units with the lowest relative uncertainty.

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COMPETING INTERESTS

Author has declared that no competing interests exist.

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